NOTCH SIZE EFFECTS IN MULTIAXIAL FATIGUE

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ABSTRACT
This paper considers three of the enduring problems of fatigue life estimation: multiaxial loading, stress concentrations and size effects. The work forms part of an extensive investigation which I and my co-workers are conducting into the use of some prediction methods which we call the Theory of Critical Distances (TCD). Data from the literature show that there is a strong effect of hole size on the fatigue limit of specimens containing circular holes. Moreover, this effect is different under different types of loading: in particular, the critical hole size is greater in torsion than in tension. Accurate predictions of this data could be obtained by combining the TCD with a critical plane theory – the Susmel-Lazzarin criterion; to my knowledge this is the first time that this type of analysis has been reported.

1. INTRODUCTION

Fatigue failures in industrial components occur, almost invariably, at stress concentration features such as notches. Therefore, any methods for the prediction of fatigue life or fatigue limit must consider the effect of such features, i.e. the creation of local regions of high stress, associated with stress gradients. Despite many decades of research it is still true to say that we do not have any agreed methods for predicting the effect of notches: the only exception, i.e. the only type of notch for which agreed methods of prediction do exist, is a crack. Thanks to the development of a theoretical framework in the form of fracture mechanics, we can agree that the fatigue limit of a body containing a crack corresponds to the stress intensity threshold for the material, ΔK_{th}. Of course even in this case there remain some complications (such as the variation of ΔK_{th} with the stress ratio R) and some areas of invalidity (such as short cracks) but at least we have a general framework in which to operate.

However, no such agreed methodology exists for notches and other stress concentrators. This is not to say that there are not many possibilities, many approaches which have been proposed for the prediction of low cycle and high cycle fatigue life, but as yet no single approach has gained general acceptance. My own work has concentrated on high cycle fatigue, i.e. on problems which can be described as nominally elastic because any notch plasticity is small and contained. In recent years I have become interested in a series of approaches which can be grouped together under the general title "The Theory of Critical Distances" (TCD). These theories have been proposed, and reproposed, at various times throughout the past fifty years or so. They have been used not only for predicting fatigue but also for the analysis of monotonic fracture, especially in polymer composite materials (1). In recent years they have been largely forgotten as methods for fatigue limit prediction, as the research community has concentrated on theories based on the specific description of crack initiation and growth from notches. However, in recent work on the strength and fracture of materials it has been realised that materials possess inherent length scales which strongly affect their behaviour. In particular, the behaviour of notched specimens changes dramatically if the size of the notch, or of the specimen, becomes similar to the characteristic length scale of the material. There are different ways of formulating this characteristic length, and of using it – implicitly or explicitly – in predictions. The characteristic length which is used in the TCD is defined as follows:

$$L = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta\sigma_o}\right)^2 \tag{1}$$

Here ΔK_{th} and $\Delta\sigma_o$ are the fatigue crack propagation threshold for cracked specimens and the fatigue limit for plain (i.e. unnotched) specimens respectively, at the appropriate R ratio. Within the TCD, there are four methods by which these parameters can be used to predict the fatigue limit, as follows:

a) The Point Method (PM) – the fatigue limit occurs when the stress range at a point located a distance L/2 from the notch root (i.e. from the point of maximum stress) is equal to $\Delta\sigma_o$.
b) The Line Method (LM) – the fatigue limit occurs when the average stress range along a line from the notch root, of length 2L, is equal to $\Delta\sigma_o$.
c) The Imaginary Crack Method – it is imagined that there is a crack, of length L, at the root of the notch, whose behaviour conforms to LEFM: the fatigue limit is associated with the threshold ΔK_{th} for this crack.
d) The Finite Crack Extension Method, which we call Finite Fracture Mechanics (FFM) – the conditions necessary for crack growth are determined using a finite amount of crack extension, which is equal to 2L.

