MICRO-MECHANICAL MODELS OF STABLE DAMAGE EVOLUTION ON POST-CRITICAL DEFORMATION STAGE AND FAILURE OF COMPOSITE MATERIALS

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The primary objective of the research is to develop two level structurally-phenomenological model for the description of the main mechanisms of stochastic damage evolution of the granular composite under quasi-static loading and failure as a result of loss of inelastic deformation stability on the post-critical deformation stage. Ceramics, rocks and matrix of modern carbon-carbon composites could be refer to the indicated type of material. This model allows to register effects of strain and damage localization; partial unloading under proportional displacement-controlled loading, unequal resistance to failure, "quantum" self-supported and self-similarity damage evolution. The existence of the critical values of the normalized radius of locality for the random micro-strain fields and damaged domains that do not depend on the stress-strain conditions is provided and the universal statistical criteria of the post-critical deformation are presented.

INTRODUCTION

Heterogeneous solids with evolutionary damage structure demonstrate stable post-critical deformation that appears as the descending branch in the stress-strain diagrams under loading. This type of inelastic behaviour could be carried out only for a local domain as a component of a mechanical system with the necessary properties. Otherwise unstable failure is observed. The main statements of the stable post-critical deformation within the framework of models of the heterogeneous solid mechanics were considered in references (1–4). The supplementary indication of the post-critical behaviour to the Ilyushin's postulate of plasticity, extended postulate of stability on the base of Drucker's approach to the deformable and loading systems; local and integral sufficient condition of the inelastic deformation stability, the generalized variation principles and the unique solution theorem of the boundary-value problem for the deformable body with work-softening and damaged domains were formulated and provided in the monograph (1). The combination of the developed structurally-phenomenological model and the stochastic description of damage evolution allows to research the damage stages of the composite under non-random loading (3). We will be limited by the correlation theory of stochastic functions for the assessment of the character of the average interactions of the micro-strain fields and damaged structural elements.

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STRUCTURALLY-PHENOMENOLOGICAL MODEL

We will use the existence conception of various scale levels of damage accumulation as a basis of the structurally-phenomenological model for the description of the mechanical behaviour of the composite bodies under complete triaxial loading and suppose the validity of the classic mechanics relations at these levels. Inelastic deformation of the composite body V with a closed surface Σ are considered as milti-scale and multi-stage structural damage evolution and described by the 4th rank damage tensor Ω and the following tensor-linear constitution equations (2):

$$\sigma(\mathbf{r}) = \mathbf{C}(\mathbf{r})\left[\hat{\mathbf{I}} - \Omega\left(j_\varepsilon^{(1)},\ldots,j_\varepsilon^{(n)}\right)\right]\varepsilon(\mathbf{r}), \quad \forall \mathbf{r} \in V, \qquad (1)$$

where σ and ε are the stress and strain tensors; $\mathbf{C}$ and $\hat{\mathbf{I}}$ are the elastic modulus tensor and the 4th rank unit tensor correspondingly; $j_\varepsilon^{(\circ)}$ are the independent special invariants of the strain tensor and n is the number of invariants. The structure of the tensor Ω and the number of independent components of it depend on the anisotropy type of material. Possible variants of the constitution equations for the anisotropic bodies with evolutionary damage structure are presented in the monograph (1).

The observable difference in the engineering practice of the character of damage accumulation in the hydraulical and pneumatical high-pressure vessels or pipe-lines can not always be explained only by the heterogeneous structure. Special experimental researches confirm that the resistance to failure also depends on the loading system rigidity. The authors (1,3) defined the set of solid, liquid and/or gaseous media, whose deformation takes place as the result of loading transmission to a deformable body or to a separate part of it as the loading system, characteristics of which are taken into account by the boundary conditions of the third kind (4):

