A STATISTICAL MODEL FOR COALESCENCE OF DISPERSED FLAWS.

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A statistical model for the formation of associations (clusters) from the several coalesced flaws which are randomly scattered in a bounded region of a material was developed. The dependence between the cluster number and the total flaw concentration was established. Two criteria of ultimate state which correspond to the values of critical concentration of accumulated flaws are substantiated for plural fracture (phenomena which is characterized by the processes of nucleation and propagation of microscopic flaws). These criteria are the threshold concentration at which the coalescence of isolated flaws is started and the concentration which characterizes the transition to the stage of total coalescence of flaws. The results obtained on the basis of the theoretical model are agreed well with experimental data for coalescence of surface cracks under low cycle fatigue of the superalloy IN738LC.

INTRODUCTION

The coalescence of dispersed flaws is an essential factor of materials damage. In the structural models for accumulation of damage (Bolotin (1), (2)) the criterion of ultimate state is the critical concentration of failed structural elements. In this case it is agreed that the maximum value of loss in material density at the expense of accumulation of microscopic flaws is an universal quantity equal approximately 1%. Surface cracks coalescence leads to their accelerated growth, sudden formation of a crack with inadmissible length or even to failure. It exists for fatigue (Ochi et al (3)), low cycle fatigue (Olschewski et al (4)), thermal fatigue (Winkler et al (5)), corrosion (Parkins and Singh (6)), under impact loading of metals and splitting of rocks (Seaman et al (7)). The cracks coalescence is a random event and depends on the following main factors: surface or volume density of flaws, crack lengths inhomogeneity, as well as relative position and force interaction between closely spaced cracks. In the present work the problem on modelling of dispersed flaws coalescence without regard to their interaction was considered and a method to determinate of critical density values of flaws with quasi-homogeneous sizes is proposed.

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Modelling of flaws coalescence

Initial principles. It is considered a bounded material region D which contains a large number N ($N >> 1$) of closely packed cells with a linear size a. In so doing, $D = Na^d$, where d is the index of space dimension. Only one flaw exponent: crack, pore, damaged grain, inclusion, fibre can be placed in each cell. The number of damaged cells is n, they are randomly dispersed in the region D (a Poisson's field of flaws) and their concentration is determined as $\tilde{n} = n / N$. If s ($s \in [2, z]$) of damaged cells are placed alongside each other (interaction between flaws is not taken into account), then are considered as merged and form flaws cluster with mass s. z is the number of the nearest neighbouring cells for one cell, which depends on the way of configuration of cells, $z = 2...14$ (Kittel (8)).

Statistical model. The probability of closely spacing of s flaws - p_s will be equal to the product of the probabilities of two independent events: 1) a flaw will be spaced in any randomly chosen cell, $p_{1s} = n / N = \tilde{n}$; 2) from z neighbouring cells $s-1$ will be damaged, $p_{2s} = [(s-1)!]^{-1} (nz / N)^{s-1} \exp(-nz / N)$, where nz / N is the average number of flaws in a sample from z cells.

The required probability will be then defined as

$$p_s = p_{1s} p_{2s} = \frac{z^{s-1}}{(s-1)!} \left(\frac{n}{N}\right)^s \exp\left(-\frac{nz}{N}\right) \quad \text{.....(1)}$$

In general case, from N cells it is possible to generate N_s clusters with mass s, so that: $N_s = N / s$. If there are n_s clusters with mass s which are formed from n damaged cells, then at $N >> 1$ takes place the approximation

$$p_s \to \frac{n_s}{N_s} = \frac{s n_s}{N} \quad \text{.....(2)}$$

The formula (1) with regard to equation (2) will be transformed to the form

$$\frac{n_s}{N} = \frac{z^{s-1}}{s!} \tilde{n}^s \exp(-z\tilde{n}) \quad \text{.....(3)}$$

The expression (3) gives the relationship between the concentration of s-mass clusters ($s \le z$) and the general flaw concentration $\tilde{n}$.

The function (3) has a maximum at $\tilde{n}_m$ (Fig. 1), such that

$$\tilde{n}_m = \frac{s}{z} \quad \text{.....(4)}$$

If healing up of flaws is not available, then the reduction of the clusters number at $\tilde{n} > \tilde{n}_m$, as well as decrease of cluster formation rate at $0 < \tilde{n} < \tilde{n}_m$ can be determined only by the coalescence of flaws and the formation of new clusters with greater mass.

