On the static assessment of notched ductile materials subjected to multiaxial loading

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ABSTRACT. The present paper initially summarises an experimental and theoretical study of the fracture of circumferentially notched samples of a commercial aluminium alloy, i.e., Al 6082, subjected to tension, torsion and mixed tension/torsion loading. Subsequently, attention is focused on the use of the so-called Theory of Critical Distances (TCD) to predict static breakage under complex loading when ductile materials contain geometrical features. Our method proved to be capable of estimates characterised by the usual level of accuracy shown by the TCD when used in other ambits of the structural integrity discipline, that is, of predictions falling within an error interval of about 20%. This result is very encouraging especially in light of the fact that our simple linear-elastic approach was used to estimate static strength even when the material in the vicinity of crack initiation sites experienced multiaxial plastic deformations and the observed cracking behaviour was rather complex.

KEYWORDS. Theory of Critical Distances; Static failure; Notch; Multiaxial loading; Aluminium.

INTRODUCTION

The Theory of Critical Distances (TCD) is an approach which has been in use for over 50 years but which has recently gained in popularity and acceptance. This has been partly due to recent work which has placed the method on a firm scientific foundation, and partly due to the increased availability of numerical methods for stress analysis, especially Finite Element (FE) analysis. Because this method can very easily be used in conjunction with linear elastic FE analysis, it can be integrated into the industrial design process. For a complete description of this method and its applications the reader is referred to a recent book on the subject [1].

The TCD is applicable to all failure modes which involve cracking, such as fatigue, ductile and brittle fracture. When a crack develops and grows from a notch (or any other stress-concentration feature), there exists a region within which the crucial failure processes take place. The fundamental assumption of the TCD is that the size of this region, given by the length dimension L, is a constant for a particular material and particular failure mechanism. In particular, it is assumed that L is independent of the geometry of the notch. The constancy of L leads directly to scaling effects, such as the well known effect of notch size at constant notch shape (including short crack effects) and can be used to explain the effect of stress gradient.

There are a number of different ways in which the critical distance L can be used to make predictions. These include modified fracture mechanics approaches such as the introduction of a pre-existing crack at the notch root or the assumption that crack growth proceeds in finite increments [2]. However, the approaches which have proved most useful in industrial practice involve equivalent stress parameters from the region close to the notch. This idea was first applied to fatigue in metallic materials by Neuber [3], who suggested taking the average value of the elastic stress on a line drawn from the notch root, along the notch bisector. The length of this line is equivalent to 2L in our terminology. Peterson [4] then proposed an even simpler method, using the elastic stress at a distance of L/2 from the notch, measured along the same line: we call this line the "focus path". It should be emphasised here that the elastic stress is the correct value to use, despite the fact that the actual stress in this region will be modified by plasticity and other effects. Recent work has demonstrated that these approaches are capable of high accuracy and reliability in predicting static failures in engineering notched materials [5-7]. According to the promising results obtained so far, aim of the present paper is to investigate whether the linear-elastic TCD is successful also in predicting static strength of notched components subjected to multiaxial loading when materials undergo plastic deformation before final breakage takes place.

EXPERIMENTAL DETAILS

The material investigated in the present study was commercial aluminium alloy Al6082 and it was supplied in bars having diameter, d_g, equal to 10 mm. The plain samples used to determine the static properties of such an aluminium alloy were machined to obtain a gauge length of 25 mm with a diameter equal to 6 mm. The material was found to have an ultimate tensile stress, σ_{UTS}, equal to 367 MPa, a yield stress, σ_Y, equal to 347 MPa and a Young's modulus, E, equal to 69090 MPa.

The tested V-notched samples had gross diameter, d_g, equal to 10 mm and net diameter, d_n, ranging between 6.1 mm and 6.2 mm. The notch opening angle was equal to 60° and four different values of the notch root radius, r_n, were investigated, i.e.: 0.44 mm (tensile stress concentration factor K_t=2.94; torsional stress concentration factor K_{tt}=1.71); 0.50 mm (K_t=2.76; K_{tt}=1.64); 1.25 mm (K_t=1.92; K_{tt}=1.32) and; 4.00 mm (K_t=1.33; K_{tt}=1.12). Static tests were run by using a conventional tensile/torsional testing machine, investigating the following five different values of the ratio between nominal tensile and nominal torsional net stress (σ_{nom}/τ_{nom}): 0 (torsion), 0.23, 0.55, 1 and ∞ (tension).

