WHY AND HOW THE ELASTIC RESPONSE OF AN IDEAL CRACK IS TO BE HARMONIZED WITH THAT OF AN ACTUAL CRACK

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A crack can be represented in fracture analysis by different sets of parameters expressed in different quantities. Their determination and subsequent converting into parameters of an ideal crack constitute the subject matter of crack modelling. This procedure is an integral part of any analytical or numerical method used in the framework of fracture mechanics. Two types of an ideal crack, i.e. simplified crack models, related to the same physical crack are contrasted with one another so as: 1) to harmonize the elastic behaviour of an ideal crack with that of a straight-through center crack in a biaxially loaded thin sheet of aluminium and 2) to demonstrate the potential of the above harmonization with respect to the prediction of the fracture initiation stress and fracture toughness when the crack length, specimen size, and load biaxiality, all are changed on condition that the whole response of a specimen to loading is predominantly elastic.

INTRODUCTION

There is a wealth of evidence against treating crack-growth resistance *R*-curves as the inherent material properties. For instance, K_R - curves for middle-cracked specimens made of a thin-sheet aluminium are shifted from the bottom upwards as the crack length and specimen size increase (Naumenko et al (1)). A similar dependence for aluminium M(T) specimens of thickness B = 6 mm and width $2W$ varying from 50 to 950 mm was reported by Naumenko and Semenets (2). Analysis of the size effects is generally restricted to a simple comparison of available *R*-curves without casting any doubt on the applicability of the stress intensity factor K as a crack-driving parameter. This parameter is of particular importance in a framework of the Conventional Methodology (CM) of fracture analysis. The data related to the comparison of one- and two-parameter approaches (1) give an additional indication of some inherent deficiency of the CM. To explain the size-scale effects of concern, if only in part, the so-called Unified Methodology (UM) developed by Naumenko (3, 4) is used in the following. The term UM implies that the effects of the crack length, specimen size, load history, boundary constraint, and load biaxiality are evaluated in the context of a single conception called by Naumenko (5) the ρ-theory.

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CRACK MODELLING AND FRACTURE CRITERION

The problem of harmonizing elastic responses of an ideal and an actual cracks is confined to the simplest case, where the in-plane geometry and the stress state are symmetric with respect to both the x- and y-axes (Fig. 1). We consider the Problem Domain (PD) in the form of a square ABCD subjected to uniform biaxial stresses $\sigma = P / 2W$ and $q = Q / 2W$. The original crack length $2c_u$ is taken as $2c_u >> B$, so that a two-dimensional state of the generalized plane stress is a good approximation. Here, B is the PD thickness. As for the evaluation the center crack parameters, a cruciform specimen (Fig. 1) is best suited to realize the above conditions along the PD boundaries. Its dimensions are: B = 1.05 mm, $W = H$ = 120 mm, $W_u = H_v$ =240 mm, R = 108 mm, and c_u = 48 mm. Due to a distinct number of the narrow slots (in amounts of 15) in each loading arm, the effect of rigid clamping on the stress distribution within the PD is at a minimum (Pisarenko et al (6)).

Crack parameters. The CM procedure for crack modelling is specified by the equation $2a = 2c_u$, where $2a$ is the length of the line (volumeless) cut and $2c_u$ is the length of the actual crack in an undeformed specimen (Fig. 2b). Thus, a crack is represented in the CM by its planar dimensions irrespective of the elastic response of the crack border and the transverse dimensions $2h_u(x)$ of the crack in an undeformed specimen (Fig. 2). The last two factors depend on the material properties, precrack-ing history, crack length, specimen geometry, its size and boundaries constraint. Another overly simplified assumption is as follows: fracture can be characterized adequately through the use of the planar crack size variation Δc and displacements v (x, y) along the y-axis with no regard to the transverse crack size $2h_u(x)$ and displacement u (x, y) along the x- axis (see Fig.1). By contrast, in the context of the UM (3, 4, 5), both the crack sizes and crack displacements in both directions are to be evaluated. To characterize a crack in a comprehensive way, one has to know the initial crack profile and elastic responses and displacements of the crack border across and along the crack plane. Such insight into the crack features is expressed in terms of the following parameters (see Fig. 2a): the initial values of the crack length $2c_u$ and the crack-mouth opening spacing $2h_u$; the nondimensional elastic compliances of the crack border in the stress-free plate along the transverse $F_{cv} = v(x=0,\ k)E/2c_u\sigma$ and longitudinal $F_{cu} = -u(x=c,\ k)E/c_u\sigma$ directions.

