Experimental and numerical analysis of elastic-plastic strains and stresses ahead of a growing fatigue crack

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ABSTRACT. Reliability and fatigue life estimation of structural and mechanical components subjected to cyclic loads require predictions of stresses and strains at critical locations such as notches and cracks. Such analyses are concerned with solving complex elastic-plastic boundary problems involving varying geometrical boundaries and cyclic loading.
The objective of the paper is an analysis of elastic-plastic strains and stresses arising in the plastic zone adjacent to the tip of a propagating fatigue crack. The analyzed below boundary value problem concerns Compact Tension specimen with growing fatigue crack subjected to non-symmetric constant amplitude cyclic loading history. A special finite element code has been developed to calculate the elastic-plastic stress-strain response ahead of a growing Mode I fatigue crack. The code consists of the cyclic plasticity Mroz-Garud model, contact elements and a module simulating the growth of the fatigue crack. The simulation was carried out for the SAE4140 steel material. The aim of the analysis was to study both the crack opening displacement field behind the moving crack tip and fluctuations of the elastic-plastic strains and stresses ahead of the crack. It has been found that both the crack opening displacement and the crack tip stress-strain fields are related to each other and occasional contacts between crack surfaces do not have as strong effect on the crack tip stress-strain affairs as it is often claimed. It was also found that the crack region adjacent to the crack tip often stays open regardless of optically visible presence of the contact between crack surfaces away from the crack tip. The effect of the plasticity induced residual stress field on the crack opening displacement field at the maximum and minimum load levels has been illustrated as well. The studies indicate that the simulation of the load history effect based only on the contact between crack surfaces, known as the crack tip closure, and manipulation of the minimum stress intensity factor is insufficient for appropriate modeling of the spectrum loading effects on the stress-strain history ahead of the crack tip and subsequently on the fatigue crack propagation process.
Limited experimental measurements of near the crack tip strains influenced by an overload were also presented and discussed in terms of its effect on the crack tip strain field at the maximum and minimum load level.

KEYWORDS. Crack tip strain history; Crack tip opening and overload effects.

INTRODUCTION

The simulation of the strain-stress response at the crack tip to the applied cyclic loading is essential for the fatigue crack growth modelling and subsequent crack growth simulation. Unfortunately, the resulting crack tip strains and stress estimations are very strongly dependent on how the crack tip geometry is modelled. It is well known that the assumption of the infinitely sharp crack tip with the tip radius of ρ=0 and the small deformation theory of elasticity leads [1] to unrealistically high stresses ($\sigma_{yy}\rightarrow\infty$) in the crack tip neighbourhood. The recent and more realistic moving boundary formulation of the crack problem [2] led to the solution with finite, albeit very high, stress (σ_{yy}=E) and radiused crack tip (ρ>0). The crack tip radius was found (1) to be depended on the applied stress intensity factor K and the modulus of elasticity E.

$$\rho = \frac{4K^2}{\pi E} \tag{1}$$

However, most often the finite element method is used at present for crack analyses. Therefore, in order to numerically simulate the geometry of the crack tip and the crack tip stress and displacement fields it is necessary to use sufficiently fine mesh in the crack tip neighbourhood which in turn depends on the unknown crack tip radius. Expression (1) can be used as a base for the initial estimation of the required element size around the crack tip.

The purpose of the present study was to analyse displacements, stresses and strains around a growing fatigue crack tip. Among others the effect of a single overload on the crack tip opening displacement field behind the crack tip and the simultaneous stress-strain response ahead of the crack tip was also analysed.

MATERIAL PROPERTIES, SPECIMEN GEOMETRY AND DIMENSIONS

The compact tension specimen shown in Fig. 1 and made of the SAE4140 steel material was used for the analysis. The stress-strain material curve was given in the form of the Ramberg-Osgood expression (2) with the material parameters E= 205000 MPa, K'=1640 MPa and n'=0.15 and ν=0.3. The cyclic stress-strain properties K' and n' were used in the finite element analysis. The specimen was analysed numerically and experimentally under the same load and the same crack depth.

