An elastoplastic model simulation to calculate local stress-strain sequences under uniaxial/multiaxial constant/variable amplitude cyclic loading

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ABSTRACT. The demand for advanced design method to accurately analyse fatigue damage of complex systems under multiaxial variable amplitude loading condition, in conjunction with the minimum safety factors due to more emphasis on lighter structures, saving materials and cost reduction has been extremely increasing. The correct prediction of fatigue lifetime under multiaxial cyclic loading seriously depends on the description of local elastoplastic stress-strain sequences at the critical point on a component [1]. Furthermore, it's universally agreed that in all engineering components local elastoplastic stress/strain will vary according to the geometrical features and degree of multiaxiality. Consequently, estimating these sequences will be more complicated by the presence of notch under multiaxial loads. Accordingly, calculating the local elastoplastic stress-strain properties with sufficient accuracy will result in a reliable and realistic fatigue assessment. However, procedural standards to plot the local elastoplastic stress-strain properties without performing experiment work are not yet available when the reversal stress/strains are involved. For the sake of accuracy, people come to believe that performing an experimental investigation is the best way to accurately estimate the local stress-strain response of components under multiaxial loading. In contrast, engineering designers argued that time and monetary requirements in the experimental investigation, in addition to the economic consideration or sometimes difficulty in accessing an accurate experimental testing machine parallel to the development of modern programing have increased attention on the use of numerical simulation model analysis. In the light of the above well-known fact, the present paper summarises an attempt to correctly formalise a novel model to be used to predict fatigue life time of unnotched components. The proposed elastoplastic model was analysed by using finite element (FE) program system ANSYS under reversed constant/variable amplitude uniaxial and torsion, as well as in-phase and out-of-phase multiaxial (tension-torsion) strain controlled loading at room temperature. From a reliability and safety point of view, a systematic validation exercise is followed by: First, using 38 experimental data sets from other technical literatures [6,7,8,9,10 &11] generated by testing 6 different materials under various loading conditions. Second, comparing the predicted local stress-strain properties with the result obtained from Jiang's model [2] that was built based on the material properties and constants obtained from the experimental test. To conclude, the local predicted hysteresis loops are compared with their experimentally determined counterparts and the results from Jiang's model. Such an extensive exercise showed that the proposed model has the capability of describing the local elastoplastic deformation features of materials under different cyclic loading. The overall scientific goal beyond this research is to formalise and verify a novel systematical study to simulate an accurate model, which can be used to estimate fatigue life time of notched metallic components under multiaxial variable amplitude loading case that will be outlined in the next steps of this research.

KEYWORDS. Elastoplastic model; Cyclic plasticity; Notched component; Variable amplitude.

INTRODUCTION

In general, engineering components are often exposed to a complex tension, torsion and combined constant/variable amplitude service load sequences resulting in a local multiaxial elastoplastic stress/strain state at the fatigue process zone. However, the stress-strain responses that are developed due to multiaxial loading conditions on notched components are much more complicated and exaggerated, particularly at the notch tip [3]. Different fatigue theories have been developed so far under different strategies, but a unique design methodology appropriate to investigate and estimate fatigue damage of notched components under variable amplitude multiaxial loading has not yet been agreed by the scientific community [4]. On the other hand, the engineeirng designers need sound methods mainly focus on multiaxial fatigue lifetime evaluation of stress raised components under variable amplitude loading conditions. Consequently, evaluation and assessment fatigue lifetime of such components have become an integral part of the design process and a complex problem that has to be addressed properly. It is universally agreed that in all engineering components, local elastoplastic stress/strain vary according to the geometrical features and degree of multiaxiality. Accordingly, fatigue problem is complicated in the presence of notches, and decreases structures lifetime. Strain based approach is recommended to perform low/medium fatigue assessment if the local elastoplastic sequences of material is correctly modeled and described [5]. This is attributed to the fact that the correct estimation of the local elastoplastic stress-strain relationship is important to accurately identify fatigue damage of a component. This paper aims at providing a systematical study to correctly model the elastoplastic stress-strain response of unnotched components under multiaxial variable amplitude cyclic loading. Due to extensive research investigation and well developed level of experimetnal fatigue assesment for unnotched components, the initail part of the research program as organised in this paper is focused on modeling and analysing unnotched component under a complex system of loading conditions, and the obtained results are compared with the experiementally performed work by others [6,7,8,9,10 & 11] as well as result from Jiang's model [2]. Further steps of the program will be focusing on the development of a reliable and theortical notched model based on the investigations from the initial part through deep insight into the fatigue mechanisms and behaviour of damged process zone under variable amplitude multiaxial loading.