Crack modelling. The above set of the experimental data was used to represent an actual crack by an equivalent elliptic hole (Fig. 2c). Its major l and minor b semi-axes have been determined by measurements of the compliances F_{cv} and F_{cu} at three levels of the load biaxiality ratio $k = q/\sigma$, namely, k = 0, 1.0 and 2.0. In doing so a purely elastic response of the stationary crack, length $2c_u$ = 96 mm, was considered in this study. During each loading-unloading cycle, σ *vs* v $(x = 0, k)$ and q *vs*

u ($x = c$, k) diagrams were recorded in pairs. Experimental values F_{cv} and F_{cu} were equated to the relevant compliances of an elliptic hole defined as $F_{lv} = v(x=0,\ k)\ E/[b(1-k)+2l]\ \sigma$; $F_{lu} = u(x=l,\ k)E/[k(l+2b)-l]\ \sigma$ and. Taking into account that the F_{lv}/F_{lu} ratio is not to be dependent on the k value, we can set up two equations providing the desired relationship between the semi-axes l and b. Absolute values of l and b were estimated through the use of the F_{cv} and F_{cu} compliances for large-size M(T) specimens of width $2W = 1200$ mm with an original crack $2c_u = 96$ mm. In this case, $F_{lv} \approx 1.0$ and $F_{lu} \approx 1.0$ since $l \approx c_u < 0.1W$. It must be emphasized that both the F_{lv} and F_{lu} compliances of the elliptic hole depend upon the dimensionless parameters $Y_1 = \rho / l$, $Y_2 = l / W$, and $Y_3 = H / W$ only. Here, ρ is the minimal radius of the hole surface curvature, $\rho = b^2 / l$. The radius ρ is treated as an inherent characteristic of any straight-through center crack in a plate of a given material and thickness when $l >> \rho$.

Fracture criterion. The ρ value is incorporated in the following criterion of brittle fracture under plane stress conditions:

$$\rho_h = \rho[1+C_l(\sigma_h/E)]^2/[1+C_b(\sigma_h/E)] \qquad (1)$$

where C_l and C_b are the stress concentration factors at the points $x = \pm l$, $y = 0$ and $x = 0$, $y = \pm b$, respectively. They take the form:

$$C_l = F_{lv}\left[1+2(l/\rho)^{0.5}-k\right] \text{ and } C_b = F_{lu}\left[k+2k(\rho/l)^{0.5}-1\right] \qquad (2)$$

The $\rho_h = \rho_R$ value is assumed to be an "inherent property of a brittle material". As for the fracture initiation labelled by a subscript "i", it is implied that the start of crack growth occurs at the same $\rho_h = \rho_i$ value irrespective of the specimen sizes, the crack length, the signs of both loads P and Q (Fig. 1), and the magnitude of the k ratio. For aluminium specimens (1): the elastic modulus $E = 73000$ MPa, the 0.2% offset yield stress $\sigma_Y = 335$ MPa, the ultimate strength $\sigma_u = 445$ MPa, $\rho = 0.262$ mm and $\rho_i = 0.305$ mm.

DISCUSSION

Fracture initiation stress σ_{ni} in the net section of large, width $2W = 1200$mm, and small, $2W = 120$ mm, M(T) specimens was less then the yield strength of 1163 AT aluminium (1). As a first approximation, assume that the σ_{ni} stress may be estimated via Eq. (1) using the next simplifications: $l = a = c_u$, $F_{lv} = F_{av} = [\sec(\pi a/2W)]^{0.5}$ and $F_{lu} = F_{au} = \beta$ where β is the nondimensional T-stress for M(T) specimen presented by Henry and Luxmoore (7) for the extended range of the crack length. Theoretical predictions resulting from such estimation were found to be close to the σ_{ni} values deduced from the tests of

TABLE 1 - Effects of size-scaling and load biaxiality on fracture toughness.[a]

λ [c]	k [b]	C_1 [b]	σ_{ni} [b] (MPa)	$K_i(\lambda, k)$ [b] (MPa·m$^{0.5}$)	$K_i(\lambda,k)/K_i(\lambda=1,k=0)$		
					1163AT alumin.(1)	AISI 1045 steel (8)	Xerox paper (8)
1	-1.0	51.9	145.0	77.6	0.97	-	-
	0.0	50.8	149.6	80.1	1.00	1.00	1.00
	1.0	49.8	154.5	82.7	1.03	-	-
2	-1.0	72.5	104.4	79.1	0.99	-	-
	0.0	71.4	106.7	80.8	1.01	1.12	1.45
	1.0	70.4	109.2	82.7	1.03	-	-
4	-1.0	101.7	74.8	80.1	1.00	-	-
	0.0	100.6	76.1	81.5	1.02	1.20	1.90
	1.0	99.6	77.2	82.7	1.03	-	-

[a] The data in question relate to the original crack length $c_u = 0.25W$.

[b] This data relate to aluminium specimens.

