Non-uniform residual stress fields on sintered materials

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ABSTRACT. The hole-drilling method (HDM) is a convenient and effective method for measuring residual stresses near the surface of an isotropic linear-elastic material. The measurement procedure is relatively simple, and has been standardized in ASTM Standard Test Method E837. The HDM is also versatile and can be performed in laboratory or *in situ*, on components ranging widely in size and shape. It is often referred to as a "semidestructive" technique, since the small hole will not, in many cases, significantly impair the structural integrity of the part being tested.
The basic principle was first introduced by Mathar in 1934. Since that time, many researchers have further developed the method, culminating in the establishment of a standardized procedure ASTM E837. At least three established methods are available for estimating non-uniform residual stress fields from relaxed strain data from the incremental HDM. They are the Power Series, the Integral and the ASTM E837-08 Methods. These methods rely on finite element calculated calibration data, utilize different approaches for stabilizing and smoothing the non uniform residual stress field, and do not have the theoretical shortcomings of the two traditional methods (the Incremental Strain and the Average Strain Methods). Since the strain variations during drill depends on many factors hardly predictable and checkable *a priori*, it seems that the best calculation method should be evaluated for each particular experiment.
This paper compares the Power Series, Integral and ASTM E837-08 methods as procedures for determining non-uniform residual stress fields using strain relaxation data from the HDM. Specimens made of AISI Maraging 300 steel by means of Selective Laser Melting are investigated and the residual stress profiles performed by means of all three stress calculation procedures are also presented and compared.

INTRODUCTION

The hole drilling method consists in drilling a very small hole into the specimen; consequently, residual stresses relax in the hole and stresses in the surrounding region change causing strains also to change; a strain gage rosette, specifically designed and standardized [1], measures these strains. Residual stresses can be calculated as suggested in [1].
The original and most common objective of hole-drilling measurements is to evaluate in-plane residual stresses that can be assumed to be uniform either with depth from the surface of a thick specimen, or through the thickness of a thin specimen. ASTM Standard Procedure E837-01 refers to these cases. For a thick specimen (a specimen whose thickness is at least 1.2D, where D is the diameter of the gage circle), ASTM Standard Procedure includes a test for the uniformity of the residual stresses being measured. The methods specified in E837-01 can only be used if the residual stresses have been confirmed to be uniform. In many cases, residual stresses are not uniform with depth from the specimen surface. The most recent version of ASTM standard [1] contemplate a methodology for the very frequent case of non uniform stress field within the specimen thickness. Since the nature of the measured residual stresses is generally not known in advance, the choice of calculation method to be used is difficult to predict. Power series and Integral methods [2-5] have also been considered in computing the experimental data. A good strategy is to try all three methods: ASTM E837-08, Power Series

and Integral. Good engineering judgment, combined with a knowledge of the phenomena that generates residual stresses, should be used to choose the most appropriate calculation method. Similar judgment is also essential when interpreting the meaning and reliability of the obtained results.

METHODOLOGY

ASTM E837-08 gives details of methods for evaluating non uniform residual stresses from incremental hole-drilling strain data. This standard compute the following combination of strains for each set j of measured strains ε_1, ε_2, ε_3:

$$p_j = \frac{(\varepsilon_3 + \varepsilon_1)_j}{2} \qquad q_j = \frac{(\varepsilon_3 - \varepsilon_1)_j}{2} \qquad t_j = \frac{(\varepsilon_3 + \varepsilon_1 - 2\varepsilon_2)_j}{2} \qquad (1)$$

and estimate the error using the following formulas:

$$p_{STD}^2 = \sum_{j=1}^{n-3} \frac{(p_j - 3p_{j+1} + 3p_{j+2} - p_{j+3})^2}{20(n-3)}$$

$$q_{STD}^2 = \sum_{j=1}^{n-3} \frac{(q_j - 3q_{j+1} + 3q_{j+2} - q_{j+3})^2}{20(n-3)} \qquad (2)$$

$$t_{STD}^2 = \sum_{j=1}^{n-3} \frac{(t_j - 3pt_{j+1} + 3t_{j+2} - t_{j+3})^2}{20(n-3)}$$

where n is the total number of hole depth increments, while the subscript j is such that $1 \le j \le n-3$.
ASTM E837-08 tabulates the coefficients $\bar{a}_{jk}$ e $\bar{b}_{jk}$ for the three standardized rosette type.
Then, standard computes the three combination stresses $\vec{P}$, $\vec{Q}$ and $\vec{T}$ corresponding to the three combination stain $\vec{p}$, $\vec{q}$ and $\vec{t}$ using:

$$\bar{a} * \vec{P} = \frac{E}{1+\nu} * \vec{p} \qquad \bar{b} * \vec{Q} = E * \vec{q} \qquad \bar{b} * \vec{T} = E * \vec{t} \qquad (3)$$

