PREDICTION OF FATIGUE CRACK GROWTH UNDER COMPLEX LOADING WITH THE SOFTWARE SYSTEM FRANC/FAM

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For economic prosperity in the rigorous global market short product development periods are essential. Thus, the application of CAE-methods and the simulation of product performance on fast computers is enforced. This presentation deals with the simulation of fatigue crack growth under complex loading conditions. The software system FRANC/FAM, developed at the Institute of Applied Mechanics at the University of Paderborn and the Cornell University, New York, with its many options for the prediction of crack growth and lifetime analysis will be presented. FRANC/FAM enables practical-revised simulations of fatigue crack growth under complex loading. It possesses modules of practical significance for strength and fracture analyses and hence allows the prediction of damages. Crack growth in the vicinity of holes and starting from multiple cracks is shown exemplary.

INTRODUCTION

Alongside short development times, low production costs and environmental compatibility, the expert dimensioning is most important for the development of technical components. Thus technical designers, developers, and engineers are increasingly attempting to verify the components' behaviour under the most realistic operating conditions in the early stages of the development phase. The rapid development of computers through constantly improving computing and memory performances enables the intensified application of computer-assisted tools during the optimisation and new development of products. The program systems which were developed in this context in recent years mainly focus on calculations under static loading without considering the special attributes of fatigue. Therefore, in order to be able to make predictions about fracture-mechanics component behaviour, the developers are in most cases dependent on complex experimental investigations consuming considerable time and money. The program system FRANC/FAM has been created to remedy the situation. It comprises fracture mechanics and strength modules of practical relevance which allow a realistic forecast of crack processes under fatigue conditions.

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CRACK GROWTH UNDER COMPLEX LOADING

In contrast to mode I fatigue loading, the crack trajectories under cyclic complex loading, i. e. superimposed mode I and mode II, as well as under superimposed monotonous loading no longer tend towards the direction of the original crack. The cracks experience a more or less large diversion and hence have to be described with substantially more complicated theories.

The crack growth rate can be described during cyclic loading as a function of the cyclic stress intensity factors (SIF) ΔK_I and ΔK_{II} as follows:

$$\frac{da}{dN} = f(\Delta K_I, \Delta K_{II}) = f(K_{I\max}, K_{I\min}, K_{II\max}, K_{II\min}) \qquad (1).$$

As in mode I, $K_{II\max}$ and $K_{II\min}$ represent the upper and lower bound of the stress intensity and ΔK_{II} the cyclic stress intensity factor ($\Delta K_{II} = K_{II\max} - K_{II\min}$) for mode II. It can be defined as

$$\Delta K_{II} = \Delta\tau\sqrt{\pi a}\, Y_{II} \qquad (2).$$

The criterion of fatigue by Henn et al [1] is a contribution to the description of crack growth under superimposed cyclic normal and shear loading. It is based on the Generalised Fracture Criterion by Richard [2] and assumes the validity of the modified law according to Erdogan and Ratwani [3]:

$$\frac{da}{dN} = \frac{C'(\Delta K_v - \Delta K_{th})^{m'}}{(1-R)K_c - \Delta K_v} \qquad (3)$$

The cyclic comparative stress intensity factor ΔK_v is applied as:

$$\Delta K_v = K_{v\max} - K_{v\min} = \frac{1}{2}\Delta K_I + \frac{1}{2}\sqrt{\Delta K_I^2 + 4\alpha_1^2 \Delta K_{II}^2} \qquad (4),$$

with

$$\Delta K_I = K_{I\max} - K_{I\min},\ \Delta K_{II} = K_{II\max} - K_{II\min} \qquad (5),$$

$$R = \frac{K_{I\min}}{K_{I\max}} = \frac{K_{II\min}}{K_{II\max}} = \text{const.} \qquad (6)$$

and a material parameter α_1 [1, 2]. For the calculation of the crack angle φ_0 the following approximation equation can be used:

$$\varphi_0 = \mp\left[A\frac{|K_{II}|}{|K_I| + |K_{II}|} + B\left(\frac{|K_{II}|}{|K_I| + |K_{II}|}\right)^2\right] \qquad (7).$$

For fatigue crack propagation the constants of A = 140,9° and B = -80,4° apply for φ_0 in degrees of angle ($\varphi_0 < 0$ for $K_{II} > 0$ and $\varphi_0 > 0$ for $K_{II} < 0$ [2]).

In order to be able to make predictions about the applicability of the concepts for the description of the fatigue crack growth under complex loading, crack growth rates, crack angles, threshold values, and crack toughness were experimentally determined among other things. These and other data were entered for the numeric investigation of fatigue crack growth.

NUMERICAL SIMULATIONS USING FRANC/FAM

The computer-based simulation of the propagation behaviour of cracks during fatigue loading requires a solution strategy, with which a multiplicity of functions must be mastered. For this reason the most close-to-reality determination of the crack paths and component life is the focus of attention.