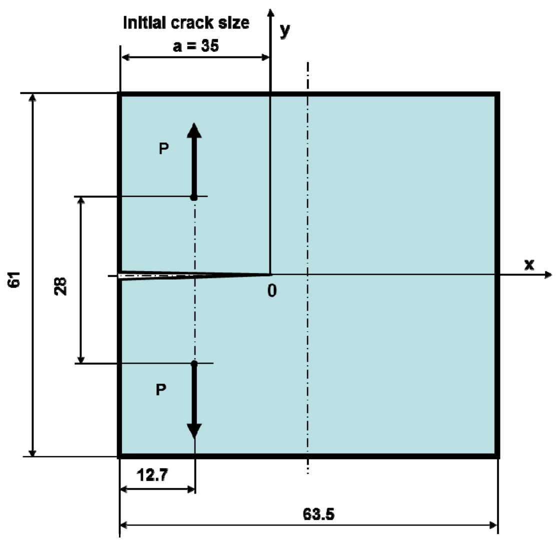

Figure 1: Geometry and dimensions of the analysed CT specimen; thickness 4mm

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{\frac{1}{n'}} \tag{2}$$

The numerical stress-strain analysis was carried with the help of in-house developed finite element software and the experimental strain measurements were obtained Croft et.al. [3] using the neutron diffraction method. The crack was propagated from its initial depth of a=35 mm up to the depth of a=38 mm at which point the overload was applied.

Details of the finite element mesh around the crack tip region are shown in Fig. 2. The plane strain state was imposed over the entire specimen. The size of the smallest finite element at the crack tip was chosen to be less than the expected crack tip radius at the maximum load. The specimen was subjected to constant amplitude cyclic load fluctuating from P_{min}=0 to P_{max}=3.8 kN as shown in Fig. 3. The load during the ascending and descending reversals was applied incrementally with increments of δP=0.38 kN. After the application of two load cycles the crack was extended by releasing appropriate number of nodes of the element adjacent to the crack tip. The actual crack increments were equal to the finite element size.

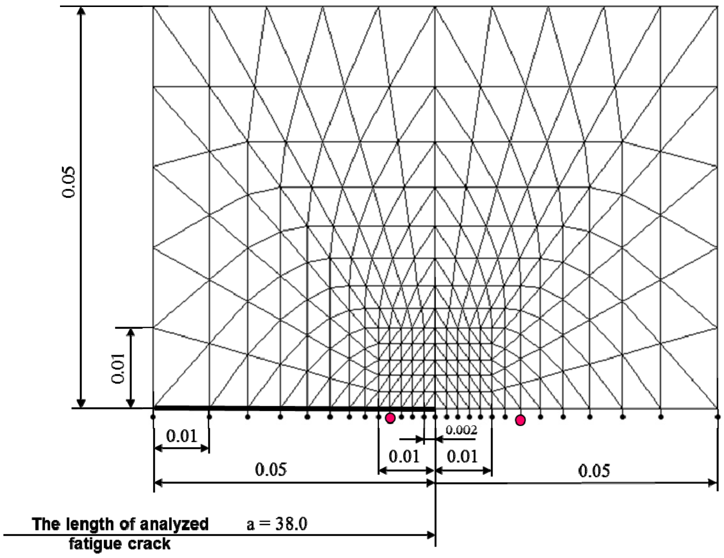

Figure 2: Configuration of the finite element mesh around the crack tip; all dimensions in millimetres.

The crack extension was achieved by incrementally releasing nodal forces of the element adjacent to the crack tip, i.e. by decreasing appropriate nodal forces to zero while holding the applied load P at its maximum, i.e. at P_{max}=3.8 kN. The applied loading history is shown in Fig. 3. The crack was grown from its initial size of a=35 mm until it reached the depth of a=38 mm when the overload of P_{ol}=7.6 kN was applied. Two more cycles were applied after the overload without further extension of the crack depth.

The cyclic multiaxial plasticity model of Mroz-Garud [4] was used for modelling the material memory and its stress-strain response to the applied cyclic loading history. In order to account for possible crack surface contact and the crack tip closure, the finite element software was enriched with a special procedure preventing the crack opening displacements to be negative and calculating resulting reactive pressure appearing between contacting crack surfaces. The finite element analyses were carried out at 10 various load levels for each ascending and descending load reversal resulting in significant numbers of executions of the FE program. Therefore, in order to grow the crack from its initial size of 35 mm to the depth of 38 mm several days of the CPU time was required.

