Characterizing mixed mode brittle fracture using near crack tip stress fields

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ABSTRACT. We describe the formulation of a generalized maximum tensile stress criterion used to predict brittle fracture under combined tensile and shear (mixed mode) loading. The criterion uses approximate near crack tip stresses. We demonstrate its applicability to a range of materials exhibiting brittle fracture. Fracture data from a number of well known mixed mode fracture specimens such as the cracked Brazilian disc, semi-circular disc, four-point bending beam, and cracked plates are used to validate the criterion. Characterization of the crack-tip fields is carried out numerically to determine tensile and shear stress intensity factors and T-stress for each specimen. Test results show that different specimens provide different fracture behavior because they exhibit differing values of T-stress for the same tensile and shear stress intensity factors.

KEYWORDS. Mixed mode fracture; Near crack tip stress; Generalized maximum tensile stress criterion.

INTRODUCTION

When cracked materials are subjected to external loading, the usual convention is to indicate the mode of loading to describe how the crack deforms. Mode I refers to tensile loading where the crack flanks open away from each other. Mode II corresponds to in-plane shear loading where the crack faces slide over each other in the same plane as the crack. Finally, mode III is the case when there is out-of-plane shear loading. A criterion for fracture of cracked materials must take account of any of these loading modes or combinations of them. In the case of brittle fracture it was the convention for a number of years [1] to describe failure using critical values of the stress intensity factors (e.g. K_I, K_{II}, and K_{III}) for each of the loading modes. For example, in the case of a synthetic soft rock called Johnstone, [1] it was suggested that for mixed mode loading (modes I and II) an empirical failure criterion is

$$\left(\frac{K_{\mathrm{I}}}{K_{\mathrm{I}c}}\right)^{\lambda}+\left(\frac{K_{\mathrm{II}}}{K_{\mathrm{II}c}}\right)^{\mu}=1 \tag{1}$$

where λ and μ are constant values assumed to depend only on the material properties. Lim et al. [1] suggested that $\lambda = \mu = 2.25$ provides the best fit to the test results.

Theoretical developments have relied on various fracture criteria such as maximum tensile strength (MTS) [2] and maximum strain energy density [3]. In the case of the MTS criterion it is assumed that failure occurs when the maximum tangential stress, $\sigma_{\theta\theta}$, at some critical distance, r_c, in a direction, θ_m, exceeds the critical stress, σ_c. For combinations of modes I and II the onset of fracture is described by

$$K_I \cos^3 \frac{\theta_m}{2} - 3K_{II} \cos^2 \frac{\theta_m}{2} \sin \frac{\theta_m}{2} = K_{Ic} \tag{2}$$

with

$$K_{Ic} = \sigma_c \sqrt{2\pi r_c} \tag{3}$$

and the direction θ_m, obtained by solving

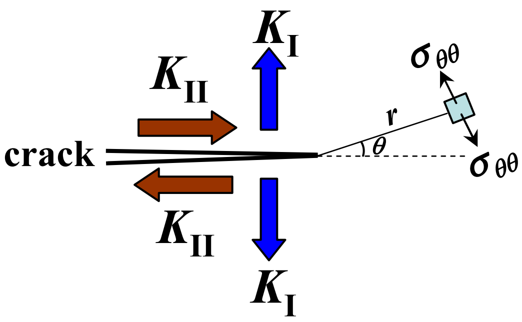
$$\frac{K_I}{K_{II}} = \frac{1 - 3\cos\theta_m}{\sin\theta_m} \tag{4}$$

The use of stress intensity factor assumes that the singular stresses dominate near to the crack tip. However, the distance over which fracture occurs ahead of the crack tip could be such that other terms in the series expansion of the stresses [4] need to be included. One additional term in the series expansion is the constant stress parallel to the crack and is often called the T-stress. For mode I loading, the T-stress has little effect on initiation of brittle fracture [5] unless it exceeds a critical value. For mode II loading however, increasing T-stress results in a significant decrease in the initiation toughness [6].

