FRACTURE BEHAVIOUR OF A SOLID WITH RANDOM POROSITY: EXPERIMENTAL ANALYSIS AND SIZE-EFFECTS

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An innovative experimental methodology has been developed to analyse the microstructural characteristics of concrete, responsible for many peculiar features of the fracture phenomenon. By means of a completely authomatised laser system, the three-dimensional morphologies of concrete can be digitised. Considering planar cross-sections of the virgin material, the pore and void distribution can be easily extracted from the laser-scanned topography. This procedure, which yields the effective depth and shape of the pores, permits to overcome the drawbacks and ambiguities of traditional image analysis techniques, where dark particles often confuse with pores. Calculating the fractal dimension by means of two different algorithms, allows us to confirm the lacunar fractal character of the stress-carrying section.

INTRODUCTION

In the study of continuous media, we are concerned with the manner in which forces are transmitted through the medium. The Cauchy definition of stress $[\sigma]$ relies on some "regularity" properties (continuity and measurability) of the medium. Euclidean measurability implies the scale independence of mechanical laws. Moreover, differentiability allows to write the equations of mechanics in terms of differential equations. If a macroscopic description of the stress field is desired, classical mechanics is sufficiently accurate. On the other hand, when singularities and inhomogeneities are present, or when the structure of the body plays a fundamental role in the definition of the physical properties, different approaches are required.

Defects are present at all scales in engineering materials and interact with each other in a complex manner. Attempts to describe these phenomena by classical methods are deemed to be incomplete. For example, size effects are not explicable in the classical framework. Different models should be considered in the presence of strain localization and large stress gradients (e.g. fracture problems). However, randomness alone cannot justify the self-organised complexity which comes into play in the

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fracture of concrete. On the contrary, the invariant features can be put into evidence in the framework of Fractal Geometry. Modelling the microstructure by means of fractal domains permits to capture the hierarchical aspect of damage accumulation and crack propagation. Carpinteri (1) assumed that the Hausdorff dimension of the stress-carrying domains in concrete were lower than 2.0 due to inherent flaws (voids, pores and cracks). Therefore, lacunar fractal domains (possessing fractal dimension Δ lower than the topologic dimension) can be used to model the stress-carrying cross sections in real materials.

The Menger sponge can be considered as a fractal model for a porous solid. It is shown at the third iteration in Fig. 1a, and its Hausdorff dimension is equal to $\Delta = \log 20/\log 3 = 2.73$. The sponge has zero volume and possesses very peculiar mass properties related to its non-compactness. In fact, if sponges of different linear size are compared, one notes that the nominal density ρ decreases with size according to a non-integer exponent equal to $\Delta - 3$. Thereby, the nominal density is not significant, because depends explicitly on the specimen size d (Fig. 1c). In particular, as d tends to infinite, ρ tends to zero (a lacunar fractal set is asymptotically a point set, with null euclidean measure). This is confirmed by natural sponges, where the larger the specimen size, the higher the probability of encountering a large void. It is worth to point out the profound difference existing between such a power-law scaling and a classical exponential decay law in the form $\rho(d) = e^{(-d/d_0)}$. In this last case, in fact, d_0 represents a characteristic length, whereas fractals are characterized by the absence of characteristic scales (Carpinteri & Chiaia (2)). Planar cross sections of the Menger sponge are Sierpinski carpets, whose iteration scheme is shown in Fig. 1b. This set presents zero area ($\Delta = \log 8/\log 3 = 1.893$) and can be considered as a lacunar cross-section inside a porous medium.

Because the Euclidean measure (length, area or volume) of lacunar sets is scale-dependent and tends to zero as the resolution increase, the Cauchy definition of stress cannot be applied. In fact, since the stress-carrying area progressively decreases with increasing the resolution of the observation, the local values of the stress are not scale-independent. The "regularity" properties of Euclidean sets are lost and are replaced by non-differentiability. On the other hand, self-similarity comes into play, providing a particular symmetry in the problem (dilatation symmetry). In the framework of Fractal Geometry, finite measures can be performed by using lengths raised to Δ. Accordingly, the stress concept needs to be revised and scaling laws must be included.

An original definition of the fractal stress acting upon lacunar domains has been put forward by Carpinteri (1). If the fractal measure d^{Δ} of the set is adopted, it is possible, by applying a renormalisation group transformation, to define a renormalised stress σ^* acting upon the fractal set: $[\sigma^*] = [F][L]^{-\Delta}$. From the mechanical point of view, this anomalous definition of the stress flux permits to explain the scale effects undergone by the tensile strength of heterogeneous materials. It can be easily proven that, according to the power-law scaling of the stress-carrying domain, the nominal strength undergoes a negative size-effect characterized by a fractional exponent, in the $\log \sigma_u - \log d$ diagram, related to the fractal dimension Δ (Fig. 1d).