$$[\sigma(\mathbf{r})\cdot\mathbf{n}(\mathbf{r}) + \mathbf{R}\cdot\mathbf{u}(\mathbf{r})]|_{\Sigma_U} = \mathbf{S}^0(\mathbf{r}), \quad [\mathbf{u}(\mathbf{r}) + \mathbf{Q}\cdot(\sigma(\mathbf{r})\cdot\mathbf{n}(\mathbf{r}))]|_{\Sigma_S} = \mathbf{u}^0(\mathbf{r}), \qquad (2)$$

$$\forall \mathbf{a} \neq \mathbf{0}:\ \mathbf{a}\cdot\mathbf{R}\cdot\mathbf{a} \geq 0,\ \mathbf{a}\cdot\mathbf{Q}\cdot\mathbf{a} \geq 0,\ \mathbf{S}^0 = \mathbf{R}\cdot\mathbf{u}^0,\ \mathbf{u}^0 = \mathbf{Q}\cdot\mathbf{S}^0,\ \mathbf{R}\cdot\mathbf{Q} = \mathbf{I},\ \forall \mathbf{r} \in \Sigma,$$

where $\mathbf{u}$, $\mathbf{S}$ and $\mathbf{n}$ are the displacement, external force and normal vectors; $\mathbf{R}(\mathbf{u},\mathbf{r})$ and $\mathbf{Q}(\mathbf{S},\mathbf{r})$ are the tensors of the loading system rigidity and compliance; $\mathbf{I}$ is the unit 2nd rank tensor. The vectors of $\mathbf{S}^0$ and $\mathbf{u}^0$ are given on the parts of $\Sigma_S \subset \Sigma$ and/or $\Sigma_U = \Sigma \setminus \Sigma_S$ of the body's surface. The presented boundary conditions are mutually reciprocal and so only one of them could be used for the whole closed surface Σ. In the cases of $\mathbf{R} \equiv \mathbf{0}$ or $\mathbf{Q} \equiv \mathbf{0}$ the eqs.(2) correspond to the stress-controlled or displacement-controlled loading.

Traditional failure criteria do not contain the loading system characteristics and correspond to $\mathbf{R} \equiv \mathbf{0}$. Therefore, these criteria could be used for the determination of the critical state of a material necessary condition of which is the loss of stability of work-softening behaviour. In spite of the fact that some criteria have real physical interpretation the phenomenological approach to failure analysis of the heterogeneous media does not describ the mechanisms of damage evolution that leads to the loss of loading capacity. The usage of the set of failure criteria allows to accept different types of failure (1,4).

The invariant measures of the damage tensor $M_m(\Omega)$ could be used for the construction of the work-softening criteria of anisotropic media. If

$$\exists \Omega_m^*:\quad M_m(\Omega) \geq \Omega_m^*, \quad m = 1, 2, \ldots \leq n, \qquad (3)$$

where Ω^*_m are the constants due to the critical damaged state and n is the number of independent components of Ω, then m-type damage act takes place. The fulfilment of each of eqs.(3) leads to the moduli degradation according to the real failure mechanisms.

Let $\mathbf{S}=\mathbf{V}-\mathbf{D}$ be determined as the 4th rank tensor of the comparative loading system stiffness. Then the stability condition of post-critical deformation is equivalent to the requirement of positively definition of the following quadric quantic (4):

$$\delta\tilde{\varepsilon}(\mathbf{r})\cdot\mathbf{S}(\mathbf{r})\cdot\delta\tilde{\varepsilon}(\mathbf{r})>0 \qquad (5)$$

for the elementary material particle, where $\delta\tilde{\varepsilon}$ are the virtual increments of the post-critical strains; $\mathbf{V}$ and $\mathbf{D}$ are the tensors of the loading system stiffness and of tangent moduli at the work-softening deformation stage correspondingly.

