The use of cohesive theories of fracture in 3D numerical simulations

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ABSTRACT. The finite element simulation of high-velocity dynamical processes involving fracture and fragmentation is one of the most demanding problems in computational mechanics. Difficulties arise especially when the nature of the problem requires a full three dimensional model. To simulate fracture, cohesive laws have been widely used in combination with finite elements, either as boundary conditions, or by enriching the set of shape functions of solid elements to include a displacement jump. An alternative successful approach introduces cohesive surfaces along boundary surfaces of continuum elements, through an automatic procedure combined with an explicit dynamic code. The presence of a characteristic time scale confers to cohesive models combined with dynamics an intrinsic rate-dependence without the need of modeling explicitly viscosity behaviors. Applications to experimental tests on brittle materials (dynamic Brazilian tests on cylinders in ceramics), aluminum (dynamic expansion of rings), graphite-epoxy composite (mixed mode dynamic loading), and the simulation of quasistatic fracture in biological fiber reinforced tissues (breaking of plaque on atherosclerotic artery) demonstrate the versatility of the method, and attest its ability to reproduce the most significant features of fracture processes.

KEYWORDS. Cohesive fracture; Dynamic fracture; Finite elements; Rate dependency.

INTRODUCTION

Fracture is a common phenomenon, observed in many materials and in many practical situations, characterized by the violation of compatibility conditions. Fracture is due to the attainment of a strength threshold in the material. In most cases, fracture is an undesired event, and structures are designed in order to avoid the nucleation and the propagation of cracks. In other cases, structures are designed in order to induce a particular, controllable fracture, when exceptional critical conditions are reached.

Figure 1 shows four examples of fracture observed in different materials under critical conditions. The image clearly shows that fracture can involve, among many others, brittle materials like PMMA (Fig. 1a), ductile materials like aluminum (Figure 1d), biological tissues (Fig. 1b) or geological materials (Fig. 1c). Fracture can characterize homogeneous and isotropic materials as well as anisotropic and non homogenous materials.

The analysis of extreme behaviors of materials and structures is not an easy task. The violation of compatibility and the attainment of the material strengths reduces or rules out completely the applicability of linearized theories to predict the mechanical response. Therefore the use of numerical approaches becomes incumbent.

One of the most interesting approaches is based on the finite element method (FEM). Because of its versatility, FEM has reached a leading role in numerical applications. The extension of FEM to the study of fracture is straightforward, even when many nonlinear features are accounted for.

In the following, we describe a successful approach to fracture simulation based on FEM, characterized by finite kinematics, able to account for free crack paths, multiplicity of branching, mixed mode loading, and irreversibility. The approach can be applied both to dynamical problems, where the equations of motion are time-discretized by the

Newmark algorithm, or to static problems, where the robustness of the approach, characterized by softening, is guarantee by the use of dynamic relaxation algorithms. Fracture is modeled by the "ad hoc" insertion of cohesive surfaces, eventually combined with mesh refinement algorithms. We will prove the versatility of the approach with different applications to mechanical problems.