In this paper, several concrete cross sections, digitised before the loading application, are analysed. The experimental procedure, which yields the effective depth and shape of the pores, permits to overcome the drawbacks and ambiguities of traditional image analysis techniques, where dark particles often confuse with pores. Several analyses have been carried out, and the fractal nature of the real stress-carrying domains has been investigated by means of three-dimensional numerical tools. It is worth to remind that, if load were applied, a larger damage would be present, and more rarefied sets would be obtained.

EXPERIMENTAL METHODOLOGY

The main purpose of the experimental methodology, entirely developed at Politecnico di Torino, is to digitize the three-dimensional topography of surfaces at the meso-scale. The surface heights measurement is performed by means of a laser profilometer, by counting the number of wave-cycles between the ray emission and the ray reception after the reflection on the specimen surface. The specimen to be analysed is rigidly framed into a solid truss, whereas the horizontal position of the distanziometer is controlled by two orthogonal micrometric step motors. The step motors interface and the data acquisition board that convert the analogical signal provided by the laser are both plugged in the same PC motherboard. A dedicated software provides extreme versatility and the full automation of the surface acquisition process. The digitised surfaces can extend over a 50mm $\times$ 100mm area, and a 2μm maximum precision can be achieved, both in vertical and horizontal direction.

In the study of the microstructural morphology of concrete it is useful to digitise planar cross sections obtained by cross-cutting undamaged specimens. These surfaces appear almost flat (Fig. 2a), with localised distribution of moon-like craters due to the intersection of the cutting plane with the inherent microstructural flaws. The presence of cavities is responsible for an effective resisting cross section that is less dense and compact than the nominal cross section. Furthermore, in real situations, the porosity is not uniform, and the relative percentage of voids depends on the linear size of the considered section.

The true stressed domain is made out of the points that do not belong to the craters, i.e. to the pore structure. Hence, from a theoretical point of view, the actual resisting section can be evaluated by considering the set of points whose heights are exactly equal to the cutting plane height. Practically, the obtained surface is not absolutely plane and presents a low uniform roughness due to the cutting process that can be confused with porosity. For this reason, another virtual plane has been considered, parallel to the cutting section, but at a lower height, which is able to intersect only the real cavities (Fig. 2b). The points whose height is greater than the virtual plane height, are considered to belong to the real stress-carrying domain, while the remaining points belong to the (complementary) void set (Fig. 2c). This procedure allows to filter out the noise produced by cutting. However, some information is lost about the finer porosity. To perform the virtual cut, it is also necessary to determine the mean real cutting plane by a detrending algorithm.

In Fig. 2d, the evolution of the fractal dimension of the effective cross section is shown as a function of the virtual plane height. While the calculation of the fractal dimension will be described in the next section, it is worth noting the rather sharp decrease of the dimension as far as the cutting noise comes into play. If one were able to quantify the effect of this noise, the virtual plane height would be unequivocally defined.

FRACTAL ANALYSIS

The fractal dimension of the effective stress-carrying domain has been calculated using two different algorithms.

Based on the concept of covering, the box-counting method estimates the fractal dimension as a function of the vanishing order of the covering area. The number of boxes N_i, needed to cover the set, is calculated for a decreasing value of the side d of the square covering element. The stress-carrying cross section is a self-similar lacunar fractal in a statistical sense. Then the following equation holds:

$$\Delta_{\mathrm{box}} = \lim_{d \to 0} \frac{\log N_i}{\log (1/d)}. \tag{1}$$

From the slope in the bilogarithmic diagram (Fig. 3a), the fractal dimension of the effective ligament, $\Delta_{\mathrm{box}} = 1.92$, can be calculated. As expected, Δ is lower than the Euclidean value (2.0).

The fractal dimension can be also evaluated by referring to the mass logarithmic density. If the effective cross section were characterised by a uniform distribution of cavities, it would be possible to calculate the density defined as the ratio between the effective area A_{eff} and the nominal area A_{nom}. In the actual case, this density can not be unambiguously calculated, because it depends on the resolution and on the size of the considered area. In fact, the complex distribution of the pores causes the probability of finding large cavities to be higher as the size of the considered area increases (like in a natural sponge, Mandelbrot (3)). The classical density is not constant, but decreases by increasing the nominal size. To obtain a scale-invariant value, it is necessary to refer to the logarithmic density, defined as:

$$\rho_{\mathrm{log}} = \frac{\log A_{\mathrm{eff}}}{\log A_{\mathrm{nom}}}. \tag{2}$$

If d is the linear size of the considered area, the fractal dimension Δ_{log} can be evaluated as the limit slope of the bilogarithmic diagram ($\log A_{\mathrm{eff}}$ versus $\log d$). In the case of Fig. 2c, the value $\Delta_{\mathrm{log}} = 1.92$ was determined, in good agreement with the box-counting method.

As a final remark, it is our opinion that this value is too high, even for an undamaged specimen. Improved cutting techniques, able to minimize the influence of noise, should reveal the presence of micro-porosity, which would lower drastically the density and compactness of the stress-carrying domains.

