MODELLING FRACTURE IN ALUMINIUM BASED METAL MATRIX COMPOSITES USING A STATISTICAL APPROACH

K. M. Mussert, M. Janssen, A. Bakker and S. van der Zwaag*

Fracture in an Al6061 based metal matrix composite (MMC) containing 20 vol.% Al_2O_3 particles is modelled using an axisymmetrical finite element model and a statistical approach for calculating strengths of ceramic via Weibull's model. In this model, the MMC is assumed to fail as soon as the particle has failed. By plotting the calculated survival probability of an Al_2O_3 particle versus the macroscopic axial stress applied on the whole MMC, the applicability of Weibull statistics on these type of new materials can be checked.
It can be concluded that for higher triaxialities, the Al_2O_3 particle fails before plasticity in the matrix occurs. Furthermore, knowing the stress distribution in a ceramic particle, Weibull's model can be used to calculate the survival probability of ceramic particles in ductile matrices, however, the Weibull modulus becomes meaningless when a non-proportional stress distribution in the matrix occurs.

INTRODUCTION

Due to advantages offered by ceramic particle reinforced metal matrix composites (MMCs), such as high strength, high stiffness and high resistance to wear as compared to the matrix materials, this type of materials have attracted increasingly more attention in the past decade. A reason for using any composite material is the extent to which the qualities of two or more constituents can be combined, without seriously accentuating their shortcomings. Aluminium and its alloys form the most widely investigated matrices, whereby the excellent ductility and formability of the matrix is combined with the stiffness and load-bearing capacity of the reinforcement.

To study the influence of ceramic particles in a ductile aluminium matrix, an axisymmetrical finite element model is used. Within this model, variables such as particle volume fraction, particle size and matrix alloy properties can be varied. It is assumed that the MMC fails as soon as the particle fails. The MMC investigated in this research has a matrix of Al6061 reinforced with 20 vol.% Al_2O_3 particles. Weibull's statistical model for treatment of strengths of ceramic is used to determine the survival probability of an Al_2O_3 particle in an Al6061 matrix under uniaxial and triaxial stress states. Since the Weibull

* Section Mechanical Behaviour of Materials, Faculty of Applied Sciences, Delft University of Technology

model is applied on a single particle, it has to be checked whether this model is representative for fracture of the whole MMC.

CELL MODEL

Micromechanical models of ductile damage and fracture can be described by the structural behaviour of relatively simple unit-cells (1). Cell model calculations are normally used to study porous solids, but in this research the model is used for metal matrix composites.

The continuum is considered to consist of a periodic assemblage of hexagonal cylindrical unit cells approximated by circular cylinders, see figure 1, which allows for a simple axisymmetrical calculation. Every cell of initial length $2L_0$ and radius R_0 contains a spherical particle of radius r_0. A disadvantage of this approach is that the relative position of the particles in the matrix is fixed. The surfaces normal to the axial and radial directions are subjected to homogeneous displacements in these directions respectively. If triaxiality shall be kept constant during the loading history, the ratio

$$\rho = \frac{\sigma_{rr}}{\sigma_{zz}} \tag{1}$$

has to remain constant, whereas the ratio of the prescribed strains, $\varepsilon_{rr}/\varepsilon_{zz}$ will consequently vary with increasing load (2).

WEIBULL MODEL

In ceramics, strength is essentially limited by the flaws which are present. Wherever a strength value is given for a ceramic it is likely to be an average, as with metals, but the amount of scatter is more pronounced. An appropriate statistical treatment for strengths of ceramics is the Weibull model (3).