For tests conducted under pure tensile and pure torsional loading, the failure force, F_u, and failure torque, M_u, were defined as the maximum value recorded during each test. On the contrary, much more tricky was the determination of the ultimate tensile loading and the ultimate torque under mixed tension/torsion loading. As an example, Fig. 1 shows the loading vs. extension and the torque vs. twist angle curves generated by testing, under a σ_{nom} to τ_{nom} ratio equal to 0.23, a notched sample having notch root radius equal to 4 mm. In the above chart the ultimate tensile force, F_u, and the ultimate torque, M_u, are not aligned along a unique vertical line. This is due to the fact that, to keep the ratio between the two nominal stress components constant during the test, the axial strain rate had to be different from the corresponding torsional strain rate. The direct analysis of all the gathered channels showed that, under combined loading, the maximum point was always reached sooner in the torque vs. twist angle curve than in the corresponding load vs. extension curve; this was true independently of both sharpness of the notch and ratio between the applied nominal loadings.

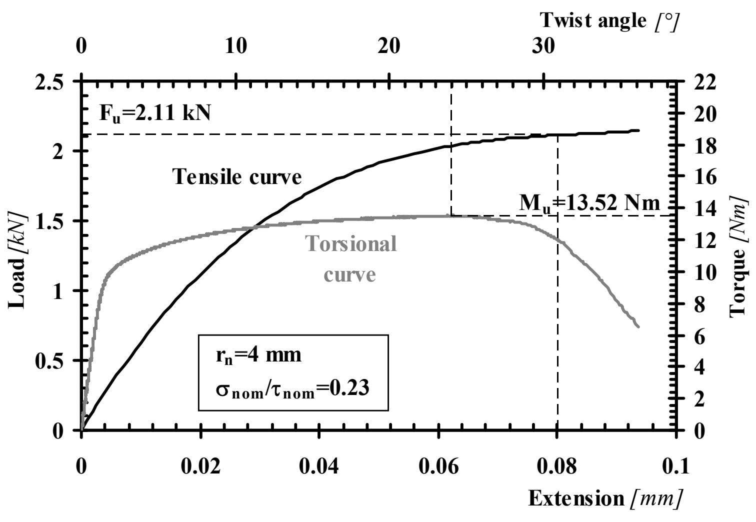

Figure 1: Load vs. extension and torque vs. twist angle curves generated by testing the notched samples having r_n=4 mm under combined tension and torsion.

Finally, three different tests were carried out for every investigated geometry/loading configuration and in Tab. 1 the obtained results are listed in terms of average value of the ultimate tensile nominal net stress, σ_u, and average value of the ultimate torsional nominal net stress, τ_u.

r_n *[mm]*	d_g *[mm]*	d_n *[mm]*	α *[°]*	K_t	K_{tt}	σ_{nom}/τ_{nom}	σ_u *[MPa]*	τ_u *[MPa]*
0.44	10.0	6.2	60	2.94	1.71	0	-	265.6
						0.23	68.9	299.8
						0.55	164.0	298.1
						1	254.7	254.7
						∞	537.3	-
0.50	10.0	6.1	60	2.76	1.64	0	-	255.1
						0.23	65.0	286.8
						0.55	148.5	274.6
						1	259.0	263.4
						∞	551.6	-
1.25	10.0	6.2	60	1.92	1.32	0	-	277.6
						0.23	69.6	301.7
						0.55	155.0	281.7
						1	271.6	271.8
						∞	523.7	-
4.00	10.0	6.1	-	1.33	1.12	0	-	312.1
						0.23	74.9	330.7
						0.55	160.1	296.2
						1	255.9	260.1
						∞	449.3	-

Table 1: Summary of the experimental results generated by testing V-notched samples of Al6082.