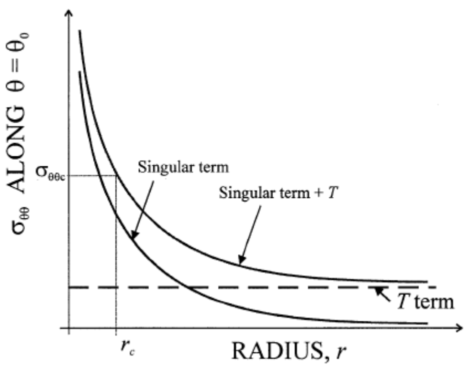
The purpose of this paper is to summarise our recent developments in characterising fracture when brittle materials are subjected to mixed-mode I and II loading. Importantly, this work is focused on materials that exhibit brittle fracture without plasticity acting as a precursor to failure. First, we provide an overview of the theory that extends the conventional analysis based on singular stresses and illustrate how T-stress modifies the MTS criterion. This new approach is called a generalised maximum tensile stress criterion. Second, the findings from a series of earlier detailed experiments, carried out on a variety of brittle materials, are illustrated. Here, different shaped specimens were used and knowledge of the stress intensity factors and T-stress for each specimen geometry is required.

THEORY

We consider elastic homogeneous and isotropic materials containing cracks subjected to mode I and II loading, as shown in Fig. 1, with Fig. 1a illustrating our notation, and Fig. 1b showing schematically both the singular stress term and how the near crack tip stress is increased by the presence of a positive T-stress.



a)



b)

Figure 1: Stresses at crack tips a) crack tip co-ordinates in polar system and two major modes of crack deformation and b) distribution of elastic tangential stress along the direction of fracture initiation, θ_0.

The stresses near to the crack tip can be described using an infinite series expansion [4],

$$\sigma_{ij} = \sum_{n=1}^{\infty} A_n r^{\frac{n-2}{2}} f_{ij}^{(n)}(\theta) + \sum_{n=1}^{\infty} B_n r^{\frac{n-2}{2}} g_{ij}^{(n)}(\theta) \tag{5}$$

where σ_{ij} is the stress tensor, $f_{ij}^{(n)}(\theta)$ and $g_{ij}^{(n)}(\theta)$ are the symmetric and anti-symmetric angular functions in the n^{th} term of stress series expansion and r and θ are the conventional crack tip co-ordinates. The first term of each summation in Eq. (5) is singular and the second term in the first summation is constant and independent of the distance r from the crack tip.

We restrict our analysis to consider brittle failure. This occurs when the maximum tangential stress, $\sigma_{\theta\theta}$, at some critical distance, r_c, in a direction, θ_m, exceeds the critical stress, σ_c then the tangential stress, $\sigma_{\theta\theta}$, is derived from Eq. (5) and is given by

$$\sigma_{\theta\theta} = \frac{1}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left[K_I \cos^2\frac{\theta}{2} - \frac{3}{2}K_{II}\sin\theta\right] + T\sin^2\theta + O(r^{1/2}) \tag{6}$$

The higher order terms $O(r^{1/2})$ can be considered negligible near to the crack tip. The crack tip parameters K_I, K_{II}, and T depend on the geometry and loading configurations and can vary considerably between different specimens.

The direction of the maximum tangential stress, θ_m, is determined from

$$\frac{\partial \sigma_{\theta\theta}}{\partial \theta}\Big|_{\theta=\theta_m} = \left[K_I \sin\theta_m + K_{II}(3\cos\theta_m - 1)\right] - \frac{16(T\sqrt{2\pi r_c})}{3}\sin\frac{\theta_m}{2}\cos\theta_m = 0 \tag{7}$$

Once the direction of initiation of fracture, θ_m, is found from Eq. (7), the value of θ_m is replaced into Eq. (6) to determine the onset of brittle fracture so that

$$(\sigma_{\theta\theta,c})\sqrt{2\pi r_c} = \cos\frac{\theta_m}{2}\left[K_{If}\cos^2\frac{\theta_m}{2} - \frac{3}{2}K_{IIf}\sin\theta_m\right] + T_f \sin^2\theta_m \sqrt{2\pi r_c} \tag{8}$$

where $(\sigma_{\theta\theta,c}) = \sigma_c$ is the critical value of the tangential stress at the critical radius r_c . For pure mode I where K_{II}, and θ_m are equal to zero and K_I can be replaced by the mode I fracture toughness K_{Ic}, then Eq. (8) reduces to Eq. (3).