Tensile Properties	304 Stainless Steel [6]	Stainless Steel [7]	SNCM630 Steel [8]	1% Cr-Mo-V [9]	45 Steel [10]	S45C [11]
E (GPa)	183	200	196	200	190	186
G (GPa)	82.8	77	77	76.9	79	70.6
v	0.3	0.3	0.273	0.3	0.202	0.28
σ_y (MPa)	325	365	951	707	370	496
σ_u (MPa)	650	----	1103	805	610	770
σ'_f (MPa)	1000	865	1272	987	843	923
b	-0.114	-0.097	-0.073	-0.071	-0.1047	-0.099
ε'_f	0.171	0.119	1.54	1.369	0.3269	0.359
c	-0.402	-0.359	-0.823	-0.802	-0.5458	-0.519
K' (MPa)	1660	1329	1056	1113	1258	1215
n'	0.287	0.244	0.054	0.11	0.208	0.217
Cyclic Torsional Properties:						
τ'_f (MPa)	709	500	858	570	559	685
b_o	-0.121	-0.097	-0.061	-0.071	-0.1078	-0.12
γ'_f	0.413	0.206	1.51	2.371	0.496	0.198
c_o	-0.353	-0.359	-0.706	-0.802	-0.469	-0.36
K'_o (MPa)	785	----	592	----	----	----
n'_o	0.296	----	0.05	----	----	----
Coefficient of Nonproportionality (Out-Of-Phase), $K'_{NP} = 1.25\ K'$ and $n'_{NP} = n'$, [5]						
K'_{NP} (MPa)	2075	1661	1320	1391	1573	1519
n'_{NP}	0.287	0.244	0.054	0.11	0.208	0.217

Table 1: Material properties from the other technical experimental work

FORMULATING THE MODEL

A plain cylindrical shaft is considered with material properties described from the previous technical literature. Almost all elastoplastic properties and constants of the materials were taken from the original papers [6,7,8,9,10 & 11]. All data sets are listed in Tab. 1. The missed torsional fatigue data (cyclic torsional properties of the materials) that was not given in the original papers were described by using von Mises criterion [12]. An elastoplastic unnotched model is analysed. To perform a comprehensive investigation, various loading cases, paths and amplitudes are applied as shown in Fig. 1. Multiaxial constant/variable amplitude loading are considered in the analysis as summarised in Tab. 2 and 3. The equivalent cyclic stress-strain amplitudes were fitted by using Ramberg-Osgood relationship [13]. Finite element ANSYS program is used to perform the analysis [14]. The result showed all considered materials are plastically responded.