MATERIAL CRACKING BEHAVIOUR

In order to investigate in detail the cracking behaviour of this material, several tests were conducted during which the specimens were closely observed using a microscope, allowing the physical appearance of the cracks to be correlated to the applied stress and strain. In more detail, it was observed that relatively blunt notches loaded in tension failed by a conventional ductile fracture mode similar to plain (unnotched) specimens. However, in tensile specimens containing sharp notches, failure occurred via the initiation, stable propagation and, finally, unstable propagation, of circumferential ring cracks. Under torsional loading, and independent of the notch root radius, static failures of the tested samples always occurred by the formation and stable propagation of ring cracks. Under mixed-mode loading there was a gradual transition between the ductile and brittle modes and between stable and unstable cracking. For all types of loading it was observed that crack initiation always coincided with peak loading conditions, and that cracks invariably grew on the plane perpendicular to the specimen's longitudinal axis.
To conclude, Fig. 2 summarises the different fracture surfaces obtained by reducing the notch root radius value from 4 mm down to 0.44 mm as well as by increasing the ratio between the applied nominal loadings from zero up to infinity.

DEVELOPMENT OF THE TCD TO ESTIMATE STATIC STRENGTH OF NOTCHED DUCTILE MATERIALS UNDER MULTIAXIAL LOADING

The TCD postulates that static failures in notched engineering materials can accurately be estimated by using information from the linear-elastic stress field acting on the material in the vicinity of stress raiser apices through an appropriate effective stress, σ_{eff}. Examination of the state of the art shows that different strategies suitable for determining such an effective stress have been proposed and validated, which include the Point Method (PM), the Line Method (LM), the Area Method (AM) and the Volume Method (VM) [1].

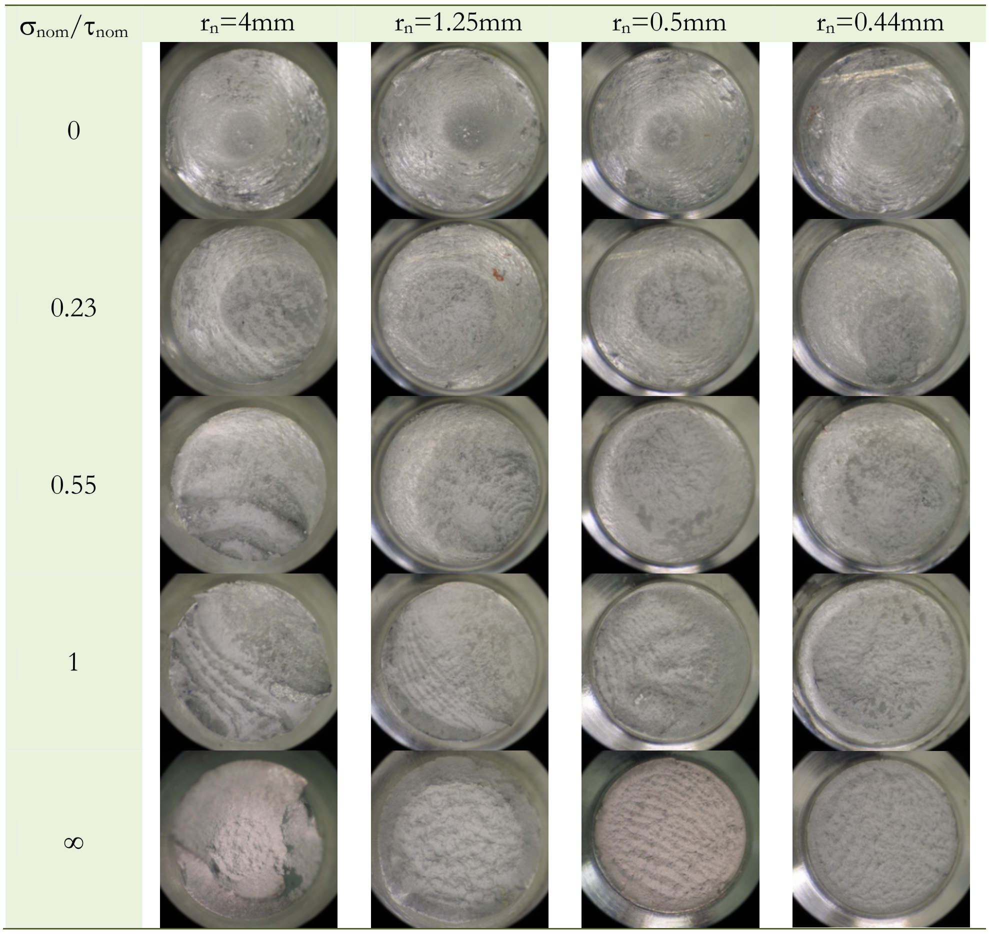

Figure 2: Matrix of the observed fracture surfaces.