Quantitative results

The adequacy of model (3) is confirmed by data from the simulation of plural fracture of solid. On numerical experiment the random co-ordinates of damaged cells were generated and the clusters number n_s was determined. The fracture of surface ($d = 2$) for $z = 6$, $N = 400$ and 900 was simulated. It was derived that the formation of large flaws and their intensive growth begin at approximately an identical level of flaw concentration ($\tilde{n} = 0.1 - 0.2$).This value of $\tilde{n}$ according to relationship (4) corresponds to the maximum of theoretical curve for $s = 1$ (Fig. 1). The highest intensification of the destruction process, that is possible to identify with the formation of an infinite cluster, is executed at $\tilde{n} = 0.33$ that corresponds to the maximum of the curve $n_2(\tilde{n})$ (eq. (4), Fig. 1).

The model (3) was used for the numerical valuation of surface cracks coalescence under low cycle loading of the superalloy IN738LC on data from work (4). The cracks placed over the area $D = 22.82\,\text{mm}^2$, the average crack length was $a \approx 0.1\,\text{mm}$. In this case: $N = D / a^2 = 2282$. The crack nucleation rate is constant and equals: for polished specimens $H = 0.03$ (mm^{-2} cycles^{-1}) and for non-polished specimens $H = 0.016$ (mm^{-2} cycles^{-1}). Then for the cracks concentration we will have the dependence $\tilde{n} = a^2 Ht$ where t is the number of loading cycles. Based on the accepted designations we will write the formula (3) as

$$F(t) = Ht \exp(-za^2 Ht) \qquad (5)$$

where $F(t) = n_1(t) / D$ is the crack density over the surface of specimen (cracks number per unit area).

The experimental data from the reference (4) and the calculation data from the formula (5) for $z = 6$ are shown in Fig. 2. The model describes well the real process of coalescence in an initial stage. The disparity at large F ($t > 200$ cycles) derives from the fact that in the formula (5) the increase of average crack length a on numbers of cycles was not taken into account (such data is not available in work (4)).

The process of coalescence according to experimental and theoretical data is characterized by following critical values of dispersed flaws concentration: $\tilde{n}_a$, $\tilde{n}_m$, and $\tilde{n}_c$ (Fig. 1).

$\tilde{n}_a$ corresponds to a moment of deviation of the dependence $n_1(\tilde{n})$ from a straight line that means a beginning of coalescence process. Because the first cluster with $s = 2$ is formed at $\tilde{n}_a$, from the condition $n_2(\tilde{n}_a) = 1$ and the formula (3) we will obtain the approximation

$$\tilde{n}_a \approx \left(\frac{2}{zN}\right)^{0.5} = \left(\frac{2a^d}{zD}\right)^{0.5} \quad \text{.....(6)}$$

For $z = 6$ at $N \in [10^3, 10^4]$, $\tilde{n}_a \in [5 \cdot 10^{-3}, 2 \cdot 10^{-2}]$. In particular, from simulation (Fig. 1) $\tilde{n}_a = 0.019$, while according to data from reference (4) the calculated values are following: $F_a = 1.2\ \text{mm}^{-2}$ ($t_a = 40$ cycles) for polished and $F_a = 0.76\ \text{mm}^{-2}$ ($t_a = 47$ cycles) for non-polished specimens (Fig. 2). The correspondence between the experimental and the calculated data can be recognized as satisfactory.

The critical concentration $\tilde{n}_m$ divides the stage of clusters formation and growth (at $\tilde{n} < \tilde{n}_m$) and the stage of their intensive coalescence (at $\tilde{n} > \tilde{n}_m$) which is finished by formation of an infinite cluster at $\tilde{n}_c$. From the formula (4) for $z = 6$ and $s = 1$ follows $\tilde{n}_m = 1/z = 0.167$. On the basis of the equation (3) we have $n_1(\tilde{n}_m) = N / ze$. For experimental data from work (4) the number of cycles t_m which corresponds to a maximum of crack density at $\tilde{n}_m$, will be determined as $t_m = (Hza^2e)^{-1} = 204$ cycles for polished specimens that conforms with observations (Fig. 2).