The total concept of FRANC/FAM (Fig. 1) is developed and implemented on this basis. The starting point for the development was the program system FRANC of Cornell University, New York (Wawrzynek and Ingraffea [4]). It supplied important aids to the solution of sub-tasks comprising an automatic network generator, a finite element (FE) solver and a module for the numeric determination of the SIF. Extensive new developments and modifications to the fracture-mechanics analysis were completed (May [5]). In particular, the simulation control for the consideration of several cracks in a structure, the lifetime determination and the examination of the technical boundary conditions were added within FRANC/FAM.

In FRANC/FAM, the fracture-mechanical calculations are executed on the basis of linear elastic fracture mechanics using the ΔK-concept. This concept is based on the elastic stress field of a crack and is applicable within small expansions of the plastic zone in front of the crack tip. For a close-to-reality simulation it is necessary to know the size of the plastic zone in order to check the validity of the ΔK concept for the calculations.

A possibility for the calculation of the expansion of the plastic zone in front of a crack tip is offered by an approximation back on Irwin, which is described in detail in May [5]. Under the assumption that yielding will occur on exceeding the yield stress σ_F and by using a suitable yielding criterion, it is possible to measure the places where the yield point is exceeded, and hence the size of the plastic zone with the well-known elastic stress distribution.

During the investigation of technically relevant crack problems particular consideration must be given to the occurrence of multiple cracks in a structure. If these cracks are able to propagate, they will grow in the real structure at different propagation rates when they are loaded differently. During the application of the incremental simulation this circumstance has to be considered for numerical crack propagation processes.

FRANC/FAM is an interactive program for the computer-assisted simulation of crack growth with different functions for linear-elastic finite element calculation and fracture-mechanical evaluation. In particular, it offers the possibility to automatically simulate crack growth independently from crack forms and locations in any plain structures, which were situated under any plain loading situation. The mentioned fundamentals enable a comprehensive and close-to-reality simulation of crack growth processes under complex fatigue loading.

CRACK GROWTH IN STRUCTURES

Using the software tool FRANC/FAM, an exemplary presentation of two practice-relevant cases of crack problems is given below. The materials data of the aluminium alloy AlZnMgCu 1.5 F53 (7075 T6) is used for the simulations (see table 1).

TABLE 1 — Material Parameters of the Aluminium Alloy AlZnMgCu 1,5 F53

Young's Modulus	E	=	70 656	MPa
Proportional Limit	$R_{p0,2}$	=	517	MPa
Tensile Strength	R_m	=	579	MPa
Crack Toughness	K_{Ic}	=	27.26	$MPam^{1/2}$
Coefficients for the modified law by Erdogan and Ratwani	C	=	0.00146	—
	m	=	0.572	—
	ΔK_{th}	=	5.5	$MPam^{1/2}$

The loads used in the FE calculations are converted into the inner forces' normal force N(t) and transverse force Q(t) of the plain structure which occur in the crack environment (Fig. 3a, t = time).

STRUCTURE WITH INNER CRACKS

In the first example the mutual influence of several internal cracks under pure tensile loading is examined. The geometrical arrangement of the cracks represented in Fig. 3a is selected. The structure is loaded with $N_{max} = 63$ kN and $Q_{max} = 0$, which corresponds to an applied normal stress of $\sigma_{max} = 70$ N/mm^2. N_{max} and Q_{max} correspond to the maximum values of N(t) and Q(t) respectively.

The simulation can be started since for all cracks the stress intensity exceeds the threshold value ΔK_{th}, and the plastic zone is small with respect to the component dimensions. After the 54th simulation step the threshold value is exceeded by crack tip 1. The calculated life up to the critical crack length of $a_c = 15.3$ mm is reached amounts to $N_P = 102\,986$ load cycles. For all cracks the final crack lengths and the pertinent cyclic comparative stress intensity factors are summarised in table 2.

While the crack tips 1 and 4 spread in their original direction, a curvature is observed with the trajectories of the crack tips 2, 3, 5 and 6. This result is basically attributed to the mutual influence of the cracks.

TABLE 2 — Crack Lengths and ΔK_V Values for all Cracks after the Simulation

crack tip	1	2	3	4	5	6
a [mm]	15.3 $= a_c$	13.3	13.0	14.4	14.7	14.6
ΔK_V [N/mm$^{3/2}$]	775.8 $= \Delta K_c$	653.9	609.4	747.2	681.5	661.9

SEVEN-HOLE STRUCTURE WITH INNER CRACK

In the second example the influence of seven holes, arranged asymmetrically with respect to the crack plane, on the propagation behaviour of an internal crack is examined. Two different load cases are regarded. In the load case A with $N_{max} = 54.0$ kN and $Q_{max} = 0$ kN the structure is loaded only by a pure tensile force, whereas load case B with $N_{max} = 6.99$ kN und $Q_{max} = 26.1$ kN indicates a clear mixed-mode loading at the crack tips. The values were selected in such a way that the threshold value is just exceeded for each load case at least at one crack tip.