This model is based on the idea that a chain consisting of nominally identical links is as strong as its weakest link. The links have individual strengths which vary statistically. The probability of survival of any one link for a stress σ is S_l. The individual link survival probability S_l is a function of stress only and it is convenient to express it in terms of a 'risk of rupture' R_l defined by:

$$S_l = \exp(-R_l) \tag{2}$$

with R_l being dependent on stress alone. Transferring the model to arbitrary volumes of ceramic, the following expression for the risk of rupture is obtained:

$$R = \int_V R_l dV \tag{3}$$

Weibull's important contribution was a postulate, based largely on empiricism, that:

$$R_1 = \left(\frac{\sigma - \sigma_t}{\sigma_0} \right)^m \qquad (4)$$

where m is the so-called Weibull modulus, σ is the applied stress and σ_t is a threshold stress at which (or below) survival is certain. It is usual for ceramics to take $\sigma_t = 0$ MPa (3); σ_0 can then be interpreted as the stress at which S = 1/e.
When modelling ceramics, a distinction can be made between fracture initiating from surface defects or volume defects. At this stage in the current research only volume flaws are taken into account.

It should be noted that for a uniform stress distribution, eq. 4 in combination with S = exp(-R) results in:

$$S = \exp\left\{ -\left(\frac{\sigma}{\sigma_0} \right)^m \right\} \qquad (5)$$

Manipulation of eq. 5 allows a straight line representation of gradient m, when lnln(1/S) is plotted against ln(σ):

$$\ln\ln\frac{1}{S} = m\ln(\sigma) - m\ln\sigma_0 \qquad (6)$$

FINITE ELEMENT CALCULATIONS

To calculate the survival probability of a ceramic particle in an MMC, the finite element method is used. For every integration point of each element belonging to the particle, the principle stresses σ_1, σ_2 and σ_3 are calculated and these values are averaged to get the principle stresses for the element. Furthermore, the volume of each element is calculated. Using the average principle stresses, the applied stress is calculated using the Drücker-Prager criterion (from this point forward this stress will be denoted as σ_{DP}):

$$\sigma_{DP} = \sqrt{\tfrac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]} + \tfrac{1}{3}\alpha(\sigma_1 + \sigma_2 + \sigma_3) \qquad (7)$$

Knowing the ratio between the compression strength and the tensile strength of the material, α can be calculated. Now, for a given σ_0 and m, R_1 can be calculated for each element. These R_1-values are added up to get the total R of the whole particle (eq. 3) with which the survival probability S is calculated.

The finite element mesh used for calculations consisted of 350 isoparametric quadrilateral 4-node elements, with a mesh as shown in figure 2. The metal matrix composite is modelled with a matrix of Al6061 (E = 69 GPa, $\nu = 0.33$, $\sigma_{ys} = 276$ MPa) and 20 vol.% Al_2O_3 particles with a diameter of 4 μm (E = 393 GPa, $\nu = 0.27$, $\sigma_{ys} = 2000$ GPa, this is a fictitious high value to prevent plastic deformation in the particle). Calculations were done with ρ-values 0 (uniaxial tensile test), 0.1, 0.3, 0.5 and 0.7 (triaxial tensile tests). The survival probability S was then calculated with m = 15 and σ_0 = 350 MPa.

To check the correctness of the calculations, lnln(1/S) is plotted versus the macroscopic axial stress σ_{zz} in figure 3. Since σ_{DP} varies from element to element this value can not be plotted in this graph. It can be seen that the calculations result in a straight line of gradient m = 15, only for ρ-values 0, 0.1 and 0.3 the last part ($\ln(\sigma_{zz}) > 5.6 \approx 270$ MPa) shows some curvature.

DISCUSSION AND CONCLUSION

In this research, the Weibull model is used in combination with finite element calculations to determine fracture of a ceramic particle in an aluminium matrix.