The theoretical model (3) does not describe the formation of an infinite (penetrating) cluster, because the mass of bounded clusters is limited by the lattice co-ordination number z. However, it is believed that a penetrating cluster is formed at a concentration $\tilde{n}_c$ which corresponds to a maximum of the function $n_2(\tilde{n})$ and equals the critical concentration of clusters $\tilde{n}_m$ with mass $s = 2$. By expression (4) for $z = 6$, $\tilde{n}_c = 0.33$ (Fig. 1). However, for a loaded material the criterion $\tilde{n}_c$ has not a practical significance, because the failure owing to flaws coalescence will occur at lesser values of concentration than $\tilde{n}_c$.

Only the criteria $\tilde{n}_a$ и $\tilde{n}_m$ are applicable in practice. If information on the average size of flaws and the flaw rate nucleation is available, then the critical stages of flaws coalescence process, then flaws with inadmissible sizes are formed can be predicted. The refinement of obtained valuations is possible by their adjusting taking into account of flaw sizes inhomogeneity.

SYMBOLS USED

D = bounded region of material

N = number of closely packed cells in D

d = index of space dimension

a = linear size of cell

n = number of damaged cells in D

s = number of merged cells which form one cluster with mass s

n_s = number of clusters with mass s

z = number of the nearest neighbours for one cell

$\tilde{n}$ = general concentration of flaws

$\tilde{n}_a$ = concentration of flaws which mark a beginning of coalescence process

$\tilde{n}_m$ = concentration of flaws which corresponds to a maximum number of s -mass clusters

$\tilde{n}_c$ = concentration of flaws which corresponds to a penetrating cluster formation

H = crack nucleation rate per unit area (mm^{-2} $cycles^{-1}$)

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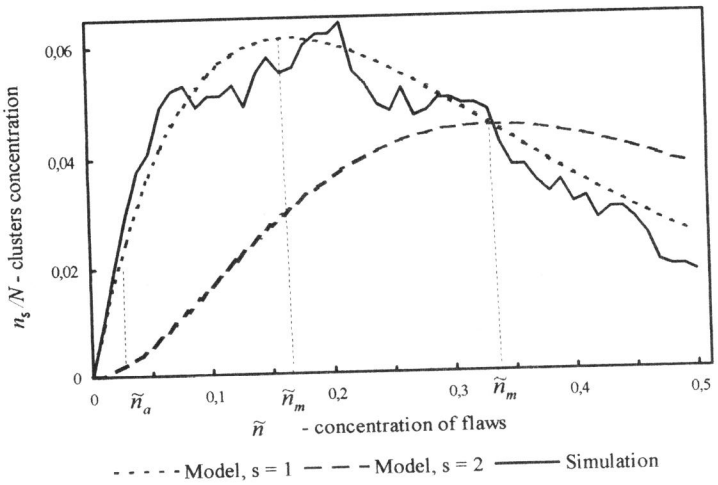

Figure 1 Evolution of the cluster concentration with general concentration of flaws

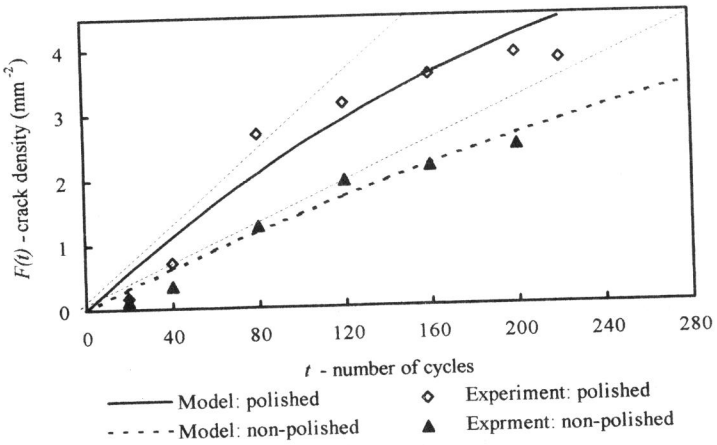

Figure 2 Model and experimental data on evolution of the crack density for Inconel 738LC under low cycle fatigue