It can be shown from Eq. (8) that

$$K_{Ic} = \cos\frac{\theta_0}{2}\left[K_{If}\cos^2\frac{\theta_0}{2} - \frac{3}{2}K_{IIf}\sin\theta_0\right] + B\alpha K_{eff}\sin^2\theta_0 \tag{9}$$

where

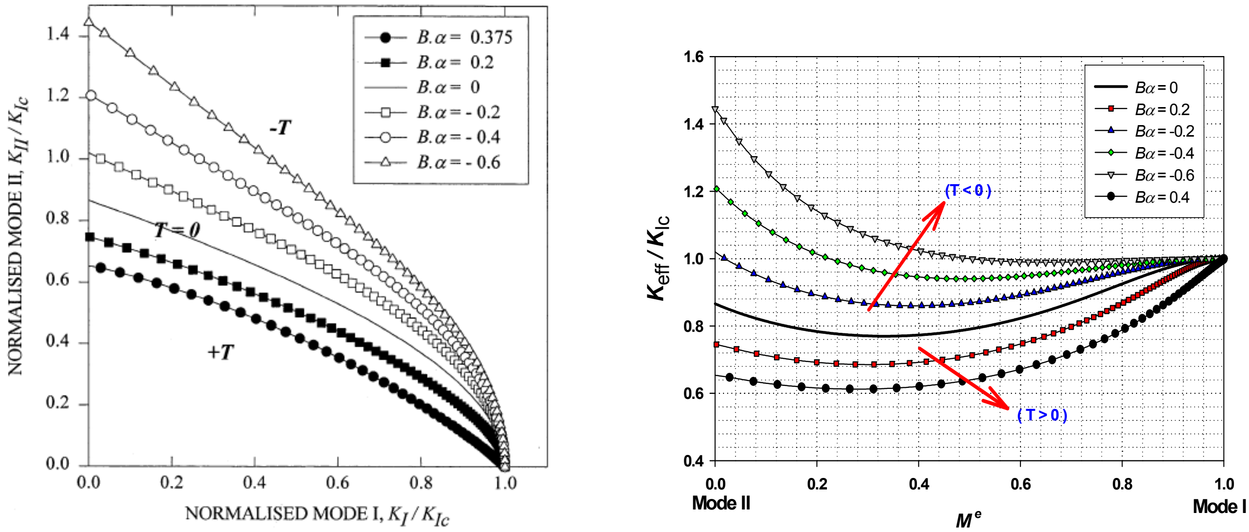
$$K_{eff} = \sqrt{K_I{}^2 + K_{II}{}^2}\ ,\ B = \frac{T\sqrt{\pi a}}{K_{eff}} = \frac{T\sqrt{\pi a}}{\sqrt{K_I^2 + K_{II}^2}} \quad \text{and } \alpha = \sqrt{\frac{2r_c}{a}} \tag{10}$$

Eqs. (8) to (9) describe a generalized MTS criterion. When K_I, K_{II}, and T are known, the condition for brittle fracture is predicted for any combination of tensile and shear in-plane loading.

It is instructive to examine how the product $B\alpha$ in Eq. (9) changes the conditions for failure. Failure curves or loci can be derived by using Eqs. (8) to (10), and these are shown in Fig. 2, with Fig. 2a), illustrating failure loci using the normalized ratios of K_{If}/K_{Ic} and K_{IIf}/K_{Ic} and Fig. 2b) showing failure curve with K_{eff}/K_{Ic} as a function of the mixity parameter, M_e where

$$M^e = \frac{2}{\pi}\tan^{-1}\left(\frac{K_I}{K_{II}}\right) \tag{11}$$

M^e is one for pure mode I and zero for pure mode II.



a) b)

Figure 2: Predictions of a generalized MTS criterion for mixed mode I/II fracture and for various values of $B\alpha$.