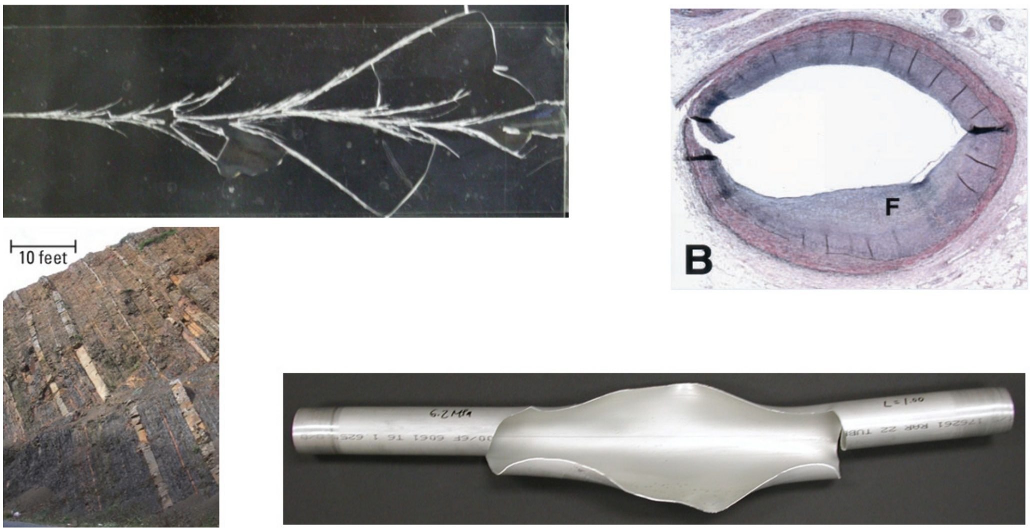


Figure 1: (a) Fracture patterns, showing extended branching, in a brittle PMMA plate loaded dynamically in Mode I [1]. (b) Localized fracture of atherosclerotic plaque in the internal part of a human artery [2]. (c) Fault of layered rock mass, showing the slipping of the top portion with respect to the bottom portion (US Geological Survey web site). (d) Ductile fracture of the wall of a thin aluminum pipe, deformed by explosion. Cracks develop from a little slit originally located in correspondence of the explosive (courtesy of Joe Shepherd, Caltech).

COHESIVE THEORIES OF FRACTURE

Cohesive theories describe the evolution of a fracture as the progressive separation of two surfaces. The separation is described by the displacement jump $\boldsymbol{\delta}$ between two points, originally coincident, on the two cohesive surfaces. In finite kinematics, the displacement jump is computed as the difference of the displacement field $\boldsymbol{\varphi}$ on the two facing surfaces (here the superscripts + and – denote the two opposite surfaces):

$$\boldsymbol{\delta} = \varphi^{+} - \varphi^{-} \tag{1}$$

while the kinematics of the cohesive surface is given by the average displacement:

$$\overline{\varphi} = \frac{1}{2}\left(\varphi^{+} + \varphi^{-}\right) \tag{2}$$

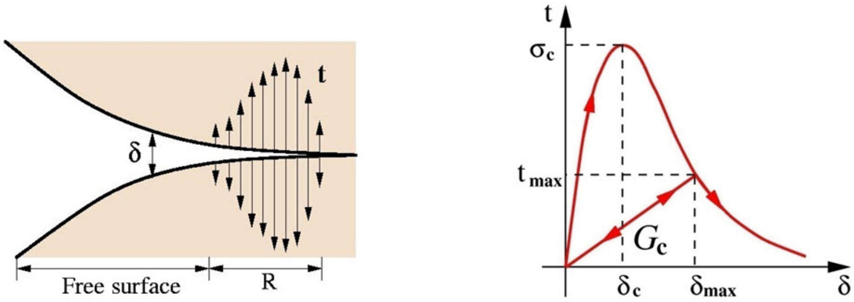
The separation is resisted by the cohesive tractions $\mathbf{t}$ acting on the cohesive zone R, located at the crack tip, see Fig. 2(a) [3, 4]. The relationship between tractions and displacement jump is defined by a cohesive law. By assuming the existence of a free energy density per unit of undeformed area ϕ, dependent on the displacement jump and on the deformation gradient on the cohesive surface, a general class of cohesive laws is obtained in the form [5]:

$$\phi = \phi\left(\boldsymbol{\delta}, \nabla_{S_0} \quad, \mathbf{q}\right), \quad \mathbf{t} = \frac{\partial \phi}{\partial \boldsymbol{\delta}} \tag{3}$$

The vector $\mathbf{q}$ contains a suitable collection of internal variables to account for history dependent behaviors. We add the hypotheses of: (i) material frame indifference; (ii) independence of the stretching and the shearing of the cohesive surface; and (iii) isotropy for the shear behavior over the cohesive surface. This allows considering a simpler cohesive law, where the dependence on $\boldsymbol{\delta}$ is stated in terms on normal and tangential components, δ_n and δ_S respectively, to the cohesive surface:

$$\phi = \phi\left(\delta_n, \delta_S, \mathbf{q}\right), \quad \delta_n = \boldsymbol{\delta} \cdot \mathbf{n}, \quad \boldsymbol{\delta}_S = \boldsymbol{\delta} - \delta_n \mathbf{n} \tag{4}$$

The simplest cohesive laws, typically used for mode I opening, are defined by two parameters, easily determined by standard laboratory tests, i.e. the normal cohesive strength σ_c and the critical energy release rate G_c, see, e.g., Fig. 2(b).