The first three methods were proposed many years ago: the PM by Peterson (2); the LM by Neuber (3) and the imaginary crack method by Lukas and Klesnil (4). The imaginary crack method was also proposed independently by ElHaddad et al (5) for the analysis of short cracks. The fourth method, in which an energy balance approach is used, calculating the strain-energy release over a finite amount of crack extension, is much more recent (6): we believe that it has the potential to make a link between the stress-based methods (PM and LM) and the actual mechanisms of fatigue crack growth from notches.

Using the same value of L, as defined by equation 1, these methods can be shown to give predictions which are similar, though not always identical. In what follows we will use the Line Method, though it is likely that any of the other methods could also be employed using the same general methodology.

The LM, and the other methods in the TCD, have been extensively tested against experimental data, (see, for example, (7) and (8)) but these investigations have concentrated on uniaxial tensile loading. In that case, the direction in which to draw the line suggests itself, by simple considerations of symmetry, as the notch bisector. For cases of stress concentrators which are not notches (e.g. corners, bends, keyways) we adopted a policy of using the direction perpendicular to the direction of maximum principal stress at the notch (which will also be the direction perpendicular to the surface at that point). Our justification was that this would be the direction of crack growth. Under conditions of multiaxial loading, which will be considered below, the use of this path may not be appropriate, nor will the use of the maximum principal stress as the characterising stress parameter. Multiaxial fatigue criteria can be divided into two types. The first type use parameters which can be defined as scalar quantities – the most common choices are the hydrostatic stress and some function of the deviatoric stress tensor; examples are the methods of Sines (9) and of Dang Van (10). The second type are the so-called 'critical plane' approaches, which avoid the complexities of the stress tensor by defining a single plane – assumed to be the plane on which early crack growth occurs – and using stress parameters defined with respect to that plane. Many critical-plane approaches have been suggested; the one which will be used here is the method suggested recently by Susmel and Lazzarin (11). The Susmel-Lazzarin (S-L) criterion can be written as follows:

$$\tau_a + \left(\tau_0 - \frac{\sigma_0}{2}\right)\frac{\sigma_{n,max}}{\tau_a} = \tau_0 \qquad (2)$$

Here τ_a is the amplitude of shear stress, τ_o and σ_o are the measured fatigue limits of the material in shear and tension respectively (expressed in terms of amplitudes) and $\sigma_{n,max}$ is the maximum value of the tensile stress in the cycle. The tensile and shear stresses are referred to a plane which is chosen as the plane of maximum shear stress amplitude. The reason for using the maximum tensile stress rather than its amplitude, is to allow for R-ratio effects, though in the present paper only R=-1 will be considered.

The S-L approach has been shown to be widely applicable for predicting the behaviour of plain specimens under multiaxial loading (11). Recently, we showed that it could be combined with the TCD (using the Point Method) in which form it was able to predict fatigue limits for large notches under various combinations of tension and shear, including pure torsion (12-14). These investigations considered large notches (i.e. notches of size much greater than L), with a range of root radii and stress concentration factors.

2. THE SIZE EFFECT FOR HOLES IN TENSION

One of the great advantages of the TCD, and of other methods using a characteristic length scale, is that notch size effects can be predicted very simply. In a qualitative sense, it is easy to see that, if the root radius of the notch is large compared to L, then the average stress over 2L will be almost equal to the maximum stress at the notch root, so the full effect of the elastic stress concentration factor, K_t, will be experienced and the fatigue limit of the notched specimen (denoted $\Delta\sigma_{on}$ and defined as the nominal applied stress) will simply be equal to $\Delta\sigma_o/K_t$. At the other extreme, if the size of the notch is much smaller than L, then the average stress over 2L will be similar to the nominal applied stress, i.e. the stress concentration effect of the notch will not be experienced and so the fatigue limit of the specimen will simply be $\Delta\sigma_o$. Between these two extremes one can expect notches for which $\Delta\sigma_{on}$ is less than $\Delta\sigma_o$, but by a factor less than K_t.