In what follows, the PM and LM will be formalised to make them suitable for being used to assess those situations involving multiaxial loading. Initially, the assumption is made that the critical distance value needed to apply the TCD is known a priori (the experimental way to determine it will be discussed below), so that, the PM and the LM can be rewritten, respectively, as follows:

$$\sigma_{eff} = \overline{\sigma}\left(\theta = 0, r = \frac{\overline{L}}{2}\right) = \overline{\sigma}_0 \tag{1}$$

$$\sigma_{eff} = \frac{1}{2\overline{L}}\int_0^{2\overline{L}} \overline{\sigma}\left(\theta = 0, r\right) dr = \overline{\sigma}_0 \tag{2}$$

In the above identities, $\overline{\sigma}$ is the linear-elastic equivalent stress calculated according to one of the classical hypotheses (i.e., Von Mises, Tresca, maximum principal stress criterion, etc.), whereas $\overline{L}$ and $\overline{\sigma}_0$ are the corresponding critical distance value and inherent material strength, respectively. Eqs (1) and (2) make it evident that, according to the TCD's philosophy, static breakage is assumed to occur when the effective stress, σ_{eff}, equals the inherent material strength.

In our theory, both $\overline{L}$ and $\overline{\sigma}_0$ are hypothesized to be material constants which can directly be determined by running appropriate experiments. In particular, the most simple way to estimate them is by using two experimental results generated by testing, under uniaxial nominal loading, specimens containing different geometrical features [6, 7]. Fig. 3 summarises the strategy suggested here as being followed to determine such constants. By using the PM argument, the above schematic chart suggests that the point at which the two linear-elastic stress-distance curves (plotted, in incipient

failure condition, in terms of the adopted equivalent stress) intersect each other allows both $\overline{L}$ and $\overline{\sigma}_0$ to directly be evaluated. It is useful to point out here also that, according to our in-field experience, the usage of a relatively blunt notch together with a notch as sharp as possible is always advisable in order to obtain accurate estimates of the above constants.
As said above, Eqs (1) and (2) represent the formalisation of the TCD proposed in the present paper to be used to perform the multiaxial static assessment of notched engineering materials. As to the form of such equations, it is interesting to conclude observing that, if the TCD is applied along with the maximum principal stress criterion, the above definitions become identical to those we have already proposed to be used to estimate static strength of notched engineering materials subjected to uniaxial nominal loading [1, 5, 7].

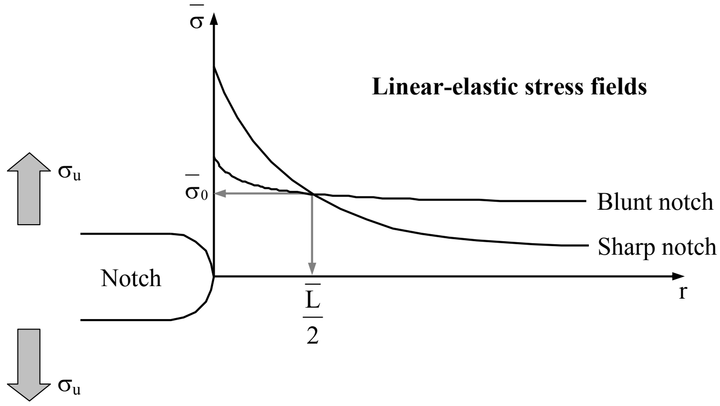

Figure 3: Experimental determination of $\overline{L}$ and $\overline{\sigma}_0$.