Inelastic deformation, work-softening behaviour and failure of heterogeneous medium under quasi-static loading as a united process could be described by the boundary-value problem that consists of the equilibrium and Cauchy's equations

$$\nabla\cdot\sigma(\mathbf{r})=0,\quad \varepsilon(\mathbf{r})=1/2[\nabla\mathbf{u}(\mathbf{r})+\mathbf{u}(\mathbf{r})\nabla],\quad \forall\mathbf{r}\in V,$$

constitution equations (1), work-softening or structural damage criteria (4), stability conditions (5) and boundary conditions (2).

Equilibrium damage accumulation occurs only in the special conditions defined by the loading system rigidity the sufficient condition of which can be put out as:

$$\int_{\Omega}\mathrm{tr}[\delta\sigma(\mathbf{r})\cdot\delta\varepsilon(\mathbf{r})]d\Omega+\int_{\Sigma}\delta\mathbf{u}(\mathbf{r})\cdot\mathbf{R}(\mathbf{u},\mathbf{r})\cdot\delta\mathbf{u}(\mathbf{r})d\Sigma>0 \qquad (6)$$

for the virtual increments of the strains $\delta\varepsilon$ and displacements $\delta\mathbf{u}$ providing $\delta\mathbf{u}^0=\mathbf{0}$ as it is noted in the monograph (1). Damage accumulation is the mechanism of elastic energy dissipation. Therefore, invalidity of eq.(6) results in disappearance of the dissipative reserves of the damaged heterogeneous medium and in the unstable composite failure.

STRUCTURAL DAMAGE MECHANISMS

As an example let us consider pseudo-plastic behaviour (inelastic deformation as the result of the damage accumulation) of the granular composite the representative volume of which fills a unit cub and contains more than 4000 homogeneous tetrahedron isotropic elasto-brittle structural elements. Each structural element can be damaged by shear and hydrostatic tension loading according to stress-strain state. Weibull distribution parameters of the random failure strength constants and the values of determinate elastic moduli are presented in the article (5). We will define the observable dependence of the post-critical behaviour on the type of loading as unequal resistance to failure. This type of deformation could be due to damage acts which are accompanied by stress redistribution and local elastic unloading resulting in partial recover of capacity of the damaged domains to resist only to hydrostatic compressible loading.

The inelastic behaviour of the composites at the work-hardening stage and the values of the limits of strength do not depend on the loading system properties. However the character of the structural damage evolution at the work-softening stage, the failure strains and the volumetric fraction of damaged domains are defined by the loading system rigidity (3). The soft loading system accumulates mechanical energy and supports the damage accumulation that at the certain level of external loading accepts non-equilibrium spontaneous character and results in macro-failure in any point of the descending branches if eq.(6) is not carried out. Partially limiting of energy inflow is possible if the loading system is rather hard. Thus, the displacement-controlled regime we will used for research of the main mechanisms of the structural damage evolution.

The fragment of the stress-strain diagram $J_\sigma^{(2)} \sim J_\varepsilon^{(2)}$ in the invariant form (where $J_\sigma^{(2)} = \sqrt{\mathrm{tr}(\tilde{\sigma}\cdot\tilde{\sigma})}$, $J_\varepsilon^{(2)} = \sqrt{\mathrm{tr}(\tilde{\varepsilon}\cdot\tilde{\varepsilon})}$; $\tilde{\sigma}$ and $\tilde{\varepsilon}$ are the deviator components of the macro-stress σ^* and macro-strain ε^* tensors that are got by means of averaging of the corresponding structural characteristics) and the normalized correlation functions of the composite structure $\tilde{K}_\lambda^{(2)}(\Delta\mathbf{r})$ are shown in Fig. 1 for the various equilibrium states ($\tilde{K}_\varepsilon^{(2)}(\Delta\mathbf{r})$ are those of the micro-strain fields which do not significantly differ from the presented relations). At the beginning of loading damage accumulation occurs uniformly in the whole of the representative composite volume. The functions of $\tilde{K}_\lambda^{(2)}$ are close-to-exponental and rapidly damping. However, the presence of weak periodicity of the functions $\tilde{K}_\varepsilon^{(2)}$ is connected with partial elastic unloading and as a consequence with the occurrence of the structural elements the strains of which are less than the composite macro-strains. Further increasing of external forces leads to local and global strain and damage localization. Work-softening behaviour is accompanied by the uniform accumulation of localized damage centres (secondary dispersible damage stage). Proceeding of the loading effects to modification of the structural damage mechanisms connected with the beginning of stable formation of a failure cluster. This stage is completed by composite macro-failure. The functions of $\tilde{K}_\lambda^{(2)}$ and $\tilde{K}_\varepsilon^{(2)}$ obtained at the stages of primary and secondary dispersible accumulation differ only by the value of $|\Delta\mathbf{r}|$. This phenomenon confirms self-similarity of the damage evolution at the various composite scale structural levels.