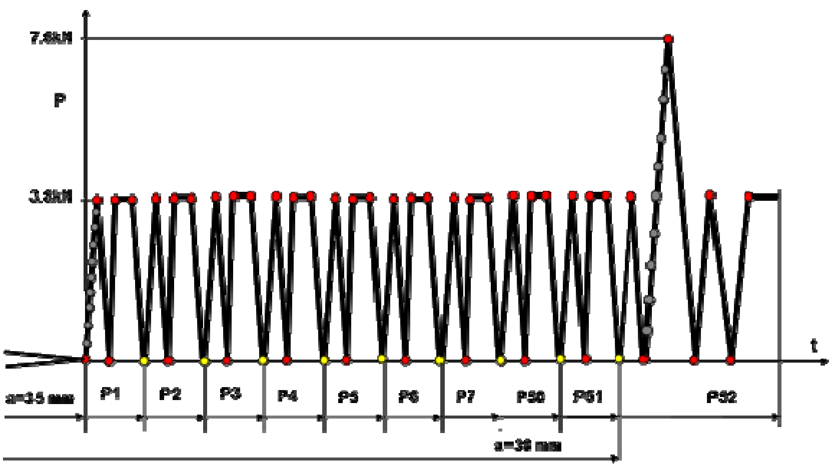

Figure 3: The applied cyclic load history.

THE CRACK OPENING DISPLACEMENT FIELD

The first set of results obtained after the crack has reached the depth of a=38 mm was the crack opening displacement field, determined for several applied load levels (Fig. 4). It is visible that just behind the tip the crack remained open even at the minimum load P_{min}=0. Even over the region of the first four elements behind the crack tip, appearing as being closed, the contact stresses were very low and without any strong indication of significant contact between crack surfaces. The results suggested that the contact stresses and the crack closure behind the crack tip was negligible. On the other hand, very visible region of no contact between crack surfaces was found behind the crack tip at the minimum load. This indicates that the region behind the fatigue crack tip was not closed but fully opened. The crack tip opening field at the maximum load P_{max}=3.8 kN was compared with analogous displacement field obtained for a stationary crack of the same length and subjected to the same load, i.e. for a crack without any prior history. The results are shown in Fig. 5. It can be noticed that the crack opening displacements just behind the crack trip were smaller in the case of the growing fatigue crack with prior loading history.