Fig.1 shows some experimental data from Murakami (15) which illustrates the effect of fatigue limit on notch size. Here the notches were small circular holes drilled into the surfaces of specimens whose dimensions were relatively large compared to the holes themselves. The hole depth was equal to its diameter. The material was a 0.46%-carbon steel, in two different heat treatments, annealed and quenched. Fatigue tests were carried out in rotating bending (R=-1).

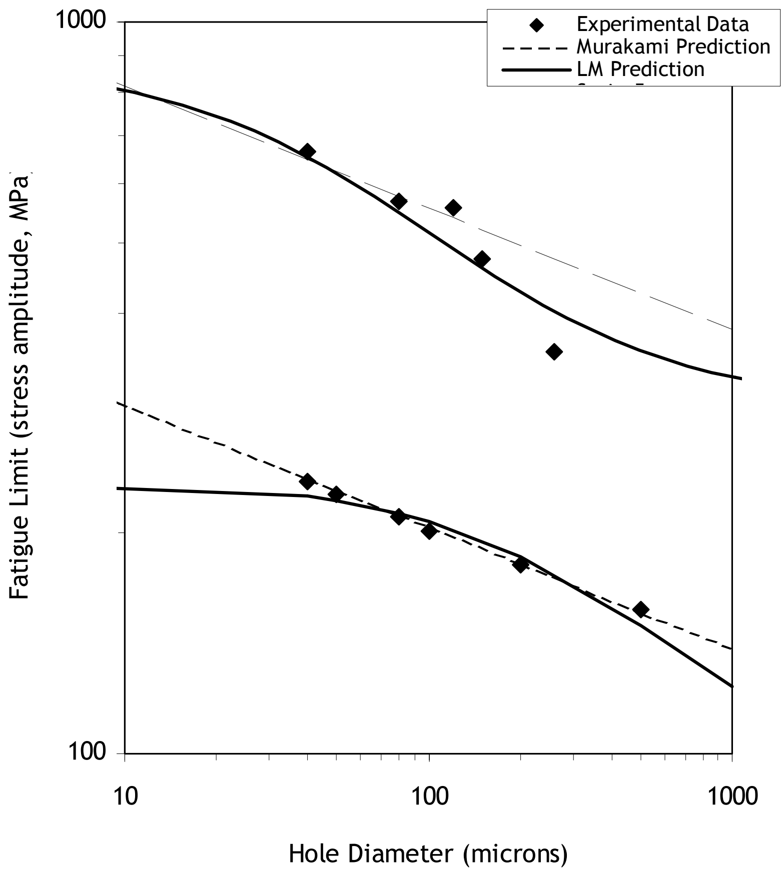

Figure 1: *Data from Murakami, showing the fatigue limit as a function of hole diameter in a 0.46%-carbon steel. The lower data points are for the steel in the annealed condition; the upper data points are for the quenched condition. Theoretical predictions using the LM(solid lines) and the method of Murakami (dashed lines).*

It is clear that the diameter of the hole has a strong effect on the fatigue limit. Predictions were carried out using the LM which, as the figure shows, was able to describe the data very well. The values of L were 0.15mm and 0.023mm for the annealed and quenched material respectively. Murakami has developed a method of prediction for small features such as these, in which the fatigue limit is expressed as a function of the square root of the projected area of the feature, and a constant C:

$$\Delta\sigma_o = C\left(\sqrt{\text{area}}\right)^{-1/6} \tag{3}$$

This is an empirical law which has been shown to work well for small defects and other features such as inclusions. As fig.1 shows, this law, which gives a straight line on the logarithmic plot, is very successful in describing the data for the annealed material, rather less successful for the quenched material. It is also obvious (as Murakami himself has stated) that this law is only applicable within a certain size range. The LM, however, is applicable for all notch sizes and will, in this case, successfully predict the levelling-off of the data to the plain fatigue limit $\Delta\sigma_o$ at very small hole sizes, and to $\Delta\sigma_o/3$ for large holes (assuming that the specimen size remains effectively infinite).

