HYPERCUBE GRAPH BASED FRACTURE MODELS OF FIBROUS BUNDLES AND COMPOSITES

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In order to Markov process hypothesis applied when arbitrary discrete composite system has damage processes on hypercube oriented graph base and hypercube rang defined by system element amount. Stochastic process of damage growth in fibrous bundles and composites was described by Chapman-Kolmogorov equations with Poisson and power rate of failure. Probabilistic models of strength, fatigue and crack resistance are given, numerical modelling was accomplished for fibrous bundle, unidirectional glass/epoxy and carbon/epoxy composites.

INTRODUCTION

The composite materials and structures are considered as discrete system of countable elements, there are granular or fibrous particles for composite and FEM-elements for structure. In simple case every element is defined by binary index expressing element status: intact or damaged. Then the discrete system state would define by binary vector and may be represented on node set of N-range cube, where N is element amount. In these conditions, the fracture of composite or structure is the damaged element accumulation and it is formalised as transfer process from node to node. The Markov process hypothesis applies when the discrete system has degrade on paths from edge subset of N-range cube because only cube edge connects nodes distinguishing one digit of binary vector. Physically this is reflected in the fact that one element has fracture in one time. The individual path includes N edges and has simple chain form connecting initial system node (all elements intact) and final system node (all elements damaged). The power set of degrading paths is branching Markov chain and scheme of this process is hypercube oriented graph (Sokolkin and Chekalkin (1)), the edge orientation is defined in line with nodal binary vector norm growth. The discrete degrading system with initially damages has input by hypercube graph base process too.

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STATISTICAL MODEL

Let discrete elements are having strength scatter, then the degrading process of composite or structure has stochastic character and fracture process is described by Chapman-Kolmogorov equations with respect to transfer probability vector and intensity matrix. The transfer probability vector components are connected with hypercube node set, the transfer intensity nonzero matrix components are connected with hypercube edge set. Initial data vector is $p_i /_{s=0} = \{1,0,0,..\}$, moving data of transfer probability vector has identity norm, considering that hypercube node set includes all statistically possible states of the discrete system

$$dp_i/ds = L_{ij}\, p_j \,;\; (L_{ij} = - L_{ji} \,;\; \lambda = n/2^n \,;\; i,j = 1...2^n) \quad (1)$$

The sparse images of transfer intensity matrix are proposed by Sokolkin and Chekalkin (1), there are Poisson process $(\beta = 1)$ and power rate of failure transfer intensity function $(\beta \neq 1,\ in\ practice\ \beta > 1)$ analysing here by sequential and parallel statistic models comparability

$$L(s) = dP/ds\ (1-P)^{-1} = \beta/\alpha\ (s/\alpha)^{\beta-1} \,;\; (R=1-P= exp\ (-\textstyle\int L(\tau)d\tau)) \quad (2)$$

The power transfer intensity has been derived from statistic two-parameter Weibull distribution and has good agreement and directly connected to element strength mean value $\alpha\Gamma(1+1/\beta)$ and coefficient of variation $(\Gamma(1+2/\beta)/\Gamma^2(1+1/\beta)-1)^{1/2}$

$$P(s) = 1 - exp\ (- (s/\alpha)^{\beta}) \,;\; (s \geq 0) \quad (3)$$

The constant transfer intensity $(\beta = 1)$ is commonly used in stochastic process analysis, there is obtained from statistic one-parameter exponential distribution. This distribution isn't shown to grains, fibres or composites strength probabilities. Transfer intensity functions of fibrous bundles and glass-epoxy and carbon-epoxy unidirectional composites are received by hypercube graph based fracture models for fibrous bundle strength *(see equation 4)* , fibre reinforced composite strength *(see equation 5)* and crack resistance *(see equation 6)*

$$L_{ii+1}(s) = (n-i)\beta(n/\alpha(n-i))^{\beta} s^{\beta-1} , \quad (4)$$
$$L_{ii+1}(\sigma) = c_i\beta((c_i+i)/\alpha c_i v_f)^{\beta}\sigma^{\beta-1} , \quad (5)$$
$$L_{ii+1}(K_I) = 2\beta((2/\pi d)^{1/2}/\alpha v_f)^{\beta} K_I^{\beta-1}, \quad (6)$$

where $\alpha = \alpha_0(\delta_0/\delta)^{\gamma/\beta}$ and $\delta = d_f(1-\sqrt{v_f})(E_f/v_f G_m)^{1/2}$. Elementary extension to cycle loading models are developed by reliability rule for independent events

$$R_N(s) = \Pi R(s) = exp\ (- N\textstyle\int L(\tau)d\tau) \,;\; (\sigma \rightarrow \sigma_{-1} \,;\; K_I \rightarrow \Delta K_I) \quad (7)$$