To explain the results obtained in figure 3, one should bear in mind that eqs. 5 and 6 only hold in case of a uniform stress distribution in the particle. In figure 4, the actual Drücker-Prager stress σ_{DP} is calculated for 15 nodes near the surface of the particle for $\rho = 0$ and 0.7 and plotted versus the macroscopic axial stress σ_{zz}. It can be seen that there is a linear relation between σ_{DP} and σ_{zz} up to 330 MPa in case of $\rho = 0$ and up to 1100 MPa in case of $\rho = 0.7$. Also, it is clear that σ_{DP} is dependent on the position in the particle, i.e. a non-uniform stress distribution exists. In eqs. 5 and 6, σ should be replaced by $\sigma_{DP}(\underline{x}) = \sigma_{zz} \cdot f(\underline{x})$, where $f(\underline{x})$ is some function of the position in the particle. Eq. 6 now becomes:

$$\ln\ln\frac{1}{S} = \ln\left(\int_V f(\underline{x})^m dV\right) + m\ln(\sigma_{zz}) - m\ln\sigma_0 \qquad (8)$$

It can now be seen that, as a result of non-uniformity, there is an additional factor, which explains the translations along the survival probability axis for increasing ρ-values.

If not only a non-uniform, but also a non-proportional distribution of stresses exists due to plasticity in the matrix, $f(\underline{x})$ becomes $f(\underline{x},\sigma_{zz})$. Now, the integral in eq. 8 is also dependent on σ_{zz} and the straight line in figure 3 will disappear. For $\rho = 0.7$ the linear relation between σ_{DP} and σ_{zz} is maintained till higher stresses, but it can be seen from figure 5 (which is just another representation of figure 3) that a survival probability of zero is already reached at a macroscopic axial stress of 500 MPa. This explains why in figure 3 the curve remains a straight line for the higher ρ-values.

So, for higher triaxialities, the Al_2O_3 particle fails before plasticity occurs in the matrix and knowing the stress distribution in a ceramic particle, Weibull's model can be used to calculate the survival probability of ceramic particles in ductile matrices. However, the Weibull modulus becomes meaningless when a non-proportional stress distribution in the matrix occurs. More calculations have to be done to investigate other m and σ_0 values, particle sizes and to incorporate surface defects and interface strengths. Furthermore, when more experimental data become available, the model has to be adjusted in terms of particle volume fraction, particle size and matrix alloy properties.

REFERENCES

(1) Koplik, J., and Needleman, A., Int. J. Solids Struct., Vol. 24, 1988, pp. 835-853.

(2) Brocks, W., Sun, D-Z., Hönig, A., Comp. Mat. Sc., Vol. 7, 1996, pp. 235-241.

(3) Weaver, G., JME, Vol. 5, 1983, pp. 768-804.