The Cartesian stress components can be recovered from the calculated transformed stresses using:

$$(\sigma_x)_k = P_k - Q_k \qquad (\sigma_y)_k = P_k + Q_k \qquad (\tau_x)_k = T_k \qquad (4)$$

and the principal stresses

$$(\sigma_{max})_k, (\sigma_{min})_k = P_k \pm \sqrt{Q_k^2 + T_k^2} \qquad \beta_k = \frac{1}{2}\arctan\left(\frac{-T_k}{-Q_k}\right) \qquad (5)$$

where β_k is the angle measured clockwise from direction 1 of strain gage rosette to the maximum principal stress direction.
The hole-drilling calibration matrices $\bar{a}$ and $\bar{b}$ are ill-conditioned because of high number of drilling steps: small errors in experimental measurements can cause much larger errors in calculated residual stresses. This is a fundamental physical limitation of the hole-drilling method, not a mathematical artifact of any given stress calculation method. The various mathematical methods use different approaches to minimize the adverse effects of experimental errors. However, they all have to face the same trade-off between spatial resolution and stress uncertainty. If good spatial resolution of the variation of residual stresses with depth is required, there will be substantial noise and uncertainty in the individual stress values. On the other hand, if low sensitivity to experimental errors is required, spatial resolution has to be sacrificed. As with any other mathematical calculation, the quality of the calculated residual stresses depends directly on the quality of the input data. The extreme sensitivity to the effects of small experimental errors in strain measurements makes hole-drilling residual stress calculations particularly dependent on having high quality measured data. Thus, meticulous experimental technique is essential, with careful attention given to getting accurate measurements that are as free as possible from noise and extraneous errors.
The Tikhonov regularization scheme used by ASTM E837-08 is an effective method for stabilizing and smoothing the non uniform stress calculation method, even when a large number of hole depth increments are used. The smoothing

slightly reduces spatial resolution, but greatly enhances stability and resistance to small experimental errors. Using the regularization matrix c:

$$c = \begin{bmatrix} 0 & 0 & & & \\ -1 & 2 & -1 & & \\ & -1 & 2 & -1 & \\ & & -1 & 2 & -1 \\ & & & 0 & 0 \end{bmatrix}$$

and the Tikhonov regularization scheme, the equations (3) become:

$$\begin{aligned} (\bar{a}^T\bar{a} + \alpha_P * \bar{c}^T\bar{c}) * \vec{P} &= \frac{E}{1+\nu} * \bar{a}^T\vec{p} \\ (\bar{b}^T\bar{b} + \alpha_Q * \bar{c}^T\bar{c}) * \vec{Q} &= E * \bar{b}^T\vec{q} \\ (\bar{b}^T\bar{b} + \alpha_T * \bar{c}^T\bar{c}) * \vec{T} &= E * \bar{b}^T\vec{t} \end{aligned} \tag{6}$$

The parameters α_P, α_Q and α_T check the quantity of regularization done: initial value is between 10^{-4} and 10^{-6}.
The strains corresponding to these stresses are not the same as the measured strains corresponding to the unsmoothed (noisy) solution.
The difference between the strains corresponding to the stress solution and the actual measured strains is called the "misfit":

$$\vec{p}_{misfit} = \vec{p} - \frac{1+\nu}{E} * \bar{a}\vec{P} \qquad \vec{q}_{misfit} = \vec{q} - \frac{1}{E} * \bar{b}\vec{Q} \qquad \vec{t}_{misfit} = \vec{t} - \frac{1}{E} * \bar{b}\vec{T} \tag{7}$$

A modest misfit is acceptable providing that it is of similar size to the uncertainty (experimental error) in the strain measurements.
Estimate the error using the following formulas:

$$p_{rms}^2 = \frac{1}{n}\sum_{j=1}^{n}(p_{misfit})_j^2 \qquad q_{rms}^2 = \frac{1}{n}\sum_{j=1}^{n}(q_{misfit})_j^2 \qquad t_{rms}^2 = \frac{1}{n}\sum_{j=1}^{n}(t_{misfit})_j^2 \tag{8}$$

If p_{rms}^2, q_{rms}^2 and t_{rms}^2 are 5% different the estimated standard strain error p_{STD}^2, q_{STD}^2 and t_{STD}^2, it is necessary to recalculate the new parameters α_P, α_Q and α_T, and to iterate:

$$\begin{aligned} (\alpha_P)_{new} &= \frac{p_{STD}^2}{p_{rms}^2} * (\alpha_P)_{old} \\ (\alpha_q)_{new} &= \frac{q_{STD}^2}{q_{rms}^2} * (\alpha_q)_{old} \\ (\alpha_T)_{new} &= \frac{t_{STD}^2}{t_{rms}^2} * (\alpha_T)_{old} \end{aligned} \tag{9}$$