NUMERICAL MODELLING

Numerical analysis was fulfilled for stochastic system *(see equation 1)* with transfer intensity matrix for bundle strength, unidirectional composite strength and crack resistance *(see equations 4 - 6)* by Runge-Kutta method on four-range accuracy scheme for 2^n-range system. Cycle loading dependence models are analysed too as fatigue strength as cyclic crack resistance for glass/epoxy and carbon/epoxy fibre reinforced composites. Data base for numerical modelling was next, E-glass fibre: $\alpha_0 = 3.45\ GPa$, $\beta = 4.0$, $\delta_0 = 10\ mm$, $\gamma = 0.5$, $d_f = 5.0\ mcm$, $E_f = 72.4\ GPa$, $v_f = 0.6$; carbon fibre: $\alpha_0 = 1.72\ GPa$, $\beta = 5.0$, $\delta_0 = 10\ mm$, $\gamma = 0.5$, $d_f = 6.9\ mcm$, $E_f = 276\ GPa$, $v_f = 0.6$; epoxy matrix: $G_m = 2.65\ GPa$. The transfer probability functions of glass fibre bundle under static tension are presented on figures 1 and 2, the real fracture process with power rate of failure has significant unlike to Poisson model *(see fig.1)*. Statistical fracture behaviour of unidirectional glass/epoxy and carbon/epoxy fibre reinforced composites are shown on figures 3 - 6. Fatigue strength *(see fig. 3 and 4)* and cyclic crack resistance *(see fig. 5 and 6)* are modelled on hypercube graph based fracture process and supported to predict high-reliability levels *(0.999)* on fatigue strength limits and cyclic resistance of initial crack growth for glass/epoxy and carbon/epoxy composites.

NOMENCLATURE

α = scale factor of fibre strength (Pa)
α_0 = scale factor on base length (Pa)
β = shape factor of fibre strength (-)
γ = size factor of fibre strength (-)
Γ = Gamma function (-)
δ = effective length of fibre (m)
δ_0 = base length of fibre (m)
λ = sparse matrix ratio (-)
σ = static stress (Pa)
σ_{-1} = cyclic stress (Pa)
τ = inner parameter (-)
c_i = damage enclosing fibre (-)
d = fibre spacing distance (m)
d_f = fibre diameter (m)
E_f = Young modules of fibre (Pa)
G_m = shear modules of matrix (Pa)
K_I = static stress intensity (Pa$\sqrt{m}$)
ΔK_I = cyclic stress intensity (Pa$\sqrt{m}$)
L_{ij} = transfer intensity matrix (N^{-1}, Pa^{-1}, $(Pa\sqrt{m})^{-1}$)
n = general number of elements (-)
N = cycle loading number (-)
p_i = transfer probability vector (-)
P = probability value (-)
R = reliability value (-)
s = loading parameter (N)
v_f = fibre volume ratio (-)

REFERENCE

(1) Sokolkin,Yu.V. and Chekalkin, A.A., "Stochastic Fracture Processes in Mechanics of Composite Materials", Proceedings of ECF-10 on "Structural Integrity: Experiments - Models - Applications", Berlin, Germany. Edited by K.-H. Schwalbe and C.Berger, EMAS, Warley, U.K., 1994.

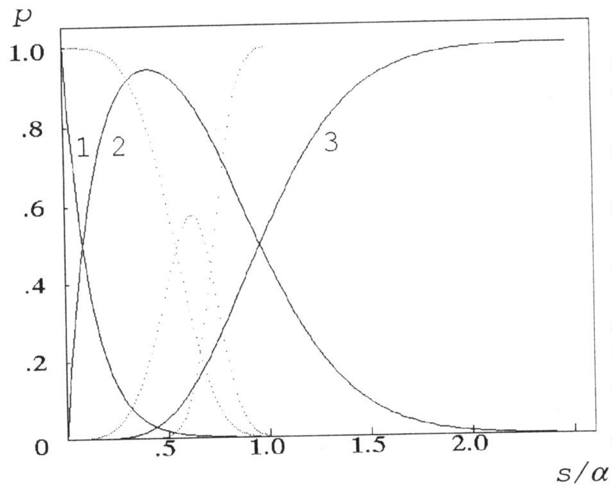

Figure 1. Transition probability function on hypercube graph base fracture model, Poisson's process is solid line, power rate of failure process is dot line: 1 - initial state, 2 - all transient states, 3 - final state.

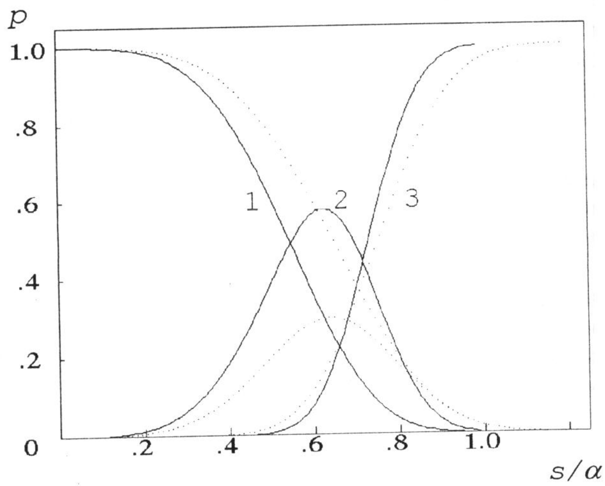

Figure 2. Transition probability function on hypercube graph base fracture model with power rate of failure process, eighth-element bundle is solid line, four-element bundle is dot line: 1 - initial state, 2 - all transient states, 3 - final state.

