ON TWO APPROACHES TO COMPRESSIVE FRACTURE PROBLEMS

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Substantiation of the continuum theory of fracture under compression for the layered composites with periodic structure is considered. Basing on the previous results obtained by the author within the scope of three-dimensional linearised theory of deformable bodies stability asymptotic accuracy of the continuum theory of plastic fracture is being proved. The particular mode of stability loss, which corresponds to the continuum approximation is determined within the model of piecewise-homogeneous medium. The investigation is carried out within the scope of small precritical deformations theory for three-dimensional (non-axisymmetrical and axisymmetrical) as well as for plane problems and plane problems and is illustrated by numerical results for the particular types of metal matrix composites.

INTRODUCTION

There are two different approaches to description of phenomena in mechanics of composites. One of them is based on the model of piecewise-homogeneous medium, when behaviour of each component of material is described by three-dimensional equations of solid mechanics provided certain boundary conditions are satisfied at the interfaces. This approach enables to investigate in the most rigorous way phenomena in the composite microstructure. However, due to the complexity its application is restricted to a very small group of problems. The other approach, or continuum theory, involves significant simplifications. Within the continuum theory a composite is simulated by homogeneous anisotropic material with effective constants, by means of which physical properties of the original material, shape and concentration of components are taken into account. Continuum theory may be applied when the scale of investigated phenomenon (for example, the wavelength of the mode of stability loss l) is considerably smaller than that of material structure (say, the layer thickness h), i.e. $l >> h$. The approach based on the model of the piecewise homogeneous medium is free from such restrictions and is, therefore, an exact one. The wide usage of the continuum theory, based on its simplicity in comparison with the model of piecewise-homogeneous medium, puts into consideration the questions of its accuracy and of its domain of applicability. The answer to it may be given only by com-

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parison of the results delivered by both continuum theory and the exact approach, based on the model of the piecewise-homogeneous medium. The last imposes no restrictions on the scale of investigated phenomena and, therefore, has a much larger domain of applicability than the first one. The results obtained within continuum theory must follow from those obtained using the model of piecewise-homogeneous medium if the ratio between the scale of structure and the scale of phenomena tends to zero, i.e. when $hl^{-1} \to 0$. If this is the case, the continuum theory may be considered as asymptotically accurate one.

The present investigation is devoted to substantiation of the continuum theory of fracture (A.N. Guz (4)) in compression for laminated composite materials with periodical structure. Within the scope of this theory the moment of stability loss in the structure of material - internal instability according to Biot (1) - is being treated as the beginning of fracture process. By the present time investigations of the continuum theory accuracy, from the model of piecewise-homogeneous medium point of view, have been done only for the problems of statics and wave propagation by Brekhovskih (2) and Rytov (9). But, there are no such investigations for problems on stability loss in composite structure yet. Basing on the results obtained by A.N. Guz and I.A. Guz (5), I.A. Guz (6-8) using the model of piecewise-homogeneous medium and three-dimensional linearised theory of deformable bodies stability (TLTDBS) developed by A.N. Guz (3), asymptotic accuracy of the continuum theory of plastic fracture is examined in this paper for composites with metal matrix. The investigation is carried out for small precritical strains for three-dimensional (non-axisymmetrical and axisymmetrical) as well as for plane problems. Consideration of small strains only is justified, since fracture of composite materials with metal matrix usually happens under small deformations.

ASYMPTOTIC ANALYSIS OF CHARACTERISTIC DETERMINANTS

Let us consider very briefly the asymptotic analysis of solutions of the stability problems for layered composites in compression. Let composite consist of alternating layers with thicknesses $2h_a$ and $2h_m$, which are simulated respectively by compressible elastic transversally isotropic and elastoplastic incompressible solids. Thickness of the latter (matrix) is assumed to be larger one. (Henceforth all values referred to these layers will be labelled by indices a and m). Suppose also that the material is compressed in plane of the layers by "dead" loads applied at infinity in such a manner that equal deformations along each layer are provided. The detailed problem statement and solution within the scope of exact approach (i.e. using model of piecewise-homogeneous medium and equations of TLTDBS) for the above materials are given in references (7, 8) and for materials with other properties of layers in references (5, 6). It is worth noting that in the case of elastoplastic layers the generalised concept of continuing loading, which allows to neglect variations of loading and unloading zones during the stability loss, is utilised.