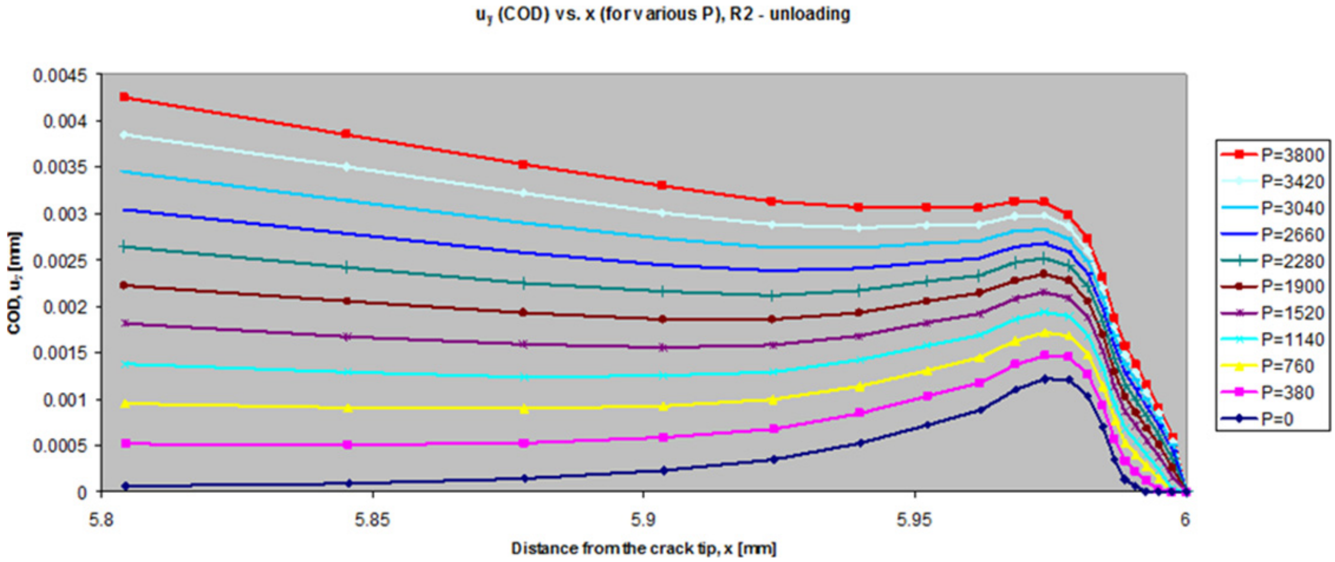

Figure 4: The opening displacement field behind the fatigue crack tip as a function of the applied load level.

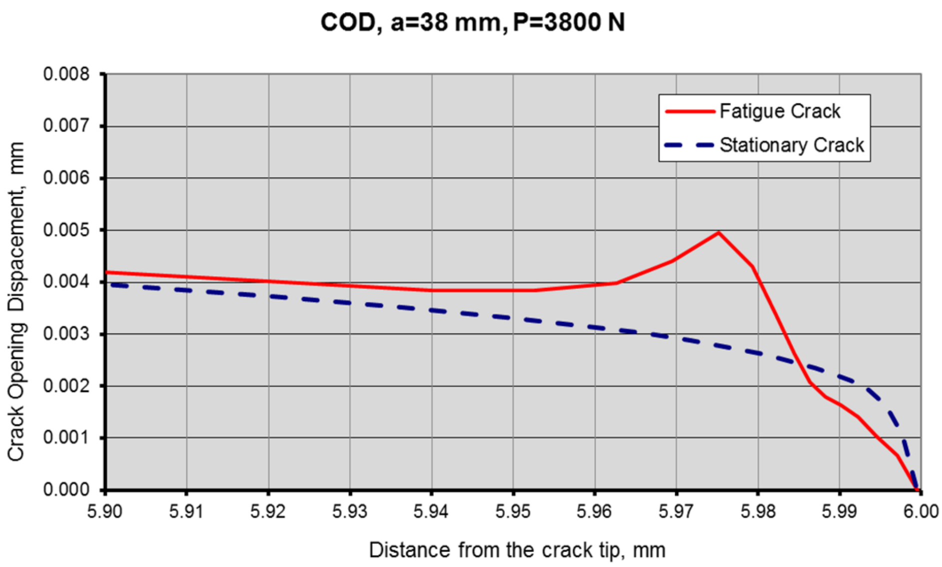

Figure 5: Comparison of crack opening displacement fields for the stationary and fatigue crack

Further away from the crack tip the crack opening displacements behind the fatigue crack tip were greater than those of the stationary crack. At large distances behind the crack tip both displacement fields were merging gradually. It means than even at the maximum load the crack tip opening displacement field near the fatigue crack was different than that one obtained for the stationary crack. It also means that the cyclic load history is influencing not only the effective minimum stress intensity factor, believed to be caused by the crack tip closure, but the effective maximum stress intensity factor is influenced as well. Because the crack surface contact at the minimum load was found to be negligible the majority of the load history effect is probably manifested by the change of the effectiveness of the maximum stress intensity factor. Therefore, it seems to be important to simultaneously analyse also the stress and strain response ahead of the crack tip.

THE STRESS-STRAIN RESPONSE AHEAD OF GROWING FATIGUE CRACK

The distributions of the stress component σ_{yy} normal to the crack plane are shown in Fig. 6. It is obvious that the cyclic load history had noticeable effect on the stress at both the maximum and the minimum load level when compared with the data obtained for the stationary crack. It was found that the decrease of the stress near the fatigue crack tip at the maximum load was greater than the change of the stress at the minimum load. The increase of the minimum stress, which might be argued to be caused by the crack tip closure, was in general only partially responsible for the decrease of the stress range at the crack tip and subsequently the decrease of the effective stress intensity range. It seems that omitting the effect of the load history on the maximum stress and strain at the crack tip induced by the maximum load level and by manipulating only the effective minimum load level might be insufficient for appropriate modelling of the load history effect.