Otherwise the principal stresses can be recovered from the calculated transformed stresses using Eq. (5).
The Power Series Method was introduced by Schajer [3, 4] as an approximate, but theoretically acceptable method of calculating non-uniform stress fields from incremental strain data. The method assumes that the unknown stress distribution is expandable into a power series:

$$\sigma(h) = b_0 + b_1 h + b_2 h^2 + \cdots \tag{10}$$

Finite element calculations are used to compute series of coefficients $\bar{a}^0(h)$, $\bar{a}^1(h)$, $\bar{a}^2(h)$ and $\bar{b}^0(h)$, $\bar{b}^1(h)$, $\bar{b}^2(h)$, corresponding to the strain responses when hole drilling into stress fields with power series variations with depth h, i.e., $\sigma^0(h) = 1$, $\sigma^1(h) = h$, $\sigma^2(h) = h^2$, etc. These strain responses are then used as basis functions in a least-squares analysis of the measured strain relaxations. In this way, the measured strains are decomposed into components corresponding to the power series stress fields. The actual stress field is then reconstructed by summing the stress fields corresponding to the individual strain relaxation components. Only the function $\bar{a}^0(h)$, $\bar{b}^0(h)$, and $\bar{a}^1(h)$, $\bar{b}^1(h)$ (values for uniform and linear stress fields) are given because the hole drilling method is not well adapted to giving accurate values for more than the first two power series terms for stresses. For the same reason, the maximum depth below the surface is limited to 0.5 r_m.

The least-squares analysis is best done by applying the "normal equations" [3] to each of the transformed strains defined in Eq. (1). The transformed stresses P(h) are calculated from strains $p(h) = (\varepsilon_1(h) + \varepsilon_3(h))/2$ using

$$\begin{bmatrix} \sum \bar{a}^0(h)\bar{a}^0(h) & \sum \bar{a}^0(h)\bar{a}^1(h) \\ \sum \bar{a}^1(h)\bar{a}^0(h) & \sum \bar{a}^1(h)\bar{a}^1(h) \end{bmatrix} * \begin{bmatrix} {}^{\circ}P \\ {}'P \end{bmatrix} = \frac{E}{1+v} \begin{bmatrix} \sum \bar{a}^0(h)p(h) \\ \sum \bar{a}^1(h)p(h) \end{bmatrix} \tag{11}$$

where °P and 'P are the first two power series components of the "P" stress field, and Σ indicates the summation of the products of the values corresponding to all the hole depths, h, used for the strain measurements. This calculation is repeated for transformed stresses Q(h) and T(h) using strains q(h) and t(h) with coefficients $\bar{b}^0(h)$ and $\bar{b}^1(h)$ instead of $\bar{a}^0(h)$ and $\bar{a}^1(h)$, and omitting the factor 1 + v. The Cartesian stress field is then recovered using Eqs. (4).
An advantage of the Power Series Method is that the least squares procedure forms a best fit curve through the measured strain data. This averaging effect is particularly effective when strain measurements are made at many hole depth increments. A limitation of the method is that it is suitable only for smoothly varying stress fields.
The use of finite element calculations as a calibration procedure has also made application of the Integral method a practical possibility. Initial developments in this area were made by Bijak-Zochowski, Niku-Lari et al., Flaman and Manning [3]. In the Integral Method, the contributions to the total measured strain relaxations of the stresses at all depths are considered simultaneously.
It is convenient to work in terms of transformed stress and strain variable. Because the use of the transformed variables decouples the stress/strain equations, it is possible to consider each transformed stress or strain independently of the others. Consider, for example, the transformed strain relaxation p(h), measured after drilling a hole of depth h. This strain is the integral of the infinitesimal strain components caused by the transformed stresses P(H) at all depths within the range 0 ≤ H ≤ h

$$p(h) = \frac{1+v}{E}\int_0^h \hat{A}(H,h)P(H)dH \qquad 0 \le H \le h \tag{12}$$

where $\hat{A}$(H, h) is the strain relaxation per unit depth caused by a unit stress at depth H, when the hole depth is h. If $\hat{A}$(H, h) is known, say from finite element calculations, and p(h) is measured, then theoretically the unknown stress field P(H) can be determined by solving the integral Eq. (12). This solution is assumed to exist and to be unique. In practice, where the strain relaxations are measured after increasing the hole depth in n discrete increments to depths h_i = 1,2, , ,n, it is convenient to use a discrete form of Eq. (12**)**

$$\sum_{j=1}^{J=i} \bar{a}_{ij}P_j = \frac{E}{1+v}p_i \qquad 1 \le j \le i \le n \tag{13}$$