To perform the asymptotic analysis we should apply the condition of applicability of the continuum theory $hl^{-1} \to 0$ to all formulae of references (7, 8) and calculate the limits analytically under this condition, which yields

$$\alpha_a \to 0, \quad \alpha_m \to 0, \quad \text{where} \quad \alpha_a = \pi h_a l^{-1}, \quad \alpha_m = \pi h_m l^{-1}$$

$$\cosh\frac{\alpha_a}{\xi_j^a} \to 1, \quad \sinh\frac{\alpha_a}{\xi_j^a} \to \frac{\alpha_a}{\xi_j^a}, \quad j=2,3$$

$$\cosh\frac{\alpha_m \Re\xi_2^m}{\left|\xi_2^m\right|^2} \to 1, \quad \sinh\frac{\alpha_m \Im\xi_2^m}{\left|\xi_2^m\right|^2} \to \frac{\alpha_m \Im\xi_2^m}{\left|\xi_2^m\right|^2}, \quad \cos\frac{\alpha_m \Re\xi_2^m}{\left|\xi_2^m\right|^2} \to 1, \quad \sin\frac{\alpha_m \Im\xi_2^m}{\left|\xi_2^m\right|^2} \to \frac{\alpha_m \Im\xi_2^m}{\left|\xi_2^m\right|^2} \tag{1}$$

On substitution of (1) into characteristic determinants derived in references (7, 8) for four considered modes of stability loss we get

$$\det\left\|\beta'_{pq}\right\| \equiv -(\xi_2^{a^{-2}} - \xi_3^{a^{-2}})\,\overset{a}{\omega}_{1111}(\overset{a}{\omega}_{1133} + \overset{a}{\omega}_{1313})\frac{\Re\xi_2^m \Im\xi_2^m}{\left|\xi_2^m\right|^4} \times$$

$$\times\, [(\frac{h_m}{h_a}\,\overset{a}{\omega}_{3113} + \overset{m}{\kappa}_{3113})(\frac{h_m}{h_a}\,\overset{m}{\kappa}_{1331} + \overset{a}{\omega}_{1331}) - \frac{h_m}{h_a}\,(\overset{a}{\omega}_{1313} - \overset{m}{\kappa}_{1313})^2\,] = 0 \tag{2}$$

$$\det\left\|\beta'_{pq}\right\| \equiv -(\xi_2^{a^{-2}} - \xi_3^{a^{-2}})\,\overset{a}{\omega}_{1111}\overset{a}{\omega}_{3113}(\overset{a}{\omega}_{1133} + \overset{a}{\omega}_{1313})\frac{\Re\xi_2^m \Im\xi_2^m}{\left|\xi_2^m\right|^4}(\overset{m}{\kappa}_{1331}\overset{m}{\kappa}_{3113})^{1/2} = 0 \tag{3}$$

$$\det\left\|\beta'_{pq}\right\| \equiv -(\xi_2^{a^{-2}} - \xi_3^{a^{-2}})\frac{\overset{a}{\omega}_{3113}}{\xi_2^a \xi_3^a}(\overset{a}{\omega}_{1133} + \overset{a}{\omega}_{1313})\frac{\Re\xi_2^m \Im\xi_2^m}{\left|\xi_2^m\right|^4}(\overset{m}{\kappa}_{1331}\overset{m}{\kappa}_{3113})^{1/2} \times$$

$$\times\, [\frac{h_m}{h_a}(\frac{h_m}{h_a}\,\overset{a}{\omega}_{3333} + 2\overset{a}{\omega}_{1133} + \overset{m}{\kappa}_{3333} + \overset{m}{\kappa}_{1111} - 2\overset{m}{\kappa}_{1133} - 2\overset{m}{\kappa}_{1313}) + \overset{a}{\omega}_{1111}\,] = 0 \tag{4}$$

$$\det\left\|\beta'_{pq}\right\| \equiv (\xi_2^{a^{-2}} - \xi_3^{a^{-2}})\frac{\overset{a}{\omega}_{1111}\overset{a}{\omega}_{3113}}{\xi_2^a \xi_3^a}(\overset{a}{\omega}_{1133} + \overset{a}{\omega}_{1313})\frac{\Re\xi_2^m \Im\xi_2^m}{\left|\xi_2^m\right|^4} = 0 \tag{5}$$

Characteristic equations (2)-(5) correspond respectively to the 1st, 2nd, 3rd and 4th modes of stability loss for the case of biaxial compression (non-axisymmetrical problem). Henceforth only this problem is analysed. Consideration of other problem statements (plane problem in the case of uniaxial compression, axisymmetrical problem in the case of biaxial compression) has proved to lead to the same conclusions.