TABLE 3 — Results of the numerical Simulation

	crack tip 1		crack tip 2		
load case	a [mm]	ΔK_v [N/mm$^{3/2}$]	a [mm]	ΔK_v [N/mm$^{3/2}$]	N_p [cycles]
A	30.0	775.8 $= \Delta K_c$	30.2	775.8	256 312
B	22.3	775.8 $= \Delta K_c$	22.7	273.6	416 527

For both load cases ΔK_v reaches ΔK_c. The calculated numbers of cycles, the crack lengths, the number of simulation steps, and the appropriate comparative stress intensity factors for the beginning of unstable crack growth are specified in table 3.

In the load case A the crack tips spread on an easily curved path. This is attributed to the influence of the holes and thus to the stress distribution (see Fig. 4a). For this load case the crack growth rates processed for both crack points are alike.

For load case B the crack trajectories turn out differently (Fig. 4b). Due to a high mode II proportion both cracks leave their original propagation direction and spread, with different growth rates. This is shown in Fig. 2. Up to a crack length of 18 mm the propagation rate of crack tip 2 is higher than that of crack tip 1. Subsequently, the growth rate for crack tip 2 significantly decreases as it comes into the influence of a hole. Crack tip 1 now grows more rapidly and becomes unstable. The simulation stops after the 35th simulation step, since ΔK_v amounts to ΔK_c.

Further simulation examples of multiple cracks in a hole environment may be found in Richard et al. [6] also.

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FIGURES

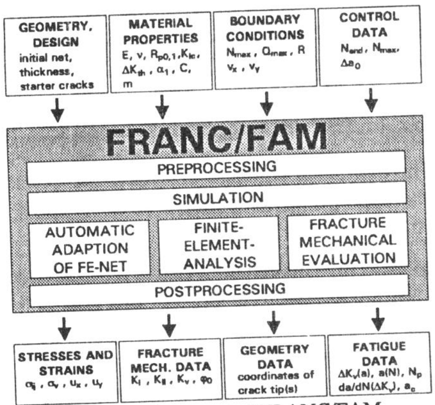

Figure 1: Overall concept of FRANC/FAM

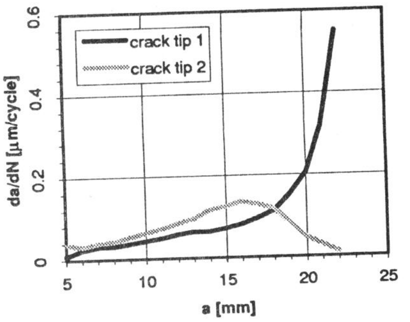

Figure 2: Crack velocity versus crack length for seven-hole structure for N_{max}=6.99 kN and Q_{max}=26.1 kN

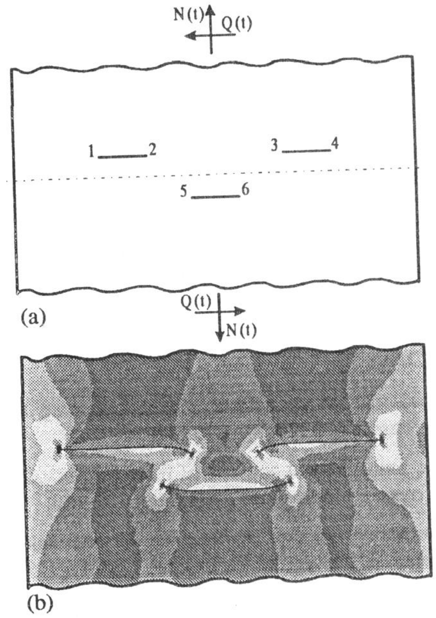

Figure 3: Structure with inner cracks
(a) geometry and loading
(b) crack paths after the simulation with N_{max}=63.0 kN and Q_{max}=0 kN

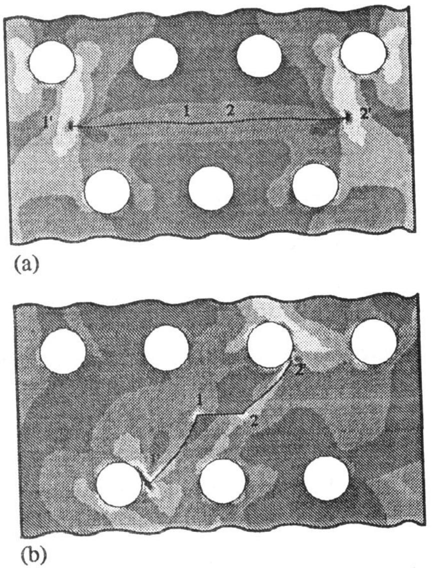

Figure 4: Seven-hole structure with inner crack
(a) load case A: N_{max}=54.0 kN, Q_{max}=0 kN
(b) load case B: N_{max}=6.99 kN, Q_{max}=26.1 kN
1,2: crack tips before the simulation
1', 2': crack tips after the simulation