Equivalent stress	$\overline{L}$ *[mm]*	$\overline{\sigma}_0$ *[MPa]*
Von Mises	0.48	483.7
Tresca	0.48	515.5
σ_1 criterion	1.64	445.9

Table 2: Values of $\overline{L}$ and $\overline{\sigma}_0$ determined, in terms of the investigated equivalent stresses, according to the strategy summarised in Fig. 3.

METHOD VALIDATION BY EXPERIMENTAL RESULTS

In order to check the accuracy of the above formalisations of the PM and LM in estimating the static strength of the tested aluminium alloy, three different strategies have been investigated by considering the following three different equivalent stresses: maximum distortion strain energy criterion (i.e., Von Mises' hypothesis), maximum shear stress criterion (i.e., Tresca's hypothesis) and, finally, maximum principal stress criterion. According to the experimental technique sketched in Fig. 3, Tab. 2 summarises the obtained values for $\overline{L}$ and $\overline{\sigma}_0$ calculated according to the above three equivalent stresses. In particular, such material properties were determined by using the results generated under tension by testing the samples having notch root radius equal to 4 mm and 0.44 mm.
Tab. 3 summarises the accuracy of Eqs (1) and (2) when applied to the experimental results listed in Tab. 1 in terms of both Von Mises' equivalent stress, Tresca's equivalent stress and maximum principal stress criterion, respectively.
It should be noted that the error is zero for the PM predictions of pure tensile loading ($\sigma_{nom}/\tau_{nom} = \infty$) for the sharpest and bluntest notches (r_n = 4mm and 0.4mm) because these two cases were used to establish the necessary material constants, as described above. To conclude, it can be highlighted that, according to Tab. 3, the use of both the PM and LM resulted in estimates falling within an error interval of about 20% when the above two methods were formalised in terms of either Von Mises' or Tresca's equivalent stress. On the contrary, the use of the maximum principal stress criterion gave satisfactory results only when applied to nominal uniaxial situations, resulting instead in poor predictions for the other loading configurations (see Tab. 3).

r_n[mm]	σ_{nom}/τ_{nom}	Prediction Accuracy (%age Error)					
		Von Mises		Tresca		σ_1 criterion	
		PM[%]	LM[%]	PM[%]	LM[%]	PM[%]	LM[%]
0.44	0	-2.3	18.8	-9.8	9.7	190.0	313.3
	0.23	-14.0	4.5	-20.4	-3.3	106.0	170.9
	0.55	-15.9	1.8	-21.4	-4.7	61.0	96.4
	1	-8.3	10.0	-12.2	5.9	42.1	62.5
	∞	0.0	16.0	0.0	16.4	0.0	0.9
0.50	0	2.5	24.6	-5.4	15.0	200.5	335.2
	0.23	-18.0	10.1	-16.2	1.9	114.9	186.6
	0.55	-8.1	11.3	-14.1	4.3	75.1	116.3
	1	-11.0	7.1	-14.6	3.3	38.0	59.8
	∞	-5.9	11.4	-6.5	11.6	-3.2	-1.4
1.25	0	-3.0	14.7	-10.4	5.9	147.9	275.1
	0.23	-11.6	4.5	-18.1	-3.2	85.5	159.5
	0.55	-9.4	7.3	-14.5	1.1	56.2	102.3
	1	-15.5	0.5	-17.1	-2.0	22.5	48.8
	∞	-16.2	1.8	-17.9	1.0	-7.4	0.1
4.00	0	-8.1	4.2	-15.2	-3.9	104.9	221.6
	0.23	-14.2	-2.8	-20.5	-9.9	60.9	131.1
	0.55	-8.5	3.4	-13.9	-2.6	43.2	88.1
	1	-6.7	5.0	-9.1	2.3	23.8	51.0
	∞	0.0	10.2	0.0	11.3	0.0	9.1

Table 3: PM and LM's accuracy in estimating the generated experimental results.

CONCLUSIONS

- ✓ The failure loads for notched cylindrical specimens of Al6082 under biaxial loading can be accurately predicted by applying the TCD (in the form of the PM and LM), using either Tresca's or Von Mises' equivalent stress;
- ✓ The use of the TCD in conjunction with the maximum principal stress criterion is justified only to assess those situations involving uniaxial nominal loading.

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