THE SCATTER OF STRENGTH IN COMPOSITE MATERIALS ARISING FROM THEIR STRUCTURAL NONUNIFORMITY

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The way of transforming discrete structural models of particulate composites into adequate continuous representations is offered. The unit cell as a discrete basis has been used for the development of a nonuniform continuous medium capable to simulate microdamage appearance and evolution. The transformation of the discrete model into the continuous one has been accomplished through the utilisation of the finite element procedure. The approach under consideration describes the complete life cycle of the material without recourse to the strength criteria and allows one to estimate not only ultimate properties but also their scatter.

INTRODUCTION

Continuous constitutive relations in themselves are not capable to predict place and moment of material failure. In practice, when strength evaluation is to be done, one commonly enlists the so called strength criteria, which are usually regarded as some independent (additional) characteristics of the material behaviour. In so doing, one, nevertheless, always feels that failure of a material is closely related to the preceding history of its loading and isolation of strength as some particular quantity looks unrealistic. A development of a unified description, where failure would be incorporated into the whole of the material mechanical behaviour as a natural event crowning its life-cycle, seems to be tempting. Previous attempts in this direction proved to be promising (1). Structural approach applied to particulate composites turned out to be a good basis for the derivation of the integral description of the life-cycle in terms of continuum mechanics. Moreover, this procedure has allowed one not only to calculate ultimate strains and stresses but also has opened a way to theoretical prediction of their scatter.

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THEORETICAL BACKGROUND

Unit cell geometry

A cylindrical elastomeric cell with a diameter equal to a height having a rigid spherical inclusion (filler particle) was selected (Moshev and Kozhevnikova (2)) as a representative element of a particulate composite system incorporated into a closely packed ensemble (Figure 1). In extension along the cylindrical axes, the restrictions, laid on every cell by surrounding, make its ends remain plane and its lateral surface remain cylindrical. This honeycomb geometry is characterized by a remarkable feature. The calculation of the cell's effective modulus as a function of the filler volume fraction for cells keeping the bonded state demonstrated that the calculated modulus-concentration dependence was close to the experimental observations over a wide range of filler concentrations (2). These results demonstrated a rather high predictive ability of cylindrical cells as constitutive elements controlling the effective behavior of particulate polymeric composites.

Constituent elements properties

All the calculations were carried out under assumption of a purely elastic matrix with no accounting for time-dependent processes. The matrix was represented by an elastomer following the neo-Hookean behavior with a low-strain shear modulus G. The inclusion was taken as perfectly rigid body. Griffith's approach was used in describing matrix separation process as it is done by Gent (3) and Gent and Tobias (4). Sliding friction between matrix and inclusion in contact debonded zones was ignored.

Effective tensile behaviour of cells

The finite element method described by Kozhevnikova et al (5) was used for calculations in the domain of large deformations.

The general appearance of the tensile curve for a cell under consideration is shown in Fig. 2. Its evolution passes through three stages. The initial stage specifies the resistance of the bonded cell. The middle one describes the progressive matrix separation from the inclusion accompanied by the stress softening. The last stage presents the resistance of the completely debonded cell until its failure. It follows from Fig. 2 that describing such tensile curves requires establishing at least five basic material parameters: the initial rigidity, g_1; the strain, e_a, at which the matrix adhesive debonds begins; the energy of debond T_d ; the final rigidity, g_2 ; and the breaking strain, e_b, of the cell.

Solid volume fraction, φ, and matrix properties (shear modulus, G; energy of the debond T_d;, breaking strain, ε_{mb}) are the basic structural features defining the shapes of tensile curves of cells through the above mentioned five parameters.

A number of boundary value problems have been solved for various combinations of φ, G, T_d and ε_{mb} to establish the corresponding magnitudes for g_1, e_a, g_2 and e_b with further transforming the results obtained into continuous constitutive relations having the form

$$\sigma_{ij} = 2\,G_c\,(\varepsilon_1\,,\,\sigma_0\,)\,(\varepsilon_{ij}\ -\theta\,/\,3\,)\,+\,\delta_{ij}\,\sigma_0\ ,$$
$$\theta\,=\,f\,(\varepsilon_1\,,\,\sigma_0\,).$$

Here, G_c is the effective shear modulus of the cell; θ is the effective volume compressibility of the cell; G_c and θ are functions of the current of the maximum main strain, ε_1, and the current mean stress, σ_0.

This approach describes the complete life-cycle of the cell.

Composite structural inhomogeneity

There exist at least two main sources of structural inhomogeneity: the geometrical local nonuniformity, contrasting with the ideal honeycomb structure of Fig. 1, and physical-chemical nonuniformity revealing in the debond, e_a, and breaking, ε_{mb}, strains scatter.

To get insight into the first source of nonhomogeneity, the synthesis of the random geometrical structures consisting of identical spherical particles with the imposed mean filler volume fraction has been developed (6). The local matrix volumes were estimated by equating the mean gap between a given sphere and adjacent neighbors to the gap in the equivalent model cell. This approach allows one to get the distributions of the local solid volume fractions for various imposed mean solid volume fractions. An analytical representation of these distributions has the general form

$$F(\varphi\,) = 1\,-\,exp\,(-\alpha\,\varphi^{m}\ )\,,$$

where $F(\varphi)$ is the probability to meet φ smaller that the indicated one. The random character of the local φ values requires the randomization of the appropriate cell parameters g_1, e_a, g_2 *and* e_b.