3. THE SIZE EFFECT IN TORSION

Fig.2 shows further data from Murakami, in which the same 046%-carbon steel, in the annealed condition, was tested in torsion, using fully-reversed (R=-1) loading. The previous bending data are also shown for comparison: the stress parameter used is the amplitude of maximum principal stress, which is equal in magnitude to τ_a for the torsion case. It is immediately obvious that the extent of the size effect is different in torsion: holes up to 100μm diameter have no effect on the fatigue limit, indeed it was reported that in many cases the fatigue failures initiated from elsewhere in the specimen. In contrast the 100μm holes tested in bending gave a fatigue limit (stress amplitude) of 201MPa, compared to 240MPa for plain specimens – a decrease by a factor of 1.2. Even 40μm holes – the smallest tested – had a slight effect in bending.

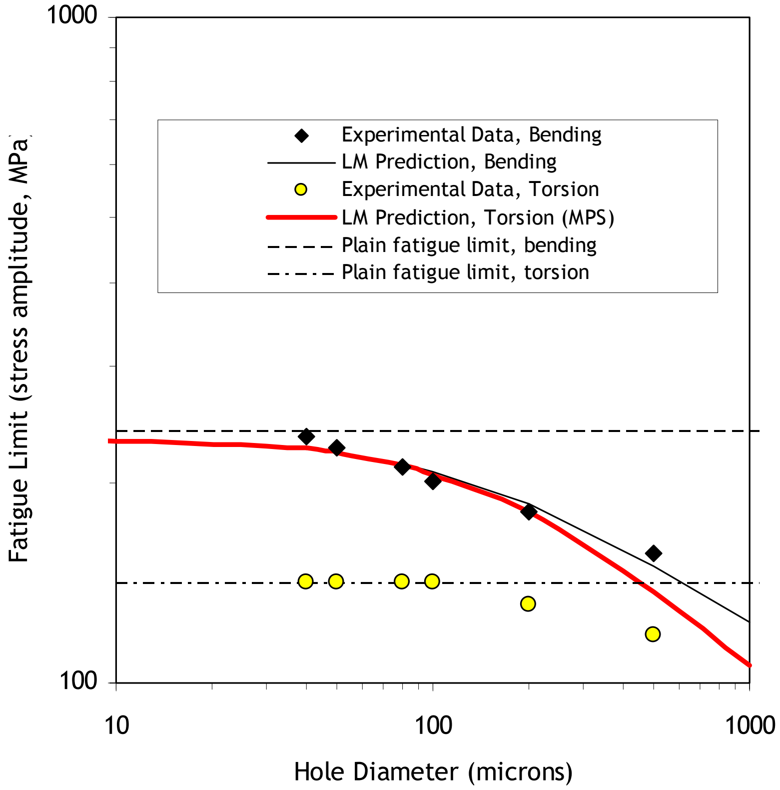

Figure 2: *Experimental data from Murakami for the same steel testing in bending and torsion. Predictions using the LM with the maximum principal stress.*

Fig.2 also shows predictions using the LM, taking the relevant stress parameter to be the maximum principal stress and using a line drawn perpendicular to the hole surface starting at the point of maximum stress. It is clear that this approach gives a very poor prediction, especially at small hole diameters where it naturally tends to the tensile fatigue limit rather than the torsion value. At large hole diameters it deviates from the prediction for bending, due to the larger K_t factor of the hole in torsion, which has a value of 4. It is likely that the prediction would be accurate for very large holes, because the stress field close to the hot spot is essentially one of pure tension.

Predictions were also made using the LM in conjunction with the Susmel-Lazzarin criterion. In this case a question arises as to the choice of the line over which to average the stresses. Fig.3 illustrates some possible choices; in this figure the axis of torsion is vertical. In the R=-1 loading there will be four equal hot-spots. Three possible lines starting at the hot spot are considered:

i) A line drawn perpendicular to the surface (which here is referred to as the 0^o direction); this is the direction that was used in the earlier prediction and it also corresponds to the direction on which cracks were seen to grow during the experiments.

ii) A line drawn so as to follow the maximum shear stress amplitude: this gives a curved line which is parallel to the hole surface at the hot spot, and turns through 45^o as it extends.

iii) A line drawn in the direction of the maximum shear stress at the hot spot: this line lies at 45^o to the free surface.