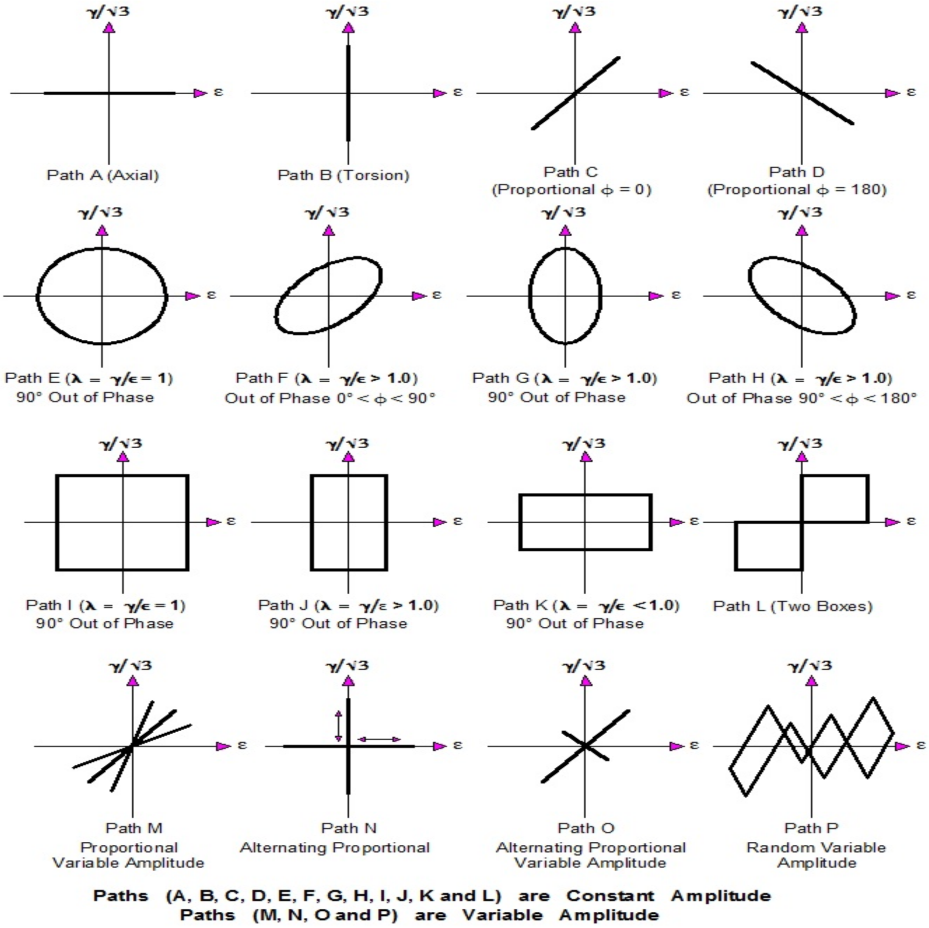

Figure 1: Schematics of strain paths (Constant and Variable Amplitudes).

RESULTS AND COMPARISON

In order to overview the accuracy and reliability of the proposed criterion, the formalised elastoplastic model has been analysed by using finite element program package ANSYS, the local elastoplastic stress-strains are estimated by post processing the developed model. A summary of predicted and experimentally reported of maximum axial stresses are illustrated in Fig. 2. Generally in the "identical" correlation of Fig. 2, all data points should lie along the 45 degree solid line, but in this paper the predicted stresses are within the factor-of-two scatter band figured by the dashed lines (Fig. 2). This correlation can be considered as a good agreement between the estimated and their experimentally reported counterparts.

References	**Materials**	**Strain Path**	**Phase angle ϕ (deg)**	**$\lambda = \gamma_a/\varepsilon_a$**	**$\varepsilon_a = \Delta\varepsilon/2$ (%)**	**$\gamma_a = \Delta\gamma/2$ (%)**	**$\sigma_a = \Delta\sigma/2$ (Mpa)**	**$\tau_a = \Delta\tau/2$ (Mpa)**
[6]	304 Stainless Steel	Path A	----	----	0.25	----	184	----
		Path B	----	----	----	0.476	----	109
		Path C	0	2.0	0.25	0.476	184	109
		Path L	----	1.7	0.25	0.43	365	199
[7]	347 Stainless Steel	Path E	90	1.7	0.577	1.0	----	----
[8]	SNCM630 Steel	Path G	90	1.5	0.6	0.9	----	----
		Path K	90	0.45	0.576	0.262	----	----
[6]	1% Cr-Mo-V	Path C	0	4.0	0.51	2.05	288	314
		Path D	180	1.5	0.99	1.54	528	230
		Path F	45	1.5	1.01	1.55	634	360
		Path G	90	1.5	1.02	1.54	669	366
		Path G	90	4.0	0.51	2.07	625	366
		Path H	135	1.5	1.01	1.52	674	349

Table 2: Type of materials, loading paths with Stress and Strain value (Constant Strain Amplitude).

Ref.	Materials	Strain Path	Strain History							
[10]	45 Steel	Path C	$\varepsilon a = \Delta\varepsilon/2$ (%)	0	± 0.2	± 0.4	± 0.6	± 0.8	± 1	0
			$\gamma a = \Delta\gamma/2$ (%)	0	±0.346	±0.693	±1.039	±1.386	±1.732	0
		Path E	$\varepsilon a = \Delta\varepsilon/2$ (%)	0	± 0.2	± 0.4	± 0.6	± 0.8	± 1	0
			$\gamma a = \Delta\gamma/2$ (%)	0	±0.346	±0.693	±1.039	±1.386	±1.732	0
		Path F	$\varepsilon a = \Delta\varepsilon/2$ (%)	0	± 0.2	± 0.4	± 0.6	± 0.8	± 1	0
			$\gamma a = \Delta\gamma/2$ (%)	0	±0.346	±0.693	±1.039	±1.386	±1.732	0
[11]	S45C	Path M	$\varepsilon a = \Delta\varepsilon/2$ (%)	-0.068	0.11	-0.15	0.15	-0.11	0.068	-0.068
			$\gamma a = \Delta\gamma/2$ (%)	-0.80	0.61	-0.41	0.41	-0.61	0.80	-0.80
		Path N	$\varepsilon a = \Delta\varepsilon/2$ (%)	0	0.41	-0.41	0	0	0	0
			$\gamma a = \Delta\gamma/2$ (%)	0	0	0	0	-1.26	1.26	0
		Path O	$\varepsilon a = \Delta\varepsilon/2$ (%)	0	0.68	-0.68	0	0.28	-0.24	0
			$\gamma a = \Delta\gamma/2$ (%)	0	1.48	-1.38	0	-0.57	0.63	0
		Path P	$\varepsilon a = \Delta\varepsilon/2$ (%)	0.0	0.10	0.30	0.43	0.32	0.06	-.12*
			$\gamma a = \Delta\gamma/2$ (%)	0.0	0.66	-0.66	0.27	1.0	-0.66	0.49*
			$\varepsilon a = \Delta\varepsilon/2$ (%)	-0.35	-0.43	-0.23	-0.05	0		
			$\gamma a = \Delta\gamma/2$ (%)	-1.0	-0.47	0.84	-0.33	0		