(a) (b)

Figure 2: (a) Visualization of the process zone at the crack tip, and definition of displacement jump $\boldsymbol{\delta}$ and cohesive tractions **t.** (b) Example of mode I cohesive law: Rose-Smith-Ferrante universal binding law. The area enclosed by the external envelope represents the critical energy release rate G_c. The peak of the curve defines the normal cohesive strength σ_c. Upon unloading, irreversibility is assumed with linear unloading to the origin and elastic reloading.

The uniaxial (mode I) cohesive law can be extended to mixed modes by introducing effective kinematic and static measures. An example is given by the following definition:

$$\delta = \sqrt{\delta_n^2 + \beta^2 \delta_S^2}, \quad \beta = \frac{\tau_c}{\sigma_c} \tag{5}$$

where the parameter β represents the ratio between the shear strength τ_c and the normal strength of the material σ_c. Thus, the free energy density is assumed to be dependent on the effective opening displacement δ, leading to a scalar cohesive law $t = t(\delta)$. The cohesive traction vector is obtained as:

$$t = \frac{\partial \phi(\delta)}{\partial \delta}, \quad \mathbf{t} = \frac{t}{\delta}\left(\beta^2 \boldsymbol{\delta}_S + \delta_n \mathbf{n}\right) \tag{6}$$

The irreversible character of fracture is included by assuming linear unloading to the origin and elastic reloading. It is important to observe that definition (3) rules out compressive behaviors. Once the fracture is open, the two surfaces may close and undergo contact, eventually including friction. Contact should not be included in the cohesive formulation, since large displacement dynamics may determine a different spatial location of each cohesive surface of a pair originally facing each other. A suitable contact algorithm has to be implemented in order to account for crack closure.

COHESIVE ELEMENTS AND INSERTION ALGORITHMS

Upon a finite element discretization of the solid, is it possible to identify potential crack patterns developing along the interelement surfaces. Therefore, a natural way to describe the fracture is to use flat cohesive elements located between solid elements where the cohesive law is enforced, see Fig. 3.

Cohesive elements do not possess mass neither body forces. They contribute to the equilibrium of the system by providing self balanced tractions to the connected nodes. Once the fracture is completely opened, their contribution vanishes.

The difficulty in the use of cohesive elements is the fact the opening of a crack induces changes in the geometry and in the topology of the system. Thus, in order to follow the nucleation of a crack and its evolution, it is necessary to modify the original discretization with an "ad hoc" remeshing procedure, possibly combined with refinement. An efficient fragmentation procedure is described in [5, 6]. The basic idea is that cracks are discrete and may develop only along the element interfaces. Unstructured tetrahedral meshes offer a good set of crack paths, and material interfaces (preferential crack paths) are modeled explicitly in FE discretizations. Cracks are explicitly modeled by a pair of cohesive surfaces inserted only when and where is necessary, providing the corresponding topology changes.

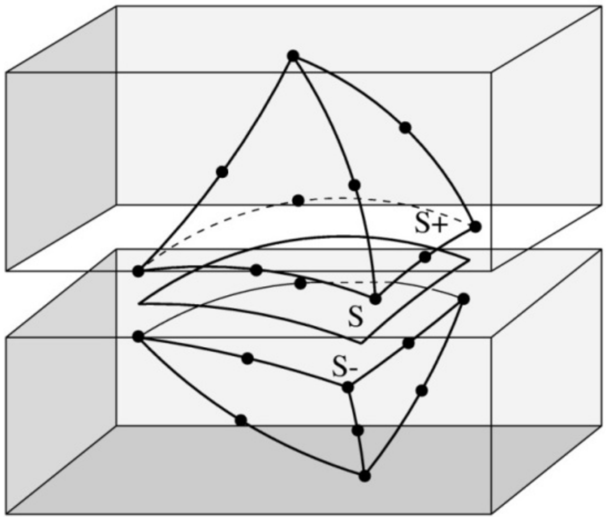


Figure 3: Insertion of a flat cohesive element between two solid elements (tetrahedral).