We also considered a line which does not start at a hot spot – this is denoted the Horizontal Path on fig.3.

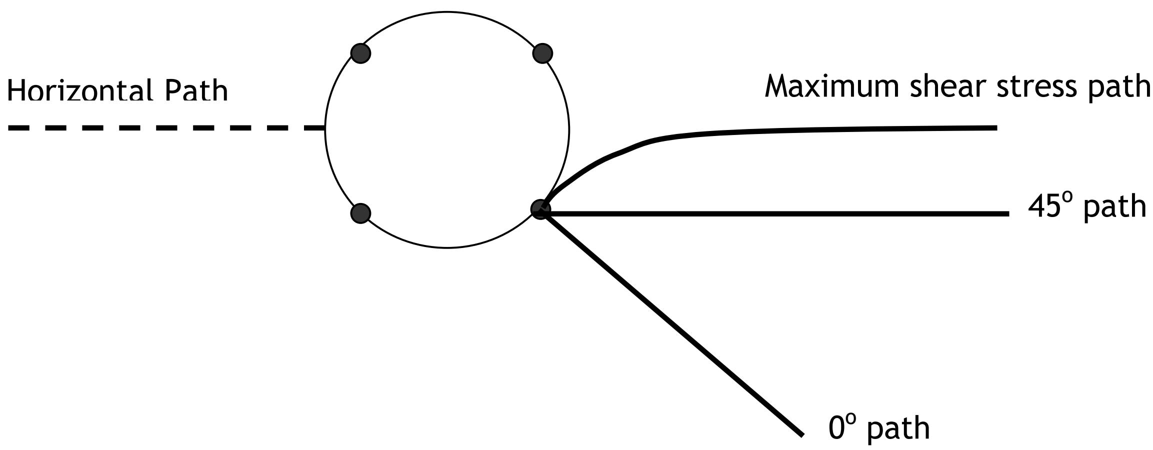

Figure 3: *Schematic showing four possible choices for the line over which to evaluate stresses for the LM prediction. The hole, loaded in torsion about a vertical axis, has four equally-spaced hot spots. Three possible paths are shown emerging from a hot spot: a fourth path runs horizontally through the centre line of the hole.*

Fig.4 shows the results of using lines (i)-(iii), taking averaged values of shear and tensile stresses on the planes represented by these lines, and using equation 2 as the fatigue limit criterion. Predictions were made both analytically (using standard Airy stress functions) and using finite element analysis – there were no significant differences between the two methods. All three lines gave rise to fairly good predictions - in fact the maximum prediction error was only 19%, which is very acceptable considering the errors inherent in the experimental data and stress analysis. Interestingly, the 0^{o} line predicts a slight increase in fatigue limit with increasing hole size for small holes. This result, which is rather counter-intuitive, arises because the gradient of tensile stress near the hot spot is very high, and the balance between tensile and shear stress changes as one moves away from the hole. The use of this line certainly demonstrates the large difference between tension and torsion, though it rather over-predicts the insensitivity of holes in torsion. The curved path representing the maximum shear stress tended to underestimate the fatigue limits, though this is perhaps a useful feature for industrial design. The best choice turned out to be the path described by a straight line, inclined at 45^{o} to the hole surface at the hot spot.