[c] For aluminium specimens $\lambda = W$ (mm) / 600 mm.

large specimens only (1). It is reasonable to suppose that the plastic zone corrections were too small to produce a considerable deviation of the large specimens response from purely elastic. Consequently, Eq. (1) is adequate at least when the specimen in question has the width $2W \geq 1200$ mm. In what follows this specimen is considered as the baseline geometry. Its sizes were scaled up by the factors of $\lambda = 2$ and 4. The same size-scaling had been used by Sinclair et al (8) for specimens fabricated from the baked Xerox paper and the AISI 1045 steel tested at -196°C. Both materials were treated as unusually brittle. With a knowledge of the σ_{ni} stress, it is possible to calculate the relevant values of different characterisation parameters used in the CM. To be correlated with very definitive results reported in (8), the fracture toughness of the 1163 AT aluminium was expressed in terms of K (Table 1). As to the baseline geometry ($\lambda = 1$, $k = 0$), the mean experimental value $K_i = 82.4$ MPa m$^{0.5}$ taken from (1) is close to K_i calculated from the σ_{ni} stress. In a qualitative sense the UM predictions agree well with the size dependence of the fracture toughness for two embrittled materials which are basically different in nature. According to the CM, the σ_{ni} stress should decrease with increasing size as one upon the square root of the scale factor, that is in the same manner for any brittle material. However, Table 1 indicates conclusively that the size-scale effect of concern is material dependent. The UM predictions show a definite decrease of the fracture toughness when the load Q in Fig. 1 is changed over from tensile to compressive. This conclusion is in qualitative accord with the theoretical and experimental results of Eftis et al (9). It is seen from Table 1 that the effect of the load biaxiality on K_i should gradually decrease as the absolute sizes of specimen are increased.

CONCLUDING REMARKS

A new approach to the one-parameter characterisation of the crack-tip fracture has been outlined. It appears to be a sound and practicable refinement in the CM, if for no other reason than to bring the elastic response of an ideal crack border into coincidence with that of an actual crack border. Owing to this in fracture analysis the crack may be treated as any stress concentrator without invoking stress singularities - the atribute of the CM. Consequently, the need to make physically sensible interpretations of the nonphysical singular fields is eliminated. This innovation readily leads to a number of evident improvements, for instance, the effects of the crack length, load biaxiality, specimen geometry and its boundaries constraint on the elastic crack-tip stress, $\sigma_1 = C_1\sigma$, could be estimated individually and in combination (see Eq. 2 and Table 1). In the long run, the UM approach may be thought of as an alternative to the two-parameter approach of the CM which has gained the status of a common recognition in the contemporary fracture mechanics.

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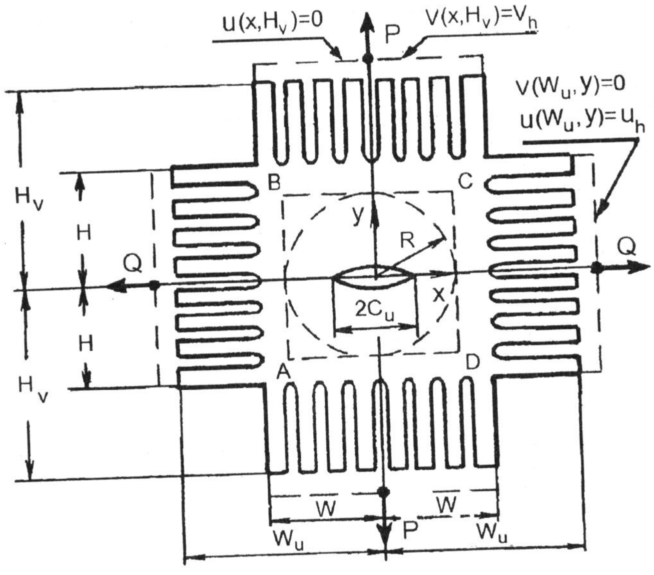

Figure. 1 A middle-cracked specimen with a low-constrained central square and rigidly clamped loading arms for biaxial tension.

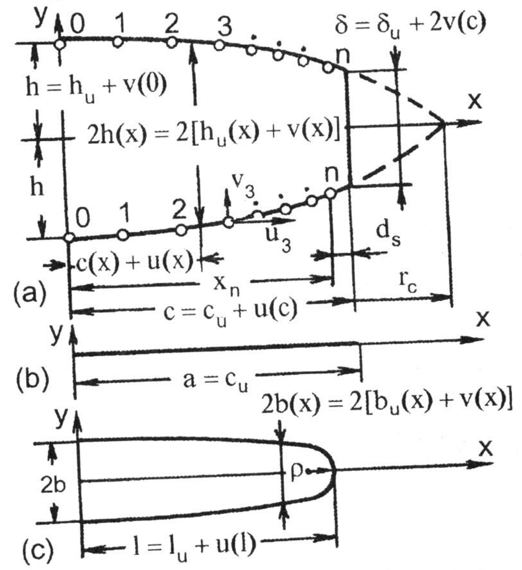

Fig. 2 Schematic sketches of (a) the deformed profile of an actual center crack, (b) and (c) ideal cracks used in the CM and UM analyses, respectively.