Let us examine characteristic equations (2)-(5). It was proved in reference (4) that for approved models of layers

$$\Re\,\xi_j^{am^2} > 0,\ j = 1,2,3; \quad \Im\xi_1^{am^2} = 0, \quad \Im\xi_2^{a^2} = \Im\xi_3^{a^2} = 0, \quad \xi_2^{m^2} = \overline{\xi_3^{m^2}} \tag{6}$$

Besides that, the roots of characteristic equations, which correspond to the considered phenomenon of internal instability, must depend on properties of both alternating layers, i.e. on the ratio h_a/h_m. This feature was discussed, for example, in references (6-8). Given the above-said and condition (4), one can observe that characteristic equations (3) and (5), which correspond respectively to the 2nd and 4th modes of stability loss, do not have such roots and, therefore, do not describe the internal instability in the long-wave approximation. Components of tensor ω and κ may be expressed using formulae of in references (3, 4). On substituting them into characteristic equation (2), which corresponds to the 1st mode of stability loss, we derive for the roots which depend on h_a/h_m

$$(1 + \frac{h_a}{h_m})^2 \mu_{13}^a \mu_{13}^m + (\sigma_{11}^{0\,a} + \frac{h_m}{h_a}\sigma_{11}^{0\,m})(\mu_{13}^a - \frac{h_m}{h_a}\mu_{13}^m) = 0 \tag{7}$$

Introducing effective values of stresses $\langle \sigma_{11}^0 \rangle$ and parameters $\langle \mu_{13} \rangle$, $\langle A_{11} \rangle$ in the moment of stability loss by well-known formulae

$$\langle \sigma_{11}^0 \rangle = \sigma_{11}^{0\,a} S_a + \sigma_{11}^{0\,m} S_m,\ \langle \mu_{13} \rangle = \mu_{13}^a \mu_{13}^m (\mu_{13}^a S_m + \mu_{13}^m S_a)^{-1},\ \langle A_{11} \rangle = A_{11}^a S_a + A_{11}^m S_m \tag{8}$$

we can obtain from equation (7) that

$$(\Pi_1)_T \equiv -\langle \sigma_{11}^0 \rangle = \langle \mu_{13} \rangle \tag{9}$$

This coincides with the results derived in reference (4) within the scope of continuum theory of plastic fracture for laminated composites.

As to characteristic equation (4), which corresponds to the 3rd mode of stability loss, we observe that in long-wave approximation this mode yields higher critical stresses than the 1st mode and, therefore, along with the 2nd and 4th modes may be excluded from consideration. Indeed, on substitution of (8) into (4) we get

$$-\langle \sigma_{11}^0 \rangle = \langle A_{11} \rangle + S_a S_m (A_{13}^a - A_{13}^m)^2 (A_{33}^m S_a + A_{33}^a S_m)^{-1} \tag{10}$$

From the condition of uniqueness of solution of a linear problem for orthotropic bodies it follows (references (3, 4)) that $A_{33}^a > 0,\ \ A_{33}^m > 0$. Besides that, real constructive composite materials show higher compressive than shear strength, i.e. $\langle A_{11} \rangle > \langle \mu_{13} \rangle$. And, finally, on substitution of the above inequalities into (10) we obtain

$$-\langle \sigma_{11}^0 \rangle = \langle A_{11} \rangle + S_a S_m (A_{13}^a - A_{13}^m)^2 (A_{33}^m S_a + A_{33}^a S_m)^{-1} > \langle A_{11} \rangle > \langle \mu_{13} \rangle \tag{11}$$

Inequalities (11) clearly show that critical stresses (9) corresponding to the 1st mode of stability loss, are always smaller than those corresponding to the 3rd mode (10).