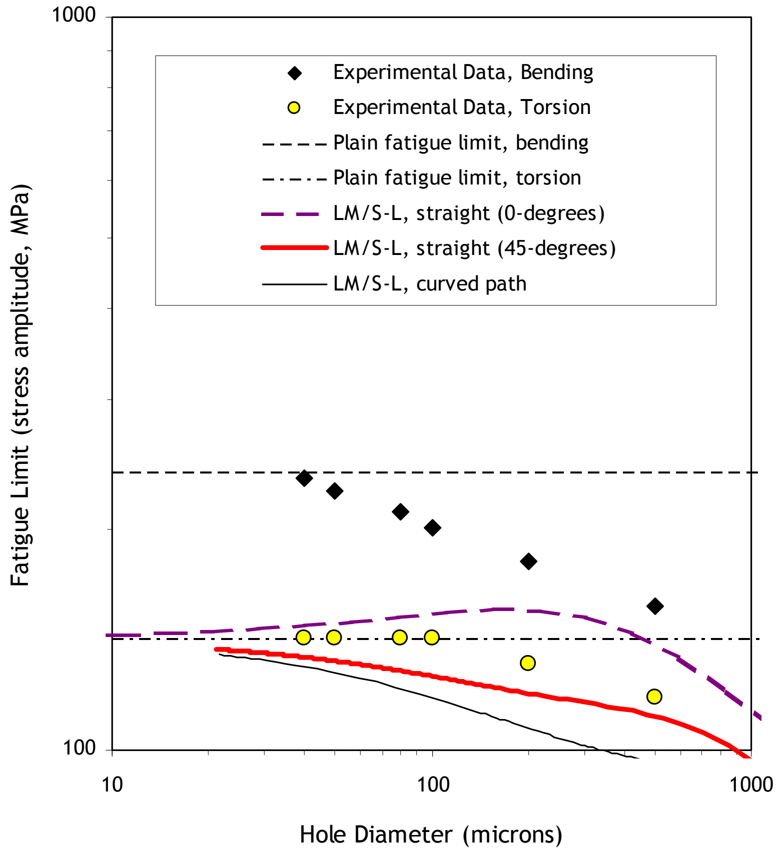

Figure 4: *Predictions using the LM with the S-L multiaxial criterion, using three different lines (see fig.3): two straight lines at different angles and one curved path representing the maximum shear stress.*

Of course any multiaxial fatigue approach must also be capable of working in the case of uniaxial loading. Fig.5 shows the results of applying the same approach (i.e. the LM combined with the S-L criterion) to the bending data, using the 45^{o} path as this proved to be the most accurate in predicting the torsion data. It can be seen that this approach also gives a successful, if slightly conservative, prediction of the bending data. The maximum error, in either the torsion or tension predictions using this strategy, was just 11%.

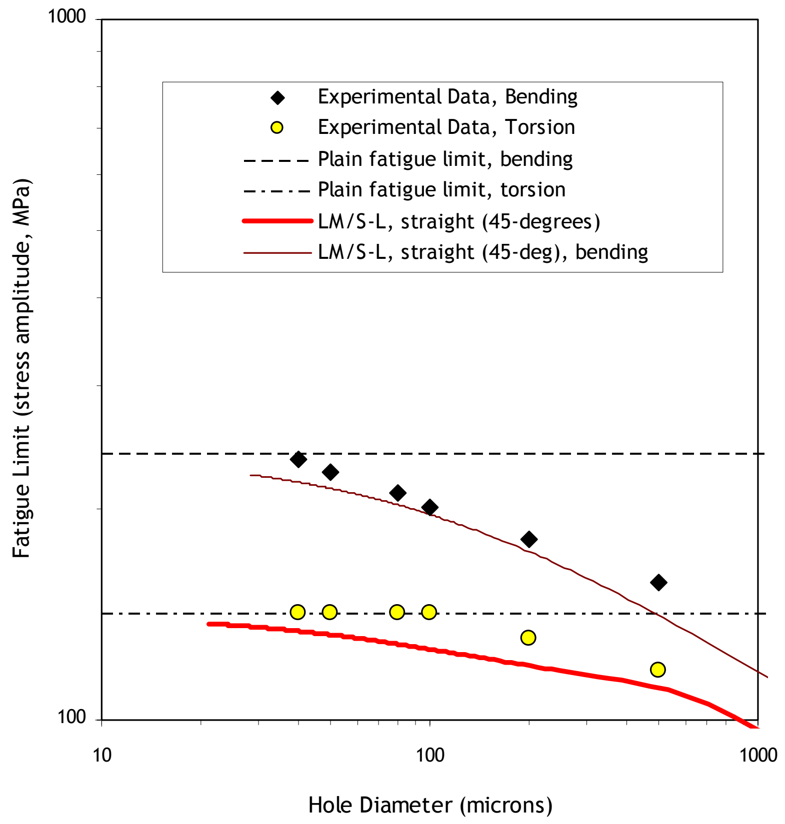

Figure 5: *Predictions for both torsion and bending data using the same approach: the LM combined with the S-L criterion on a path at 45^o to the hole surface at the hot spot.*