Fig. 2a) and b) show, for a given α, that the failure curves are lower than the standard MTS curve for positive T-stress and higher for negative T-stress. Although not shown here, the presence of the T-stress also changes the angle at which the crack initiates and grows from the original crack. Further details of this can be found in [5] and [7].

EXPERIMENTAL EVIDENCE

Early evidence for the effects of T-stress on brittle fracture was explored using simple Perspex (or PMMA) plate specimens containing angled cracks [8, 9] and reviewed extensively in [5]. More recently, brittle fracture of PMMA, [6, 10] and other materials, such as glass [11] and an assortment of rocks, including limestone and marble [7, 12] have been explored using a variety of specimen shapes. Fig. 3 shows a selection of the specimen shapes, ranging from asymmetrically four point bending (FPB), biaxial loaded plates containing angled cracks, compact tension-shear specimens and semi-circular and centrally cracked discs. Finally, a diagonally loaded square plate (DLSP) with a central crack inclined to the loading direction has been recently developed [13].

A key feature to interpreting data from tests using these specimens is obtaining values of the parameters, K_I, K_{II} and T that characterize the near crack tip stress field. With the exception of special cases, such as the angled crack in a biaxial loaded plate [5], it is often necessary to undertake elastic finite element analyses to obtain these parameters.

In summary K_I and K_{II} and T-stress in these specimens are often shown as:

$$K_I = \begin{cases} Y_I \dfrac{P}{RB}\sqrt{\dfrac{a}{\pi}} & BD \\ Y_I \dfrac{P\sqrt{\pi a}}{2RB} & SCB \\ \dfrac{Y_I P\sqrt{\pi a}(l_1 - l_3)}{2BWl_1} & FPB \\ Y_I \dfrac{P\sqrt{2\pi a}}{2wB} & DLSP \end{cases} \qquad K_{II} = \begin{cases} Y_{II} \dfrac{P}{RB}\sqrt{\dfrac{a}{\pi}} & BD \\ Y_{II} \dfrac{P\sqrt{\pi a}}{2RB} & SCB \\ \dfrac{Y_{II} P\sqrt{\pi a}(l_1 - l_3)}{2BWl_1} & FPB \\ Y_{II} \dfrac{P\sqrt{2\pi a}}{2wB} & DLSP \end{cases} \qquad T = \begin{cases} \dfrac{T^* P}{\pi RB(R-a)} & BD \\ \dfrac{T^* P}{2RB} & SCB \\ \dfrac{T^* P}{BW} & FPB \\ \dfrac{T^* P\sqrt{2}}{2wB} & DLSP \end{cases}$$

where Y_I, Y_{II} and T^* are the normalized forms of K_I, K_{II} and T, respectively. Their values are illustrated in Fig. 4 as functions of the mixity parameter, M^e for selected values of specimen dimensions.