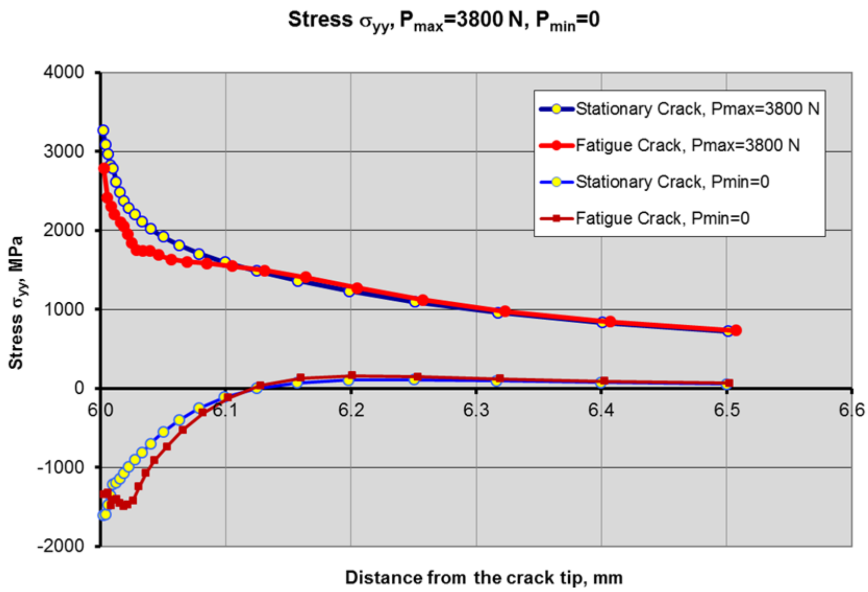

Figure 6: Comparison of stress distributions ahead of stationary and fatigue crack.

Therefore it was very informative to analyse also the stress-strain response ahead of a growing fatigue crack tip. In order to avoid ambiguities associated with the accuracy of highly deformed finite elements adjacent to the crack tip the 4th element away from the crack tip was chosen for extracting (Fig. 7) the local stress-strain response. However, even four elements away from the crack tip the strains were very high but it is felt that the qualitative differences between the fatigue and the stationary crack were correctly captured. It has been found that in the case of the fatigue crack the maximum stress and the stress and strain ranges were less than those obtained for the stationary crack while the difference between minimum stresses was insignificant. This confirms once again that the cyclic loading history influences significantly the crack tip stress-strain response at the maximum load and much less at the minimum load. It can also be concluded that the decrease, due to the load history effect, of the maximum crack tip stress with almost unaffected minimum stress, leads to simultaneous decrease of both the effective maximum stress and the effective stress and strain range ahead of the propagating crack tip.

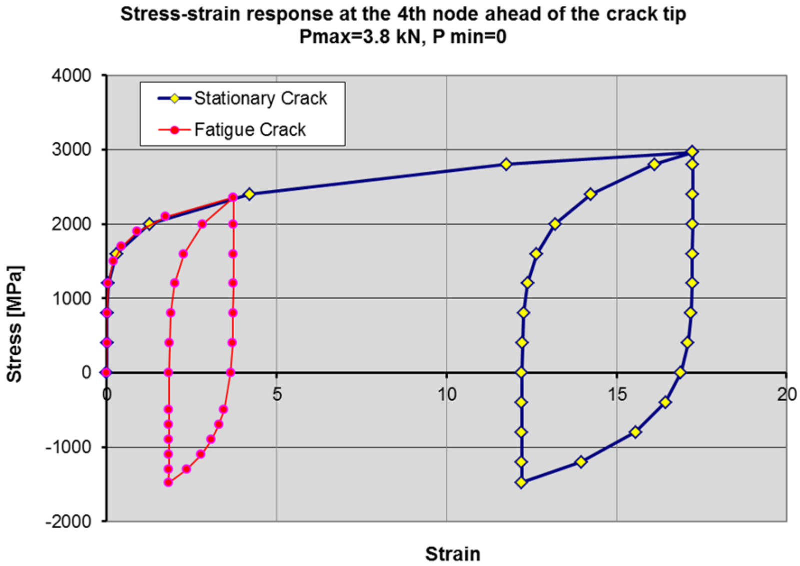

Figure 7: Cyclic stress-strain responses ahead of fatigue and stationary cracks.