NUMERICAL RESULTS FOR METAL MATRIX COMPOSITES

Now, using the results of the previous section, the accuracy of the continuum theory (i.e. ratio of results obtained in the context of exact approach and continuum theory) can be calculated. Values of critical strains (or other critical parameters, e.g. critical stresses) calculated within the scope of exact approach may be found in numerous publications, e.g. in reference (8). Following these papers values of critical strains for the 1st mode of stability loss under the condition of applicability of the continuum theory are easily obtained. Some papers even show them explicitly. Comparing the above-mentioned values of critical parameters the asymptotic accuracy of the continuum theory of fracture for metal matrix composites in compression can be estimated and conclusions about reasonability of utilisation of this theory can be made properly.

As an example, let us consider a composite consisting of alternating layers of linear-elastic isotropic compressible filler characterised by Young's modulus E and Poisson's ratio ν, and elastoplastic incompressible matrix with power-mode dependence between the equivalent stress and the equivalent strain in the form $\sigma_I^0 = A(\varepsilon_I^0)^k$. Dependences of parameter Θ on A/E calculated following reference (8) are given on Fig. 1 and Fig. 2.

SYMBOLS USED

h = half-thickness of the layer (m)

S = volume fraction of a particular type of layers

α = wave-generation parameter for modes of stability loss

Θ = ratio of results obtained in the context of exact approach and continuum theory (%)

κ = tensor in linearised constitutive equations of TLTDBS for incompressible solids

$(\Pi_1^i)_T$ = theoretical strength limit (GPa)

$\langle \sigma_{11}^0 \rangle$ = effective values of stresses (GPa)

ω = tensor in linearised constitutive equations of TLTDBS for compressible solids

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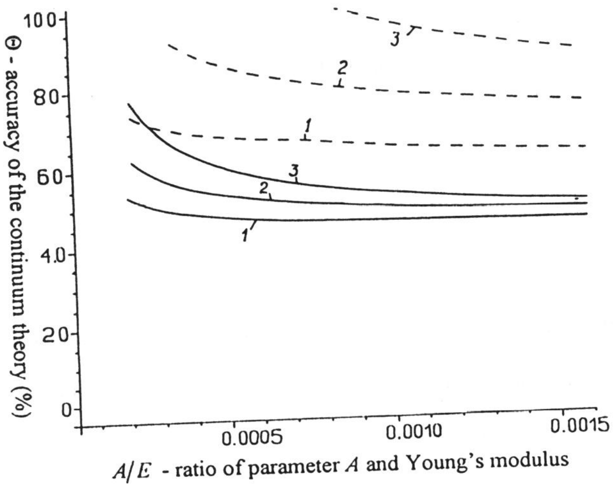

Figure 1 The case of biaxial compression: $\nu = 0.21$; $k = 0.1$ (curves 1), 0.43 (curves 2), 0.7 (curves 3); $h_a/h_m = 0.02$ (continuous curves), 0.05 (hatched curves)

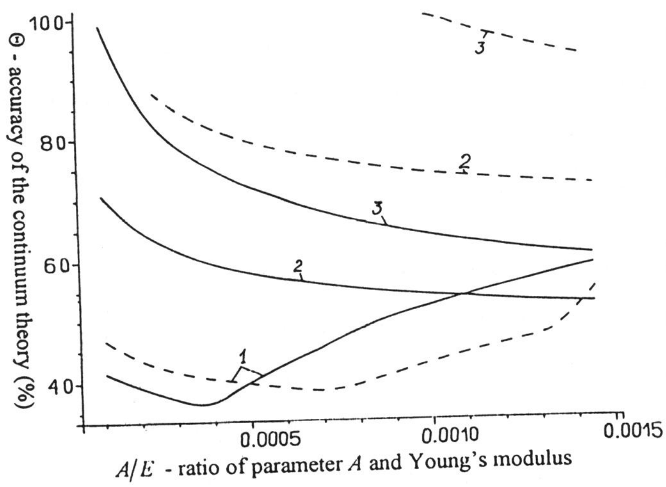

Figure 2 The case of uniaxial compression: $\nu = 0.21$; $k = 0.1$ (curves 1), 0.43 (curves 2), 0.7 (curves 3); $h_a/h_m = 0.03$ (continuous curves), 0.06 (hatched curves)