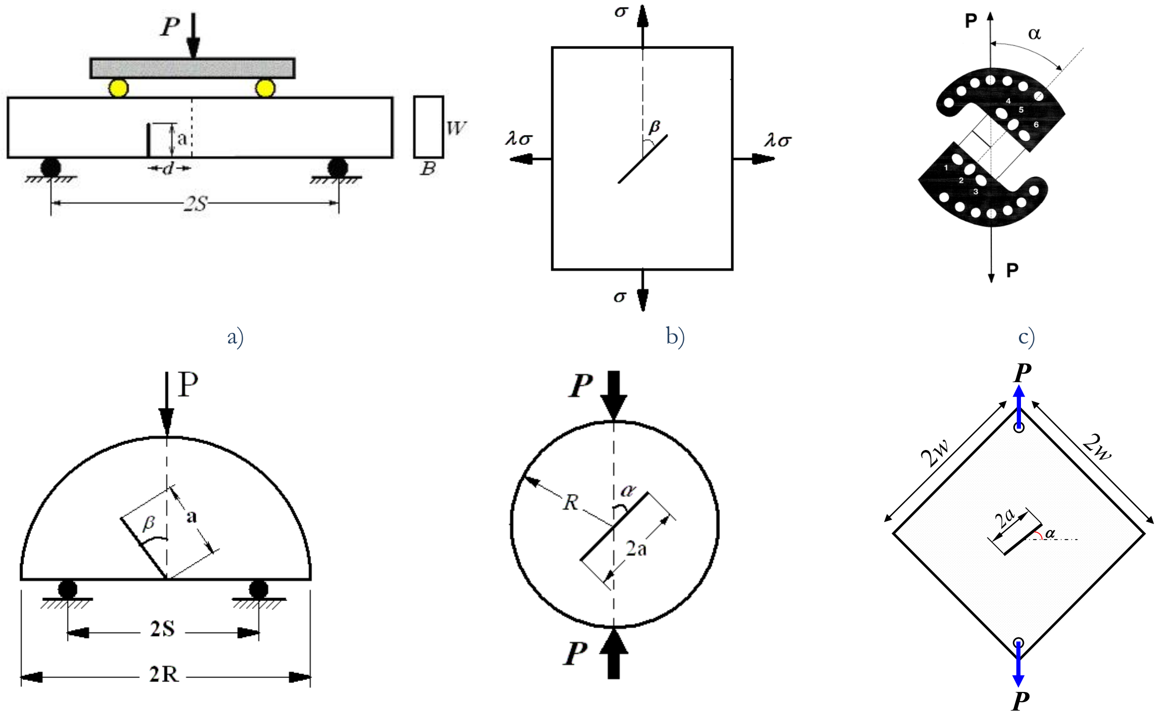


Figure 3: Typical mixed mode I-II fracture toughness specimens (a) asymmetric four point bend (FPB) specimen, (b) plate containing an angled internal crack, (c) compact tension-shear specimen, (d) semi-circular bend (SCB) specimen and (e) centrally cracked circular disc under diametral compression (BD) specimen, (f) DLSP specimen.

There are a number of important observations about the distribution of these parameters. The Brazilian disc is the only specimen that provides an almost linear increase in mode II stress intensity factor when there are changes in the crack angle. However, the value of the normalized T-stress, T^*, is always negative for all levels of mode mixity for this specimen. In contrast, the semi-circular disc specimen, while exhibiting a small negative T-stress at or near to mode I, provides positive and increasing levels of T-stress with decreasing levels of mode mixity. In general, the asymmetric four point bending specimen has little or no T-stress irrespective of mode mixity.

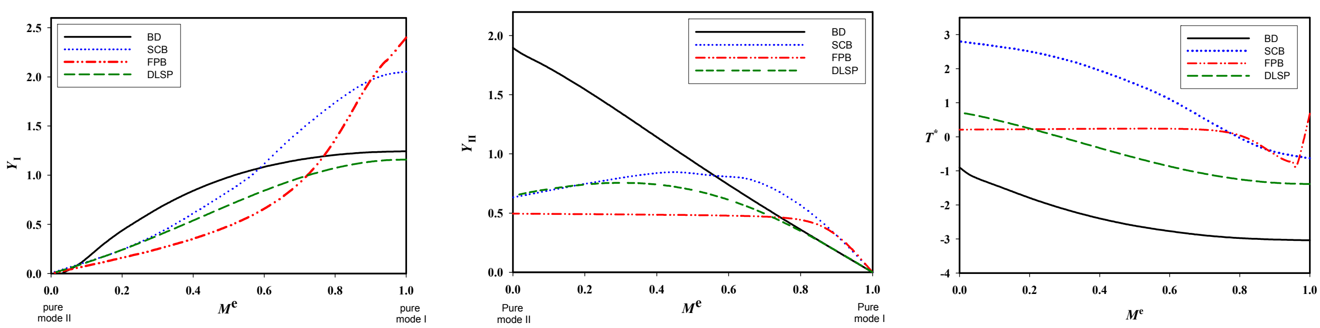


Figure 4: Variations of normalized K_I, K_{II} and T-stress for BD, SCB, FPB and DLSP specimens.