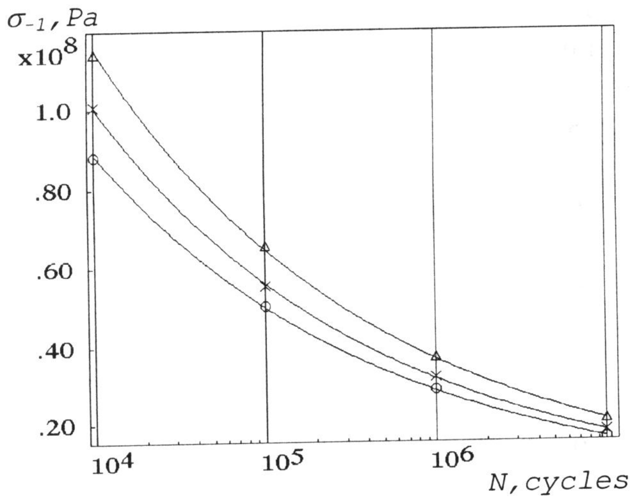

Figure 3. Probabilistic fatigue curve of unidirectional glass/epoxy composite, reliability levels: ° - 0.999, × - 0.99, Δ -0.9.

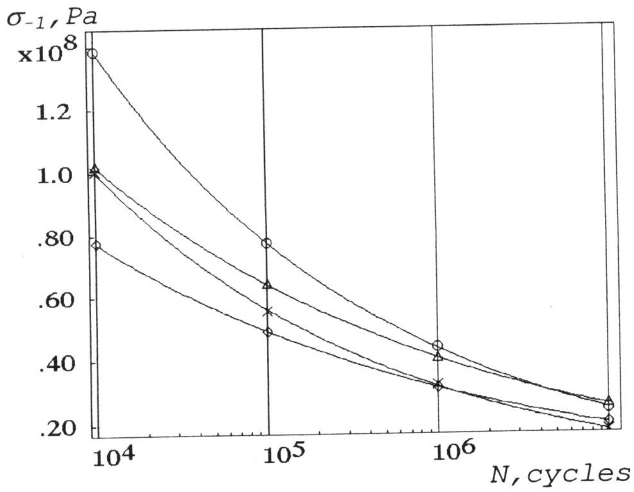

Figure 4. Probabilistic fatigue curves of unidirectional glass/epoxy (°,×) and carbon/epoxy (Δ,◊) composites, reliability levels: °,Δ - 0.5, ×,◊ - 0.99.

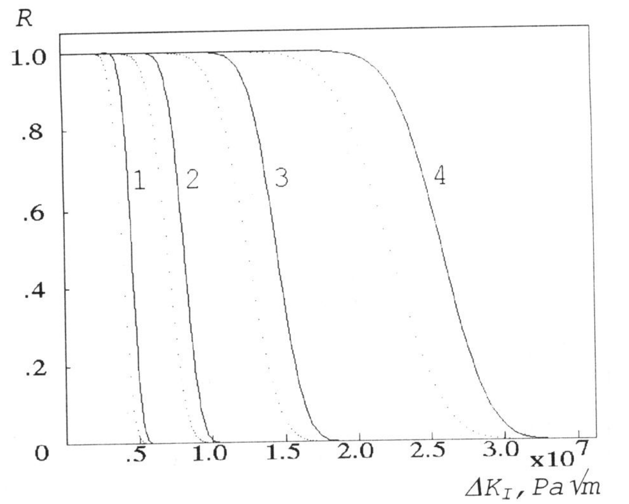

Figure 5. Reliability function of unidirectional glass/epoxy composite with crack mode I, six-fibre critical size near crack tip is solid line, three-fibre critical size hear crack tip is dot line, base cycle loading number: 1 - 10^7, 2 - 10^6, 3 - 10^5, 4 - 10^4.

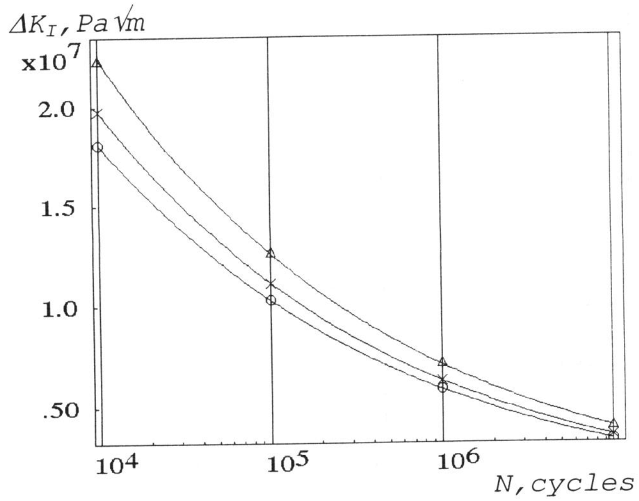

Figure 6. Probabilistic cycle fracture resistant curve of unidirectional glass/epoxy composite with crack mode I, reliability levels: ° - 0.999, × - 0.99, Δ - 0.9.