Crack nucleation follows the attainment of a critical condition (insertion criterion) based on stress, strain, or energy measures. In our application, we use the maximum effective traction criterion:

$$t = \sqrt{t_n^2 + \beta^{-2} t_S^2} > \sigma_c \tag{7}$$

where we compare the effective traction acting on each element interface with the material strength. Obviously, on material interfaces the material strength is due to the adhesive connecting the two materials. Each insertion of a cohesive element involves only one body and one cohesive element at time. Topological changes may lead to the formation of new bodies (fragments).

It is important to point out two features of cohesive theories. The intrinsic length scale:

$$\ell_c = \frac{EG_c}{\sigma_c^2} \tag{9}$$

where E is the Young modulus of the bulk, is introduced by the physics of the fracture phenomenon, which refers to a surface behavior and not to a volume behavior. It can be considered as a measure of the extension of the process zone, located at the head of the crack tip, where all the inelastic processes are located. The characteristic length (9) is responsible of the sensitivity to the size of the specimen. An intrinsic time scale arises only when dynamics is involved. Combining cohesive fracture with inertia it is possible to define a time scale [8]:

$$\tau_c = \frac{\rho c \delta_c}{2\sigma_c} \approx \frac{\ell_c}{c} \tag{10}$$

where c is the longitudinal wave speed. The characteristic time may be thought as the time necessary for a wave to cross the process zone. The presence of an intrinsic time introduces material sensitivity to the rate of loading. Although the bulk may be not described as a viscous or rate dependent material, cohesive behaviors in dynamics show a marked rate dependency, in good agreement with experimental observations.

EXAMPLES OF APPLICATION

A first example of application, in dynamics, refers to the behavior of advanced ceramics [9]. A set of fracture experiments performed on discs of special ceramics, loaded dynamically in a split Hopkinson bar, were available from the specialized literature. Results included the loading history and crack patterns. Details of the numerical simulation can be found in the original paper. Here we report an image where the experimental results are compared to the numerical simulation results in terms of crack patterns, see Fig. 4.

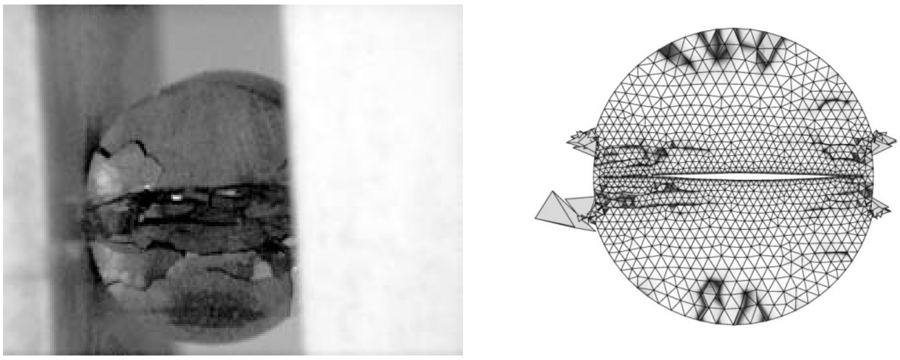


Figure 4: Snapshot of the experimental crack pattern at the final stage of the loading, compared to its numeric counterpart [9].

A second example shows the ability of the approach to deal with ductile fracture. The sudden application of a magnetic field at their center induces a fast radial expansion in thin aluminum rings. The thin structures show instabilities, forming necks at several locations, and subsequently the rings split into small fragments, see Fig. 5 [10].

To model the concentration of plastic deformation, an adaptive refinement of the mesh is combined with the fragmentation procedure. The numerical simulations are able to capture the observed rate dependency of the fracture process. By increasing the expansion velocity, the number of fragment increases, repeating nicely the experimental observations. The distribution of the fragment mass at elevated speeds is very close to the experimental mass distribution [10].