ON “QUANTUM” SELF-SUPPORTED DAMAGE ACCUMULATION

In the composite structure local domains are detected the non-equilibrium damage of which is not affected neither by the averaging size of structural elements $\tilde{k}$ nor by increasing of loading steps. Similar local unstable “quantum” energy dissipation proceeding without increase of external loading level has the form of separate more or less expanded slopes in the stress-strain diagrams (Fig. 1). These slopes are characteristic of the materials having the tendency to self-supported damage accumulation both at work-hardening and work-softening deformation stages. This effect occurs if the release of the elastic energy accompanying the damage act is greater than its dissipation. Therefore we have not succeeded completely in the controlling damage character even in the displacement-controlled monotonous loading regime at the structural level.

STATISTICAL CRITERION OF POST-CRITICAL DEFORMATION

Let us consider the integral length scale of statistically homogeneous and statistically isotropic random field $\xi(\mathbf{r})$ as the following:

$$R^\xi = \int_0^\infty \left|\tilde{K}_\xi^{(2)}(\eta)\right| d\eta, \quad \eta = |\Delta\mathbf{r}| \quad \text{......} \quad (6)$$

that is equivocally determined by $\widetilde{K}_{\xi}^{(2)}(\Delta\mathbf{r})$ and equivalent to the size order with a distance of non-zero correlation between the values of $\xi(\mathbf{r})$ at the points of $\mathbf{r}$ and $\mathbf{r}+\Delta\mathbf{r}$. We will define this integral length scale as the normalized radius of locality. The character of the non-dimensional relations $R^{\lambda}/\widetilde{k} \sim J_{\varepsilon}^{(2)}$ qualitatively and quantitatively coincides with that of $R^{\varepsilon}/\widetilde{k} \sim J_{\varepsilon}^{(2)}$ for the various proportional macro-deformation regimes (Fig. 2), where $\widetilde{k}$ is an averaging size of structural elements.

The small decrease of the resistance to failure at the initial work-softening stage is accompanied by decreasing of $R^{\varepsilon}/\widetilde{k}$. The change of the mechanism and the scale level of the damage evolution are connected with general damage localization. The macro-strains of the composite are defined by the micro-strains of the damaged structural elements. The decrease of a volumetric fraction of the domains with alternating in respect to composite representative volume averaging strains leads to monotonous growth of R^{ε}. The appropriate to the limit of strength non-dimensional normalized radii of locality are marked by the points of A_i that are considerably distinguished from each other by the value of $J_{\varepsilon}^{(2)}$ owing to the unequal resistance to failure of the damaged composite. However, practical coincidence of the corresponding R^{ε} allows to suppose the existence of the critical values R_{cr}^{λ} and R_{cr}^{τ} of the random fields $\lambda(\mathbf{r})$ and $\varepsilon(\mathbf{r})$ that do not depend on the stress-strain conditions. These critical values are the material constants and so the following conditions

$$R^{\lambda} \geq R_{\text{cr}}^{\lambda} \quad \text{or} \quad R^{\tau} \geq R_{\text{cr}}^{\tau}$$

could be considered as universal statistical criteria of the work-softening behaviour.