No data are known on the second kind of nonuniformity. This one is not taken into account.

Computation procedure

Replacing the discrete representation of cells behaviour by the continuous one and accounting for the local cell nonhomogeneity allows one to examine particulate composites as piecewise continuous systems. The finite element potentialities seem to be most appropriate for computation of mechanical behaviour of such bodies.

Evidently, it is impossible to compose structure, where one finite elements represents the behaviour of one structural cell. It would require accounting for thousands, possibly millions, finite elements that is beyond the possibilities of modern machinery.

An implementation of the intermediate averaging procedure becomes inevitable. It can be performed through increasing the number of cells within one finite element. So the number of the finite elements in the volume of the body can be reduced to a reasonable level. Obviously the mechanical variability of finite elements becomes less than that of individual cells and it is to be taken into account in calculations.

Numerical experiments have demonstrated that, in extension, the structural nonuniformity of the system provokes an appearance and progressive evolution of a large-scale nonuniformity leading to the loss of the elastic longitudinal stability and macrocrack origination in the most compliant part of the body. In this approach, the description of the failure does not need to be referred to strength criteria, crack formation being inherent property of the system..

RESULTS OF NUMERICAL EXPERIMENTS

To verify the potentiality of the approach under study, tensile curves have been estimated in plane strain for composites using a matrix with G =1 MPa and ε_{mb} = 200% as the base. Two filler concentrations (20 and 40%) and two particle diameters (50 and 150 μm) have been tested for specimens 10 mm wide and 70 mm long. The corresponding tensile curves (means of 9 repeated experiments) are shown in Figure 3 and the data on the mean breaking stresses and strains and their mean scatters are summarised below

Filler content, %	20	20	40	40
Particle diameter, μm	50	150	50	150
Mean breaking strain, %	79.0	85.8	18.6	20.3
Relative scatter of the breaking strain, %	2.8	3.2	8.1	9.1
Mean breaking stress, MPa	14.7	14.4	7.8	8.2
Relative scatter of the breaking stress, %	.0	1.1	3.8	17.0

DISCUSSION AND CONCLUSIONS

The topic of this paper is focused on a special point of the theoretical calculation of the scatter of ultimate stresses and strains in tensile tests of particulate elastomeric composites. It is demonstrated that calculations based on the model accounting for real structural nonhomogeneity, apparently, provide the background for reaching this goal. Figure 3 and the above numerical data corroborate well strong influence of the filler concentration on the values of ultimate stresses and strains known from experience. The increase in filler content from 0.2 to 0.4 provokes considerable drop of ultimate characteristics accompanied by marked rise in the scatter of these magnitudes, whereas change in the size of particles from 150 to 50 μm does not affect appreciably their values.

Modelling structural inhomogeneity seems to be the only way in the search of constitutive relations capable of more adequate presentation material mechanical behaviour involving failure phenomenon. Theoretical prediction of the strength and strength scatter is a good illustration of the capability of this direction.

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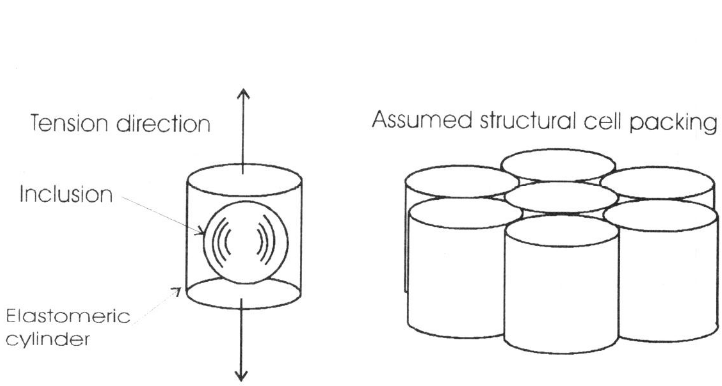

a b

Figure 1 Schemes of the unit cell and the cell parking

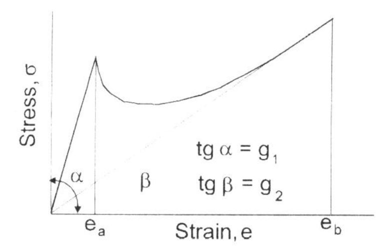

Figure 2 Typical tensile curve for unit cell

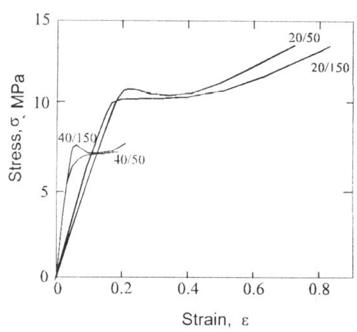

Figure 3 Tensile curves for various filler loading and particle diameters