Figure 5: Numerical simulation of a fractured ring, at expansion speed 208 m/s [10].

A third example concern a transversally isotropic epoxy-graphite composite plate loaded in mode II [11]. Experiments were able to measure intersonic mode II crack propagation in materials characterized by a preferential direction of fracture. Numerical simulations, using anisotropic bulk material and anisotropic cohesive models, were able to replicate the global response in terms of crack speed versus crack extension. Other interesting features of intersonic mode II crack propagation, such as the formation of a double shock, were captured by the numerical simulation, see Fig. 6.

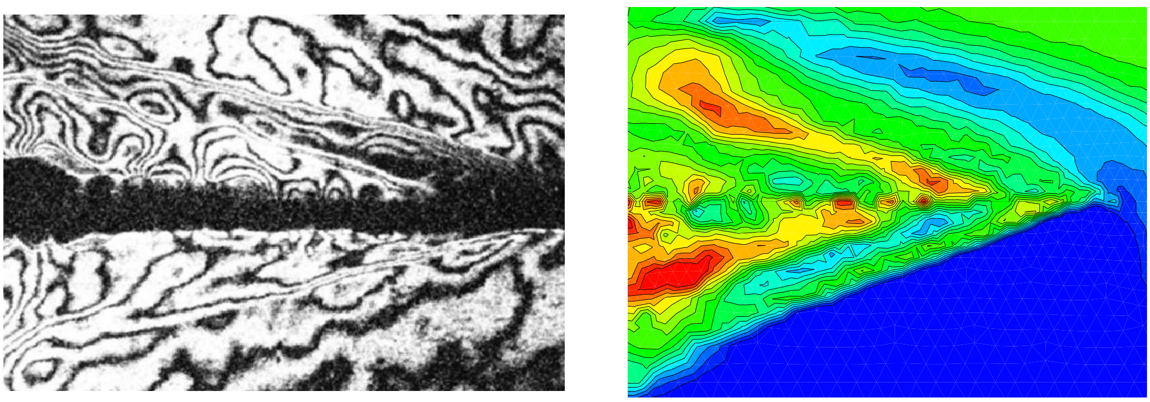


Figure 6: Comparison between experimental (Coker and Rosakis, 1998, 2001) and numerical shock wave structure [11].

A fourth and last example concerns fracture propagation in anisotropic fiber reinforced biological materials. In [12], the overpressure induced rupture of atherosclerotic plaque from an artery wall has been studied. The class of cohesive laws (3) has been modified in order to account for the presence of material directions. Also the insertion criterion accounts for the anisotropy of the material.
A model of an atherosclerotic human iliac artery has been reconstructed by manual segmentation from magnetic resonance images. An increasing pressure is applied quasi-statically to the internal surface of the vessel. The simulation has been conducted up to large extension of cracks, leading to the exposition to the lumen of the internal layers of the artery, see Fig. 7.

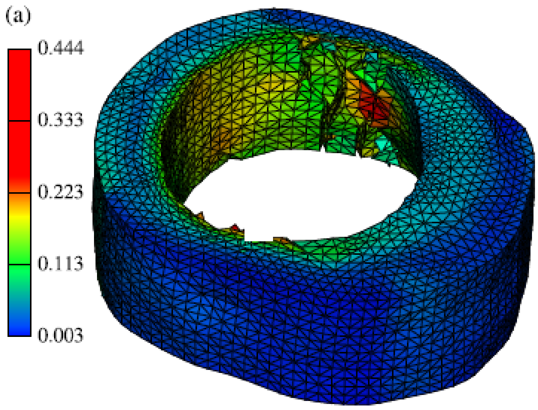


Figure 7: Final configuration of the over-expanded and fractured atherosclerotic artery. Contour levels of tresses in MPa [12].

CONCLUSIONS

We described a numerical approach for the simulation of crack nucleation and propagation under dynamic or quasistatic loading within FEM. Crack are simulated through cohesive models, and crack surfaces are inserted adaptively when and where is necessary. Cohesive laws are able to distinguish between mode I and mixed mode of loading and may account for material preferential directions. Examples of application in brittle, ductile, polymeric and biologic tissues prove the versatility of the approach.

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