CONCLUSION

The constitution equations, stability and damage criteria and a new formulation of the boundary conditions are given. The normalized radius of locality establishes equivocal quantitative connection between the damaged structure and the character of mechanical behaviour of the heterogeneous medium. This value is a sensitive parameter of the deformation describing evolution of the micro-strain fields and structural damage of the granular composite under loading.

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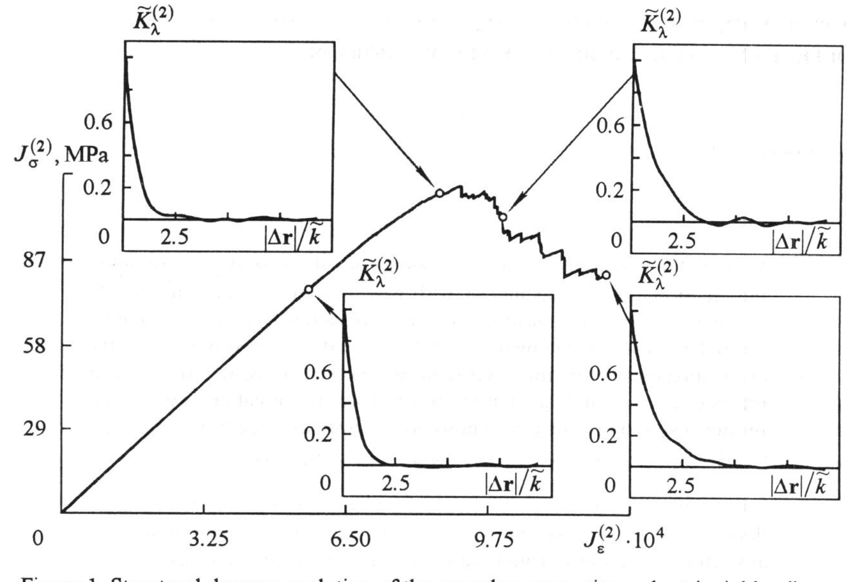

Figure 1. Structural damage evolution of the granular composite under triaxial loading ($\varepsilon_{11}^* = \varepsilon_{22}^* = -0.5\varepsilon_{33}^*,\ \varepsilon_{33}^* > 0$)

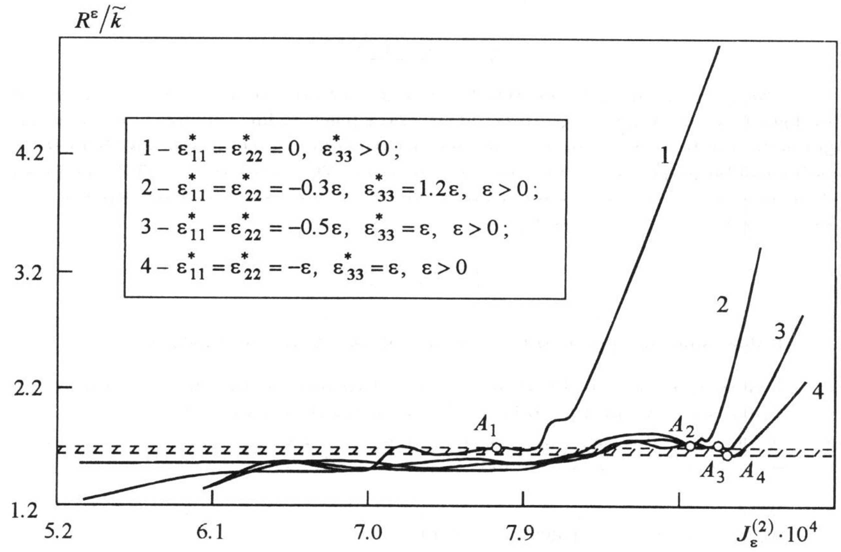

Figure 2. The normalized radius of locality as a function of the quadratic invariant of the macro-strain tensor