EXPERIMENTAL STRAIN MEASUREMENTS AHEAD OF A FATIGUE CRACK TIP

The elastic strain components ahead and behind of the fatigue crack tip were measured with the help of the neutron diffraction method described in reference [3]. The strains measured before and after application of the overload were of particular interest. The distribution of only the strain component ε_{yy} normal to the crack plane is shown in Fig. 8.

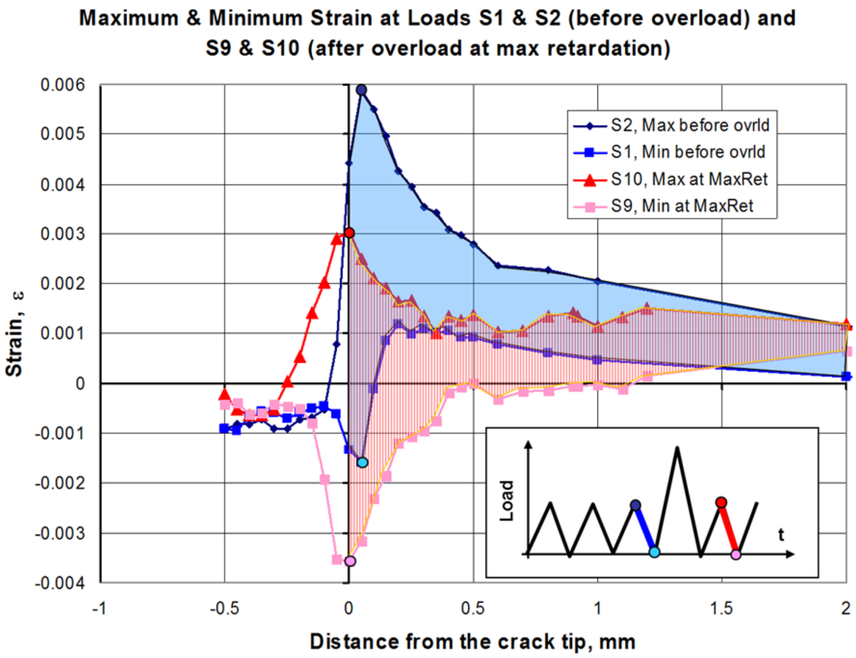

Figure 8. Maximum and minimum strain distributions ahead of growing fatigue cracks and the effect of single overload.

The overload has reduced both the maximum and minimum strain ahead of the crack tip. However, the reduction of the maximum strain after the overload was greater than the reduction of the minimum strain. Therefore application of the overload led to both reduction of the maximum strain and the strain range. Because strains and stresses near the crack tip are relatively high they lay on the flat part of the stress-strain curve and therefore significant changes in the strain magnitude result in relatively small changes of the stress. For these reasons the overload effect and the load history effects in general are more distinctly manifested in changes of crack tip strains and crack opening displacements rather than stresses. It has been also found that the overload had induced high level of stress tri-axiality ahead of the crack tip limiting significantly the possibility of plastic cyclic deformation. It is believed that the reduction of the plastic flow near the crack tip might also be responsible for the decrease of the post overload fatigue crack growth rate.

SUMMARY AND CONCLUSIONS

A numerical and experimental analysis of displacements, stresses and strains around the growing fatigue crack tip has been carried out. It has been found that the relatively large segment of the crack remained open even when the load was reduced to zero. The contact stresses between finite elements just behind the crack tip were found to be small and they were probably due to numerical inaccuracy. The analysis of the displacement field indicates that the past loading history of the growing crack was visibly manifested in changes of the displacement profile under both the maximum and minimum load. The stress distributions ahead of the fatigue crack indicate on the other hand that the prior loading history influences mostly (reducing) stresses at the maximum load with rather weak effect on stresses at the minimum load level.

It was also found that application of the overload had induced almost tri-axial/hydrostatic stress state ahead of the crack tip and reduced significantly the plastic flow. This effect was also manifested in reduction of the crack opening displacement range behind the crack tip.

The experimental strain measurements have revealed that application of the overload reduced both the effective maximum and minimum strain ahead of the crack tip. However the reduction of the maximum strain was higher than the reduction of the minimum strain therefore the strain range was also reduced due to the application of the overload.

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