APPLICATION OF FRACTURE MECHANICS TO REINFORCED CONCRETE BEAMS UNDER BENDING

D. Sumarac* and Lj. Atanaskovic♦

In the present paper application of Fracture Mechanics to reinforced concrete beams, under bending, theoretically and experimentally is investigated. During the increasing loading (force) of a three point bend reinforced concrete beam, the crack reaches the reinforcement, passes it, and then the crack along the reinforcement develops, causing debonding. Simultaneously vertical crack continues to develop. This crack is propagating with the driving force at the crack tip due to externally applied load. At the same time the reinforcement behaves as elasto-plastic spring decreasing the stress intensity factor at the crack tip. Theoretical model allows calculation of collapse load, crack length and crack opening displacement. It is based on Carpinteri's approach in which the debonding is implemented. Experimental investigation is comprised of measurements of all theoretically obtained results. Special attention is focused on the determination of vertical crack size. This is achieved using graphite and optical fibers.

INTRODUCTION

The present paper focuses on the determination of ultimate loading of three point bend reinforced concrete (RC) beams using concept of fracture mechanics. It consists of two parts, theoretical and experimental. Theoretical model is based on Carpinteri's (1) approach, which is extended allowing the crack opening in the elastic domain. Using fracture mechanics parameter such is SIF (stress intensity factor), it is possible to determine the size of the crack, crack opening displacement, the force and the stress in the reinforcement. For the experimental measurements, several transducers were developed based on the optical and graphite sensors. Also the program written in Pascal was developed for communication with the PC for the sake of numerical and graphical acquisition of all measurements: crack length, crack opening displacement, vertical settlement of supports, deflection of the middle point of beam and intensity of force P. Measurements were done on two groups of three point bend specimens: four RC beams with mild steel reinforcement, and four RC beams with the reinforcement with ribs (high yield steel).

*Department of Civil Engineering, University of Belgrade
♦Ratko Mitrovic, Bulevar doo, Belgrade

THEORETICAL MODEL

Consider the specimen shown in Fig.1, which is three-point bend reinforced concrete (RC) beam with the initial notch at the middle. Due to increasing of loading the crack, emanating from the notch will propagate. Then the force in the reinforcement will depend of the level of loading or, in another words, of the crack opening displacement. The most important assumption in this model is that the debonding between reinforcement and concrete takes place. Debonding is a consequence of a development of horizontal (conical) crack propagation. The length of the debonding size is denoted as ℓ_0. Because of the presence of debonding it is possible to have crack opening displacement in the elastic domain. In this paragraph, starting from Fracture Mechanics procedure, force in the reinforcement, its stress, crack length, crack opening displacement as a function of intensity of force will be determined.

SIF at the crack tip for the three point bending beam with the vertical crack without reinforcement is, Tada et al (3):

$$K_I^P = \frac{6P\ell}{4bd^2}\sqrt{\pi a}F_P(\alpha), \tag{1}$$

where:

$$F_P(\alpha) = 1.107 - 2.120\alpha + 7.71\alpha^2 - 13.55\alpha^3 + 14.25\alpha^4, \tag{2}$$

and $\alpha = a/d$. SIF due to force of intensity X acting at the place of reinforcement is:

$$K_I^X = \frac{2X}{b\sqrt{\pi a}}F_X(\alpha,\beta), \tag{3}$$

where is:

$$F_X(\alpha,\beta) = \frac{3.52(1-(\beta/\alpha))}{(1-\alpha)^{3/2}} - \frac{4.35-5.28(\beta/\alpha)}{(1-\alpha)^{0.5}} + \frac{3.52(1-(\beta/\alpha))}{(1-\alpha)^{3/2}} +$$
$$+\left[0.83 - 1.76(\beta/\alpha) + (1.3 - 0.3(\beta/\alpha)^{3/2})\Big/\sqrt{1-(\beta/\alpha)^2}\right](1-\alpha+\beta) \tag{4}$$

and $\alpha = a/d$ and $\beta = c/d$. Once SIF are known it is possible to calculate crack opening displacement using well known procedure in Fracture mechanics. From the expressions (1) and (3), using (2) and (4), crack opening displacement at the place of reinforcement due to external force P will be:

$$\delta_{10} = \frac{6P\ell}{bdE_b}\int_\beta^\alpha F_P(\alpha)\cdot F_X(\alpha,\beta)d\alpha. \tag{5}$$