* Continued in the following εa or γa line

Table 3: Type of materials, loading paths with Stress and Strain value (Variable Strain Amplitudes)

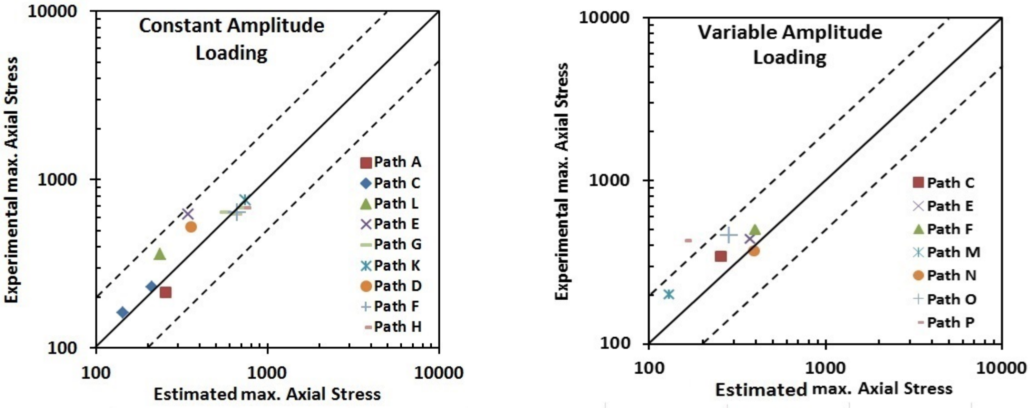

Figure 2: Error charts to compare the predicted local elastoplastic stresses with the experimentally reported.

According to the scenario described above, and based on the well-documented stress-strain paths of Fig. 3: first, all the local stress-strain sequences under different loading paths (constant amplitude) show the predicted curves agreed very well with the experimental investigation result and are located within a good manner. Second, for the applied variable amplitude multiaxial loading, the predicted local elastoplastic cyclic σ-ε and τ-γ plots are fully support and in very reasonable agreement with the experimental results as well as Jiang's model. To conclude, it can be pointed out that all results show that the formalized method is capable of predictions falling within accepted range and are proved to be most satisfactory.

However, a slight discrepancy can be observed between the predicted local stress-strain sequences and their experimental counterparts. These differences are mainly due to inconsistency in the microstructural features of materials [6] because Socie [6] proved that different materials show different sensitivities to additional hardening under multiaxial nonproportional loading and the amount depends on the microstructure and dislocation motion in the material during plastic deformation that can be determined from the laboratory test.