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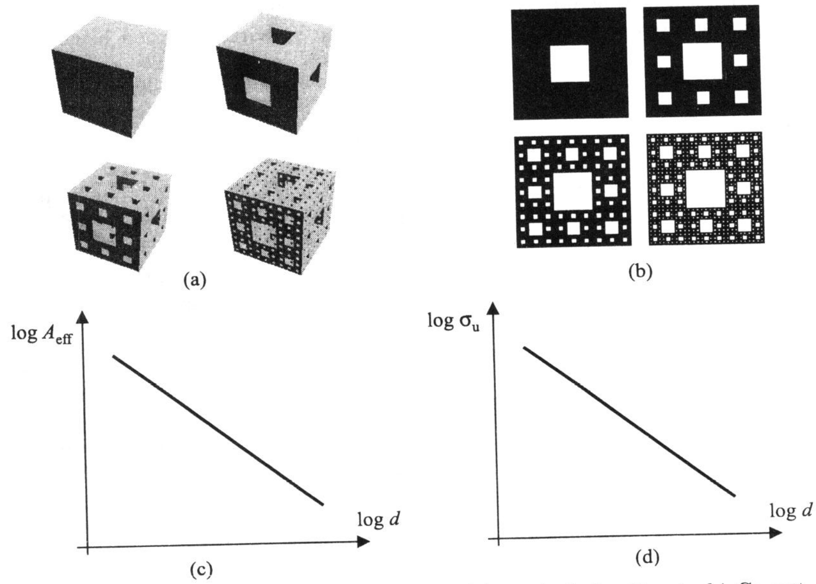

Figure 1 Generation of the Menger Sponge (a), and of the Sierpinski Carpet (b). Scale-dependence of the nominal density for lacunar sets (c), and of the nominal strength defined over the same sets (d).

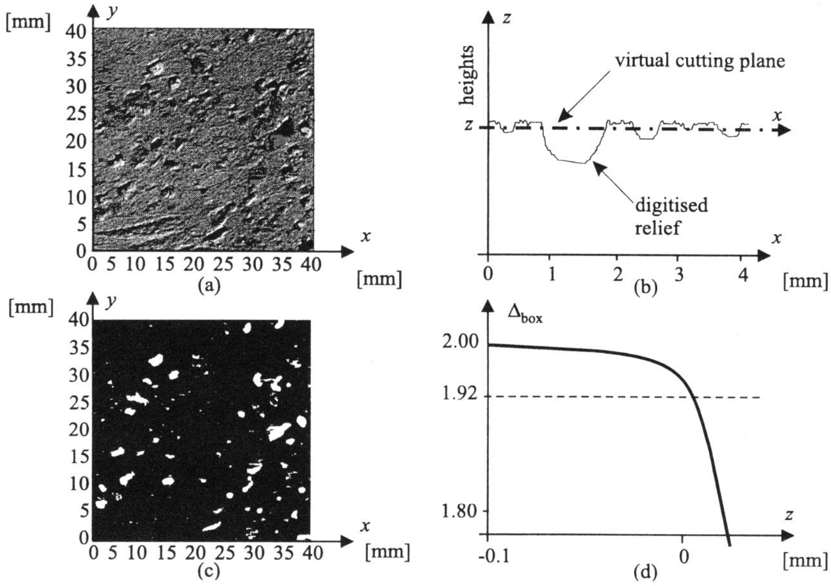

Figure 2 Digitized concrete surface (a). Scheme of the virtual section (b). Stress-carrying cross section (c). Fractal dimension vs. virtual section height (d).

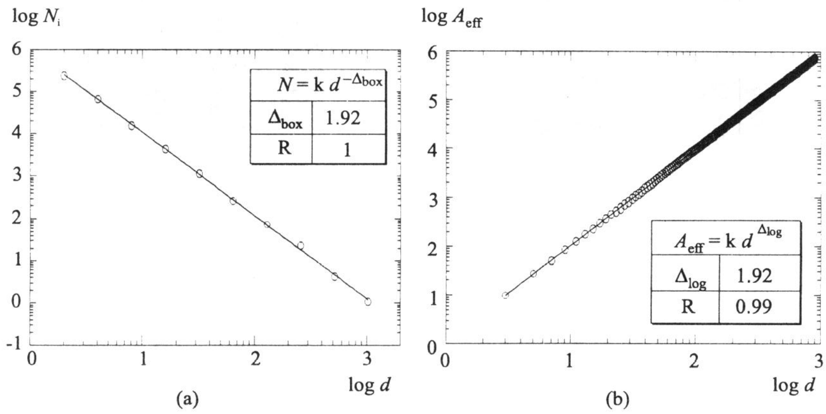

Figure 3 Box-counting method: number of covering boxes vs. resolution (a). Logarithmic density: effective area vs. resolution(b).