Recently [10], a variety of mixed mode test results for Perspex, using some of the test specimens shown in Fig. 3, were collated and examined in terms of the generalized MTS criterion. The test results are shown in Fig. 5 along with the curve for the conventional MTS criterion. As a result of the comparatively low magnitude of T-stress, the fracture data obtained from the FPB and DLSP do not show a significant variation from the conventional MTS criterion. In contrast, the BD and SCB test data are appreciably different from the conventional MTS failure locus, due to the presence of higher magnitudes of T-stress. Predictions of the failure loci using the generalized MTS criterion provide the curves shown in Fig. 6. In order to make these predictions it was necessary to estimate the size of the process size, r_c from

$$r_{\mathrm{c}} = \frac{1}{2\pi}\left(\frac{K_{\mathrm{Ic}}}{\sigma_{\mathrm{t}}}\right)^2 \tag{12}$$

where σ_t is the material tensile strength. Assuming values for K_{Ic} of 1.8 MPa m$^{0.5}$ and σ_t of 71 MPa, the process zone size is about 0.1mm. It was assumed in using this value was independent of geometry and mode mixity.
The predicted curves in Fig. 6 show the same trends as the data illustrated in Fig. 5. These results demonstrate that the specimen geometry has an important effect on the observed fracture behavior of a material.

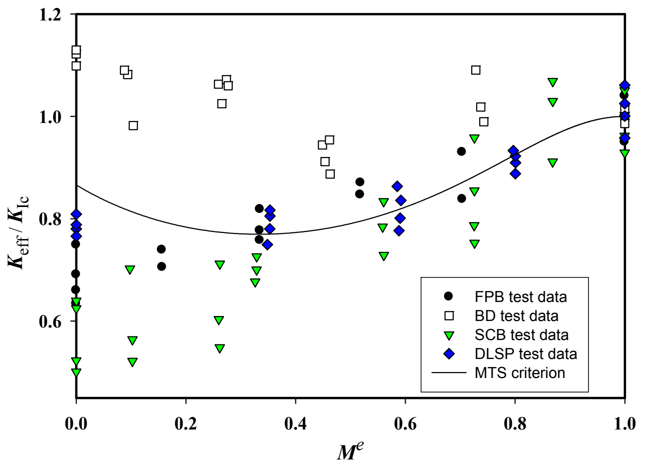


Figure 5: Mixed mode I/II fracture toughness data for Perspex tested with four different specimen shapes.

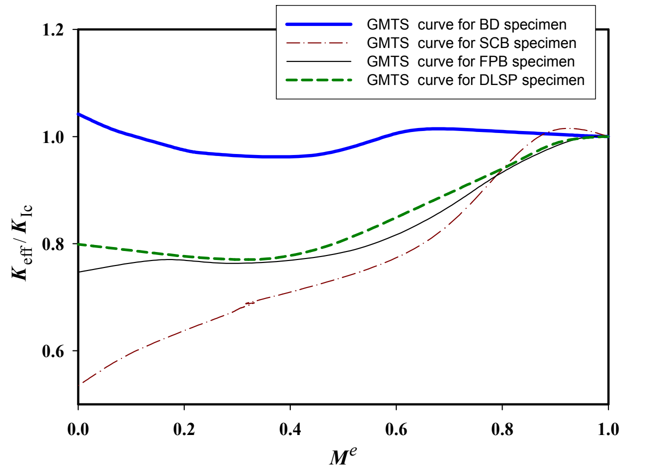


Figure 6: Comparison of fracture curves derived from the generalized MTS criterion for Perspex.

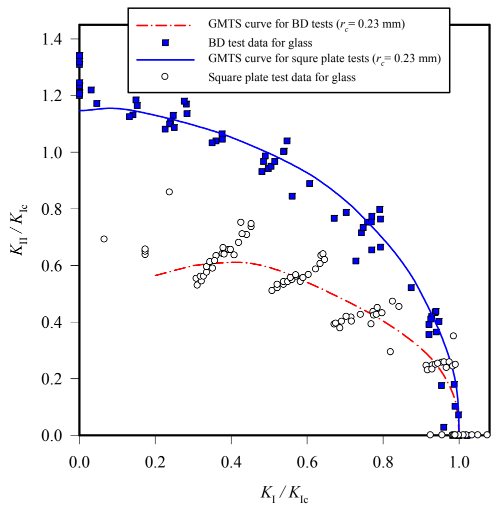


Figure 7: Application of the generalized MTS criterion for mixed mode fracture resistance of soda-lime glass.