Predictions made for the torsion case using the horizontal path were rather unusual: they gave a lower fatigue limit for small holes, rising to much higher values (greater than the plain-specimen fatigue limit) for hole diameters greater than 1000μm. This behaviour occurs because, for this line, the stress at the hole surface is very small, and rises to a peak value at a distance of about one hole diameter (at which point it is about 15% larger than the nominal stress) and then decreases to the nominal stress at large distances. In principle, then, it would be possible for fatigue failures to occur from this region of peak stress, but in practice this did not happen – cracks initiated either at the hot spots or in places elsewhere on the specimen, remote from the hole. The reason for this is probably that the regions of peak stress occur only over rather small areas, so the probability of a crack initiating in these regions is low due to statistical size effects.

4. DISCUSSION

These results raise several interesting questions, the first of which is "Why is the size effect different in torsion and in tension?". The explanation can be found by comparing the stress fields created by these two types of loading, as shown in fig.6, which displays results obtained from FEA for the case of a hole loaded with a nominal stress of 100MPa in either tension or torsion. Consider a hole of diameter d=L=150µm: this is a critical value because holes of this size have almost no effect in torsion but quite a strong effect in tension. When making the LM/S-L prediction we use the shear and normal stresses averaged over a distance which, for this particular hole, will be r=2d.