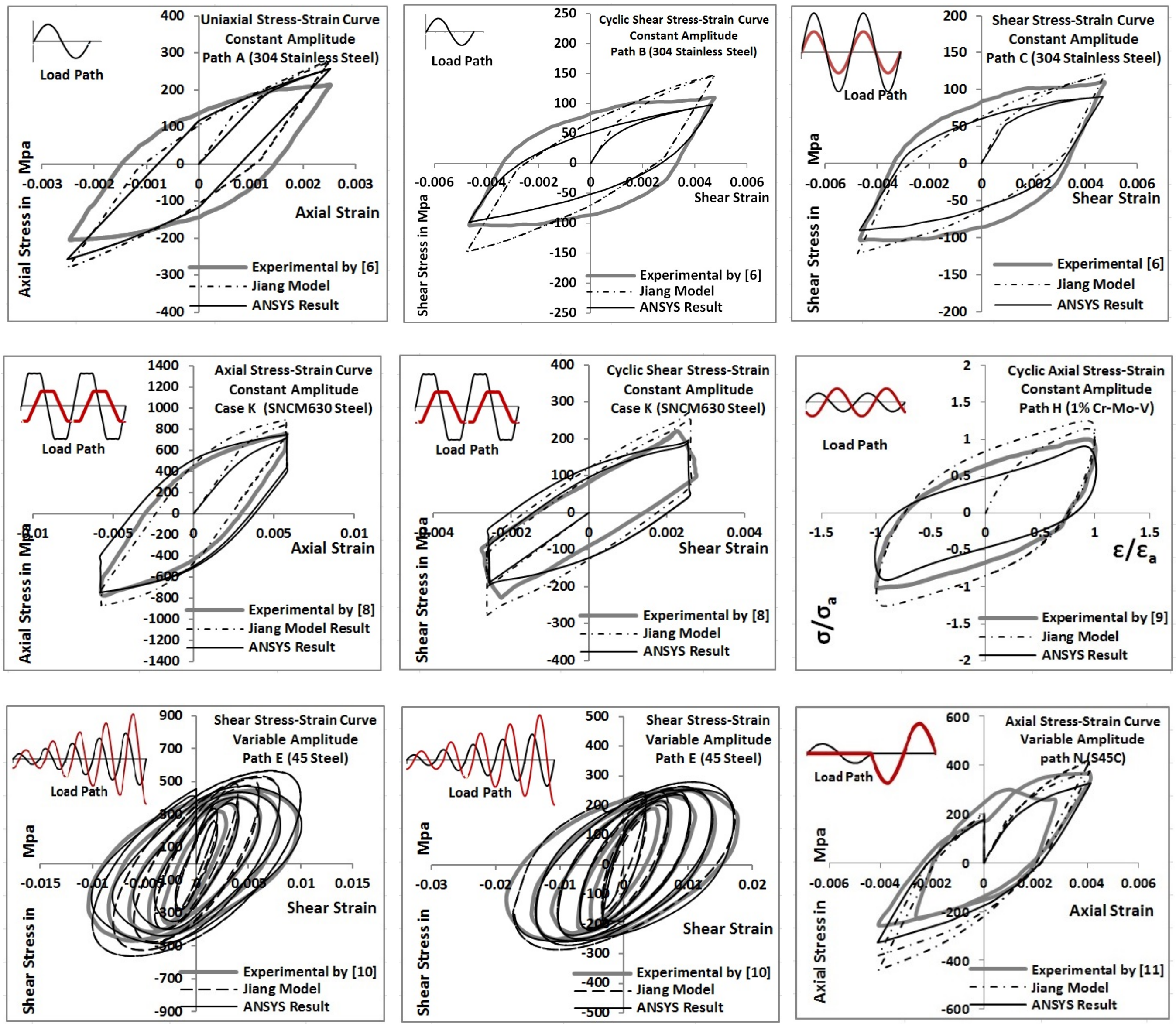

Figure 3: Compare the predicted Cyclic stress-strain relationship with the experimental reported and result of Jiang's Model

CONCLUSION

In this paper, various phenomena in relation to the elastoplastic stress-strain path of some materials under a complex system of cyclic loading has been modeled and studied particularly when the multiaxial displacement does not increase/decrease in a proportion (constant amplitude) but changes in a different manner (variable amplitude), the results are presented and discussed. The following conclusions can be drawn:

1. According to the comparison and exercise, it can be assumed that the predicted local stress-strain sequences are located within a closest range and a similar manner compared with their experimentally reported counterparts and Jiang's model. This is attributed to the fact that the formalised model is in a good agreement for all investigated loading paths and amplitudes.
2. A slight difference can be observed between the estimated curves and others under non-proportional loading case mainly probably due to non-proportional additional hardening. Socie [6] reported that materials have responded with additional hardening under nonproportional loading in contrast of proportional case, and the amount of additional hardening varies from none for some materials to more than double in some others based on the materials' microstructure [17].
3. The formalised model can be considered as a reliable relation with reasonable accuracy developed between monotonic tensile properties and uniaxial/multiaxial fatigue assessment of engineering materials. That serves to provide fast fatigue problem solution without involving time and cost in fatigue test. In contrast, Jiang's criteria require uniaxial and torsional fatigue data to determine material constants from experimental fatigue investigation.
4. The proposed elastoplastic models can be used to analyse and describe cyclic behavior of various metallic materials under different loading paths and amplitudes.
5. As briefly verified in the previous sections and in the light of accurate results and well agreements obtained from considering unnotched model, the subsequent target steps is extending into investigation of notched model and verify the accuracy and reliability of model with the data sets reported by technical literature [16].
6. Success in providing a model simulation to accurately estimate elastoplastic stress-strain property of materials, and by taking full advantage of the Modified Manson-Coffin Curves [5], fatigue lifetime of a component can be correctly estimated. That will be outlined in the next steps of this research program.

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