where P_i is measured strain relaxation after the *i*th hole depth increment; P_j is equivalent uniform stress within the *j*th hole depth increment; $\bar{a}_{ij}$ is strain relaxation due to a unit stress within increment j of a hole i increments deep; n is total number of hole depth increments.
The relationship between the coefficients $\bar{a}_{ij}$ and the strain relaxation function $\hat{A}(H,h)$ is

$$\bar{a}_{ij} = \int_{H_{j-1}}^{H_j} \hat{A}(H,h_i)\, dH \tag{14}$$

In matrix notation, Eqs. (13) and similar equations for the other two transformed stresses become Eqs. (3), where the matrix $\bar{b}$ contains the coefficients corresponding to the strain relaxation function $\hat{B}$(H, h) for hole drilling into a pure shear stress field. Matrices $\bar{a}$ and $\bar{b}$ are lower triangular. Therefore, a stepwise approximate solution for the stress variation with depth can be found by solving Eqs (3), using simple forward substitutions. The resulting stress values are the equivalent uniform stresses within each hole depth increment. The equivalent uniform stress does not equal the arithmetic average stress within each depth increment. Non-uniform strain response sensitivity biases the value in favor of the stresses close to the specimen surface [3].
The Cartesian stress components can be recovered from the calculated transformed stresses using equations (4) and the principal stresses using Eqs. (5).

COMPARISON OF METHODS

Fig. 1 shows a comparison of the experimental results of all three stress calculation methods from hole-drilling measurements into rectangular plate made of AISI Maraging 300 steel by means of Selective Laser Melting process [6,7].

Calculations for the Integral Method uses the matrices reported in [4]. The Power Series Method calculation uses the coefficients tabulated in [3]. The ASTM E837-08 Method uses the coefficients tabulated in [1].

The Integral Method gives a good stepped approximation to the actual stress variation with depth, and the Power Series Method gives a close straight-line fit. It can be observed the high variability of the ASTM results with respect to the others. Therefore, for smoothly varying stress fields, the Power Series Method with many small hole depth increments is probably the best choice. The least squares procedure used by the Power Series Method tends to average out the effects of random measurement errors. This increases the robustness of the calculation. The Integral Method is suitable when residual stress field changes abruptly, and when strain relaxations are measured after only a few hole depth increments. Meticulous experimental technique using a drilling procedure that does not introduce additional localized stresses is essential whatever stress calculation method is used. Small strain measurement errors can cause significant variations in calculated stresses, particularly for stresses far from the surface. In this case, the Integral method seems to be the most suitable for residual stresses calculations because it gives the most satisfactory balance between a realistic result and a stable solution.

The quality of the solution can be judged by observing that all calculation methods give stress solutions that do not exactly correspond to the given strain data. For example, the Power series method always calculates stresses that are linear with depth. If the actual stresses are not linear, then the strains corresponding to the assumed linear stresses are not the same as the actual (measured) strains. Similarly, the Integral Method with regularization gives smoothly varying stresses. The strains corresponding to these stresses are not the same as the measured strains corresponding to the unsmoothed (noisy) solution.

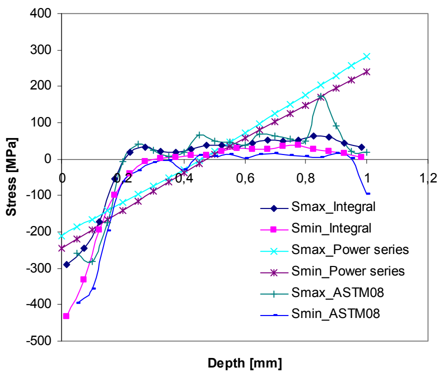

Figure 1: Residual stresses values (specimen 1 – location 1).

CONCLUSIONS

This paper studied three calculation procedures to determine non-uniform residual stress fields from incremental strain relaxation data from the hole drilling method: ASTM E837-08, Power Series Method and Integral Method. The Power Series Method is suitable for use with smoothly varying stress fields. It is relatively robust numerically because the least-squares procedure used tends to smooth out the effects of random errors in the experimental strain data. The Integral Method is suitable for calculations with irregular stress fields.

Since the nature of the residual stresses being measured is generally not known in advance, the choice of calculation method to be used is difficult to predict. If the residual stress field has been found to be non-uniform as per ASTM, a good strategy is to try all three non-uniform stress calculation methods. Good engineering judgment, combined with a knowledge of the stresses expected, should be used to choose the most appropriate stress calculation method.

Released strains into rectangular plate made of AISI Maraging 300 steel by means of Selective Laser Melting have been measured and residual stresses values have been computed according to ASTM method (case of non uniform stress field), *Power series method* and *Integral method*. In the present experiment, the Power series method seems to be the most suitable for residual stresses calculations.

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