Coefficient of crack opening displacement at the place of reinforcement due to force of intensity $X=1$ acting at the same place from (3) and (4), is:

$$\delta_{11} = \frac{8}{\pi b E_b}\int_\beta^\alpha \frac{[F_X(\alpha,\beta)]^2}{\alpha}d\alpha. \tag{6}$$

Having expression (5) and (6), and designated, as before, with ℓ_0 the debonding length, it is possible to calculate the compatibility condition at the place of reinforcement. Compatibility condition means that the crack opening displacement given by (5) from which is subtracted expression (6), multiplied by X, should be equal to total elongation of debonded reinforcement of length ℓ_0 due to force X. In another words it reads:

$$\delta_{10} - X\delta_{11} = \frac{\ell_0 X}{F_a E_a}. \qquad (7)$$

Form the expression (7) it is possible to determine the unknown force in the reinforcement:

$$X = \frac{3P\ell\pi}{4d} \frac{FF(\alpha)}{FX(\alpha) + \dfrac{\pi b\ell_0 E_b}{8F_a E_a}}, \qquad (8)$$

where $FF(\alpha)$ is the integral which appears in the expression (5), and $FX(\alpha)$ the integral which is part of the expression (6). Once the force in the reinforcement X is known, it is possible to determine elongation at the place of reinforcement, which represents crack opening displacement:

$$\delta_A = \frac{X\ell_0}{F_a E_a}. \qquad (9)$$

Also if X is known, then the total SIF at the crack tip will be:

$$K = K_I^P - K_I^X, \qquad (10)$$

where SIF on the right hand side of above expression are given by equation (1) and (3). Equating expression (10) with the expression for K_{IR} :

$$K = K_I^R, \qquad (11)$$

it is possible to determine, for a given force P, the crack length a. For K_{IR} it is taken:

$$K_{IR} = 0.83(1 + 10\alpha^2) \qquad \left[MNm^{-3/2}\right]. \qquad (12)$$

In the above expression initial value (for $\alpha=0$) is $K_{IR}^0=0.83\ MNm^{-3/2}$ which is taken from the paper Hilsdorf and Brameshuber (2). The expression given by eq.(12) represents the crack growth resistance curve for given reinforced concrete beam. From the expression (8), (9), (11) and (12) it is possible to calculate, for a prescribed loading P, force and governing stress in the reinforcement, crack length and its opening. For those calculations it is assumed that, $\ell_0 = (a-a_0)/2$, where $a_o=2cm$ is the initial length of a notch.

The displacement of the middle point of a beam can be calculated as:

$$\Delta = \Delta_{WC} + \Delta_C - \Delta_R, \qquad (13)$$

where is:

$$\Delta_{WC} = \frac{P\ell^3}{48E_b I}, \qquad (14)$$

the displacement of the middle point of a beam without crack. In the above expression I=bd^3/12 is the moment of inertia of the cross section.

$$\Delta_C = \frac{9P\pi\ell^2}{2bd^2E_b}\int_0^\alpha \alpha[F_P(\alpha)]^2 d\alpha, \qquad (15)$$

is the displacement of the middle point of a beam due to presence of a crack. Finally,

$$\Delta_R = \frac{6X\ell}{bdE_b}\int_\alpha^\beta F_F(\alpha)F_X(\alpha,\beta)d\alpha, \qquad (16)$$

is the displacement of the middle point of a beam due to force of intensity X acting at the place of reinforcement.