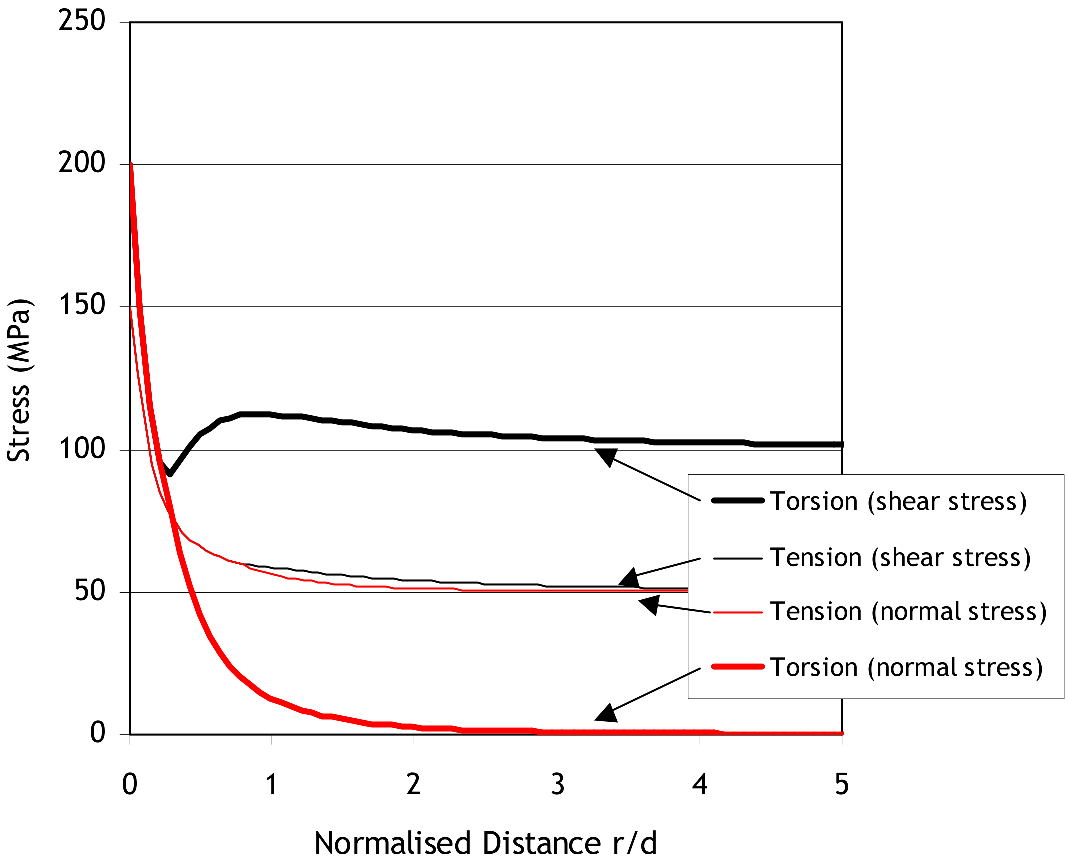

Figure 6: *Results from FEA showing shear and normal stresses on 45^o paths for a hole in tension or in torsion. The nominal applied stress was 100MPa: distance r from the hole surface is normalised by the hole diameter d.*

Over this distance the average value of the stresses in the tension case will be significantly larger than their nominal values of 50MPa, but in torsion the shear stress, whose value plays a dominating role in the S-L criterion (eqn.2), remains almost constant until much smaller distances, of the order of 0.2d. The rising value of normal stress near the hole does exert some effect but its role in the equation is relatively minor. The consequence of this is that the stress concentration effect of the hole in torsion will not really be felt until we are averaging over distances of less than 0.5d, i.e. when the hole diameter itself is of the order of 4L, which is 600µm.

The second question we might ask is "why does the 45^o path give the best predictions?". The usual justification for the choice of the critical plane is that this is the plane of initial crack growth, i.e. of growth during the Stage 1 phase, which being shear-dominated tends to occur on planes of maximum shear stress and continues for distances of the order of the grain size. Thus we might expect initial crack growth to occur at 45^o to the hole surface, and then to curve around to 0^o as the crack length increased and Stage 2 (tension-dominated) growth took over. However, no such 45^o growth is apparent in Murakami's photographs of these specimens; it seems that crack growth closely follows the 0^o path in both tension and torsion.

Another explanation, then, may be that the 45^o plane is critical not because it is the plane of crack growth but because it is the plane of dislocation motion, the active slip plane which will allow the mechanisms of crack initiation and growth to take place. It is well known that dislocation motion is strongly affected by the gradient of the local stress or strain: under high strain gradients dislocation motion is inhibited, and this leads to a number of interesting phenomena. For example, the measured hardness of a material increases with decreasing size of indentation. The explanation given for this phenomenon, which is known as Strain Gradient Plasticity (16) is that, in order to create a high gradient of plastic strain, extra dislocations are required (so-called 'geometrically necessary dislocations'); the existence of these extra dislocations increases the amount of work hardening, hence increasing the flow stress. The effect of this in the presence case would be that dislocation motion would be more difficult around a small hole than around a large hole for the same applied stress, so fatigue cracking would be consequently more difficult as well. Materials exhibit strong effects of Strain Gradient Plasticity at size scales of the order of 100μm – very similar to the critical sizes of the holes in the present investigation.

A final question that arises from the work is "Can the same approach be applied more generally, to all types of notches and multiaxial stress states?". Of course this is a question which can only answered by making the attempt, but we already have some indications that similar approaches can be successful. For example, Susmel and Taylor applied the Point Method, in conjunction with the S-L criterion, to a range of experimental data on large notches in tension/shear and tension/torsion, including some experiments on sharp notches inclined to a tensile stress field to produce Mode I/II mixtures. Accurate predictions (usually within 20%) were achieved (12,14). We also successfully analysed data on specimens with circumferential notches tested in torsion (13), though this work raised some questions about the appropriate stress parameter to use and also about whether the critical distance, L, will be the same in torsion as in tension. Further work is certainly needed, but all the indications are that the Theory of Critical Distances has a useful role to play in the analysis of multiaxial fatigue behaviour.

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