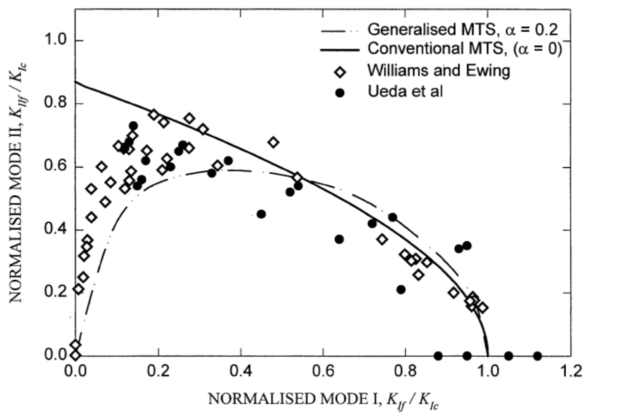


Figure 8: Mixed mode fracture toughness of Perspex using plates containing angled cracks.

Our final example of recent work comes from a re-examination of mixed mode fracture in soda-lime glass [11]. Earlier work had shown that for Brazilian disc (BD) specimens the fracture response was always higher than predicted by the conventional MTS criterion. As we have seen in Fig. 4, this geometry has a negative T-stress for all levels of mode mixity and consequently we might expect an apparent increase in toughness. For the conventional MTS criterion the predicted mode II toughness is about 0.87 of the toughness in tensile loading alone (mode I). If, as in the BD specimen, there is a large negative T-stress, the mode II toughness increases. This is borne out by the results shown in Fig. 10, where a variety of tests, all using the BD specimen, are shown. Also illustrated is the predicted failure locus calculated for a critical distance, r_c, equal to 0.23mm.

Included in the re-evaluation of earlier data for soda-lime glass [11] were results from tests using plates containing cracks angled relative to the loading direction. These test results are shown in Fig. 7. For these plate specimens, there are simple analytical equations for the crack tip parameters, K_I, K_{II} and T [5]. These were used to provide a generalized MTS prediction again using r_c=0.23mm which agrees approximately with the experimental results.

An interesting feature of these plates is that as the crack angle tends towards the same direction as the vertical loading (see Fig. 3b) both the mode I and II stress intensity factors tend to zero and the fracture condition relates solely to the T-stress (i.e. the stress parallel to the crack). Fracture is a consequence of the net section stress being near or equal to the critical stress, σ_c. Results in Fig. 8 show this effect more clearly for Perspex test data [5] obtained from plates containing angled cracks. Fracture results at positions corresponding to near zero values of K_I and K_{II} arise from fracture dominated by T-stress and deviate significantly away from the conventional MTS criteria. Such a characteristic feature is not entirely evident in the results for glass in Fig. 7. Nevertheless, the predicted locus using the generalized MTS criterion also begins to deviate significantly away from the conventional MTS curve

CONCLUDING REMARKS

There are now many examples where failure of brittle materials subjected to combined tension and shear loading (mixed mode I and II) can be characterised by the near crack tip stress fields. In this paper we have shown that the crack tip field can be described approximately using two important singular, K_I and K_{II} terms united with the non-singular, T-stress. These stresses combined with a simple fracture criteria (a critical stress at a critical distance); have been shown to be sufficient to describe the fracture loci of many mixed mode tests. It is also demonstrated that the fracture response of laboratory test specimens is geometry dependent. This is because the value of the T-stress may be different even though the stress intensity is the same. This is demonstrated by the differences between cracked Brazilian and semi-circular discs.

There remain some significant challenges. This work has not covered the effects of out-of-plane shear loading, mode III. For example, earlier work using Perspex for combined mode I and III conditions [14] revealed potential effects of non-linear deformation contributing to increases in fracture toughness, particularly for non-proportional loading.

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