EXPERIMENTAL INVESTIGATIONS

Experimental measurements were performed on three point bend specimens. Experiment is designed as the force controlled. The specimens are divided into two groups of four beams, with mild steel reinforcement and with the reinforcement wit ribs (high yield steel). Each beam had notch at the bottom with the length of 2cm. All beams were reinforced with the two bars with diameter of 10mm, in the tension zone, and with the two bars of diameter of 6mm in the compression zone. Concrete mix was made of two-size particle with the maximum diameter of 8mm and with the compressive strength of 30MPa. The beams were tested at an age of 28 days.

For the sake of better determination of crack front position, the graphite and optical fibbers are placed along the height of a beam, Sumarac et al (4). Three graphite elements of 3mm diameter were placed perpendicular to the crack plane. The electrical resistance of graphite elements was up to 2Ω. At the moment when the crack passes the position of graphite, it will be broken and electrical resistance will jump to infinity. Then it is sure sign that the crack is of the size, which matches the position of graphite. At the same place where the graphite element is placed, the optical fiber with UV acrylate cure protection is put. The core diameter of the fiber was 0.0625mm. The principle of measurements is similar. In the case of optical fiber, the exchange of intensity of light was recorded. To make the optical fiber to be brittle, the protection coating was removed on the length were the crack was expected. At the moment of breaking the optical fiber could not transmit the intensity of light.

Besides the size of the crack, the measurements of vertical deflection of the middle point of beam, and settlement of supports were measured using inductive displacement sensors. Also crack opening displacement and intensity of force were measured automatically and saved on PC. For this purpose the program written in Pascal was developed for communication with PC for the sake of numerical and graphical acquisition of all measurements.

In Fig.2 force-displacement of the middle point of a beam diagrams are plotted. Curves with the solid lines are representing experimentally obtained results on four RC beams with the high yield reinforcement. Dashed line curve is representing theoretically obtained results from the expression (13). From the diagrams it is evident that the ultimate loading is about P=36kN. According to some codes (Yugoslav and Russian) the ultimate loading for this beam is of lover value. This fact is the reason why Fracture Mechanics concept is needed to be incorporated in the regulations for design of reinforced concrete beams.

SYMBOLS USED

K_I = stress intensity factor for mod I ($MNm^{-3/2}$)

X = force in the reinforcement (kN)

a = crack length (cm)

P = applied force (loading) (kN)

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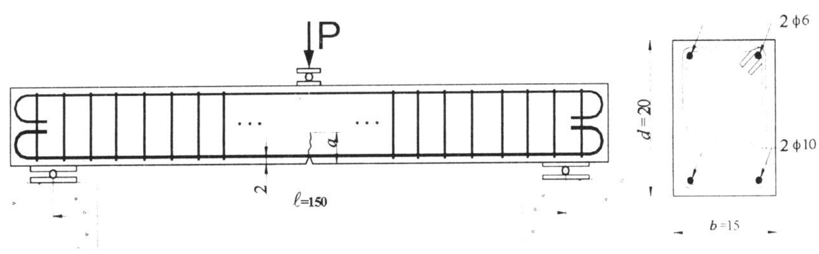

Fig.1. Three-point bend reinforced concrete beam specimen: loading and geometry

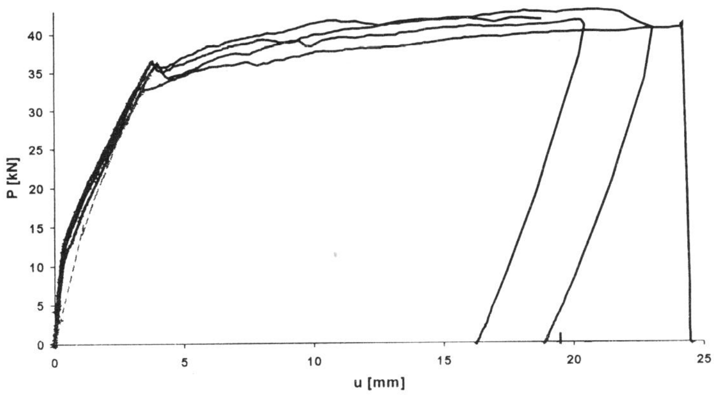

Fig.2 Theoretical (dashed line) and experimental (solid lines) results