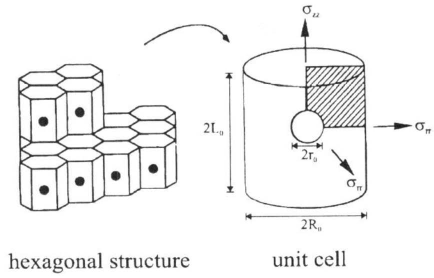

Figure 1: Micromechanical modelling of a matrix containing a spherical particle

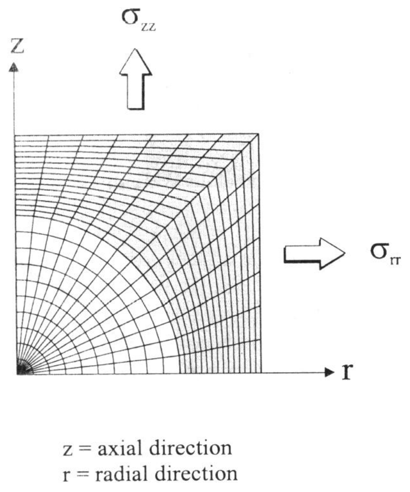

Figure 2: Finite element mesh used for calculations

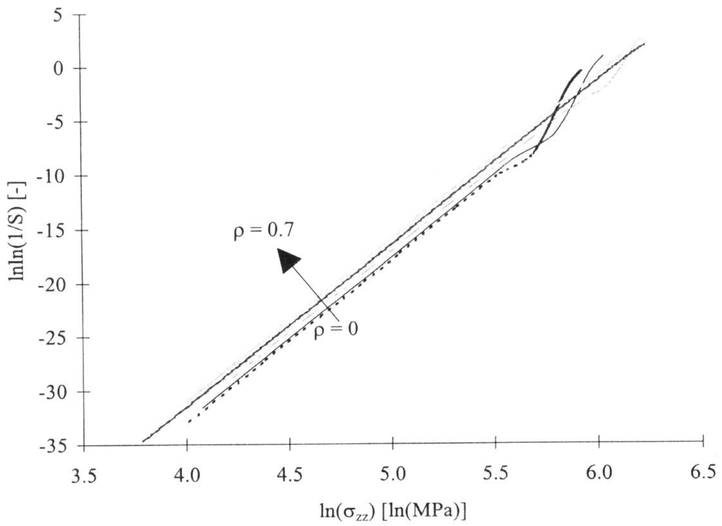

Figure 3: Survival probability S as a function of macroscopic axial stress σ_{zz} for $\rho = 0$, 0.1, 0.3, 0.5 and 0.7, for a particle with a diameter of 4 μm, $\sigma_0 = 350$ MPa and m = 15

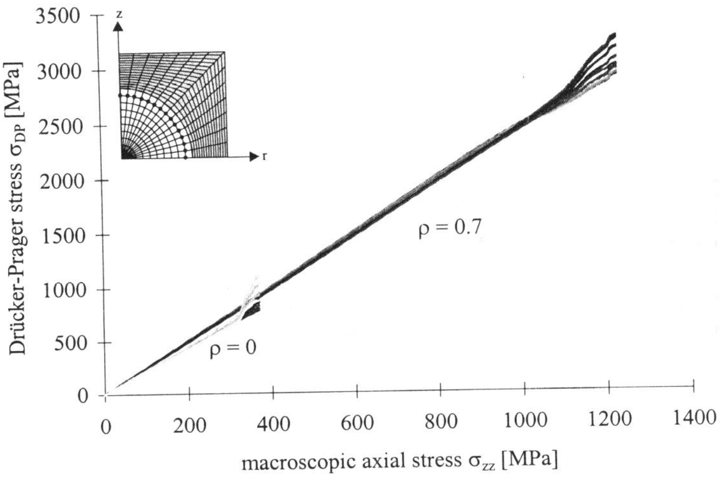

Figure 4: Drücker-Prager stress σ_{DP} as a function of macroscopic axial stress σ_{zz} for various nodes in a particle with a diameter of 4 μm

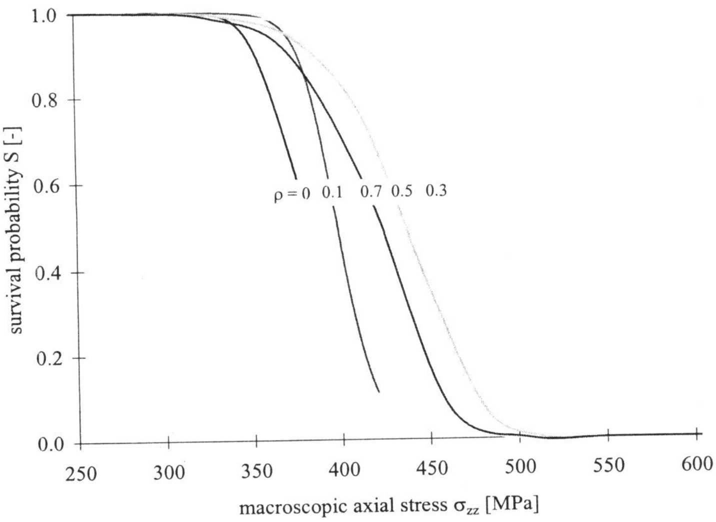

Figure 5: Survival probability S as a function of macroscopic axial stress σ_{zz} for different stress ratios ρ and a particle with a diameter of 4 μm