STRESS INTENSITY FACTORS FOR A SEMI-ELLIPTICAL CRACK UNDER VARIOUS STRESS DISTRIBUTION.

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Stress intensity factors for a semi-elliptical surface crack under various stress distribution were analysed by the body force method (1,2). A semi-elliptical crack is assumed on the y-z plane in a semi-infinite body, where the origin of the coordinate is at the surface of the semi-infinite body. Various stress fields, on y-z plane, are assumed, such as constant, linear and parabolic, etc... in the z-direction. The numerical results are arranged in a set of equations so that we can estimate easily the stress intensity factor for a crack subjected to a complex stress field. A practical example of the prediction of the growth of a crack within a complex stress field is given.

INTRODUCTION

The general framework of the present study is the modelling of 'mechanically' short crack propagation, e. g. cracks propagating from a notch root, or cracks within the influence of a shot peening residual stress field ... Existing set of equations (3), does allow the calculation of the stress intensity factor for a combination of a constant stress and a linear stress field. But it happens, that this could not be sufficient (4). An engineering example of this is the prediction of the crack propagation from a notch. In many components, notches are plastified at the first loading, the subsequent cycles being purely elastics. The crack growth can therefore be predicted from linear elastic fracture mechanics analysis, provided that a correct solution for the stress intensity factor is used (5). Two opposite effects are induced. The first one, which is beneficial, results from the compressive residual stresses at the notch root generated by the plastic deformation during the first cycle.

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The second effect, which is detrimental, results from the stress concentration induced by the notch. This study provides a set of equations that allows the computation of the stress intensity factor for such complex stress fields.

STRESS INTENSITY FACTOR COMPUTATIONS.

A semi-elliptical crack is assumed on the y-z plane in a semi-infinite body, where the origin of the coordinate is at the surface of the semi-infinite body (Fig 1). The crack is characterized by its depth b, and the a/b ratio, where $2a$ is the surface length of the crack. The stress field is given in the following form , where σ_o is a constant and where $m < 4$:

$$\sigma(z)=\left(\frac{z}{b}\right)^{m}\sigma_0 \qquad (1)$$

The stress intensity factor is given in the following form, where F_I is called the stress intensity magnification factor :

$$K(\theta,m,b/a)=\sqrt{b}\,\sqrt{\pi}\,F_I\,\sigma_0 \qquad (2)$$

F_I has been computed for $m<4$ and for crack aspect ratios of $0.5 \leq b/a \leq 2$.

The computations have been conducted by essentially the same method developped in Ref. (2) for a semi-elliptical crack in a semi-infinite body. However, the method has been revised in this study in terms of the determination of F_I . With increasing the numbers of elements in meshing a cracked area, F_I has been determined with high accuracy by extrapolating the calculated body force densities from inner points to crack front. All these calculations results in a set of curves, $F_I(\theta)$, for various combinations of the parameters $(m,\ b/a)$. The effect of the crack aspect ratio (b/a) on the stress intensity factor magnification factor F_I., is plotted on figure 2 This magnification factor is varying, for $\theta = 0$, from a maximum close to 0.9 for $b/a=0.5$, to a minimum close to 0.45 for $b/a = 2$. The crack tip stress intensity factor of a tunelling crack (b/a > 1) is strongly reduced as compared with that of a semi-circular crack of the same length.

EQUATIONS SET

These equations can be easily introduced in a post processing routine of finite element analysis of an uncracked body. The variables are $(\theta,\ b/a,\ m)$ and the functions are $(F_I,\ F_0,\ Gm,\ C_0,\ C_2,\ C_4,\ Nm,\ Pm)$. F_I is given by :

$$F_I(\theta,m,b/a)=\left\{1+\frac{2\,(-1+Gm)\,\mathrm{ArcTan}[1000\,m]}{\pi}\right\}F_0 \qquad (3)$$

The function Fo, is the stress intensity magnification factor for a semi-elliptical crack in a semi-infinite body subjected to a constant stress. For polynomial stress fields one additionnal function is needed, Gm, to describe the evolution of the stress intensity magnification factor F_I.

$$F_0(\theta, b/a)=C_0+\theta^2 C_2+\theta^4 C_4 \quad \text{.......(4)}$$

With :

$$C_0\left(\frac{b}{a}\right)=1.225-\frac{0.8512\,b}{a}+\frac{0.3414\,b^2}{a^2}-\frac{0.0561\,b^3}{a^3} \quad \text{.......(5)}$$

$$C_2\left(\frac{b}{a}\right)=-0.54781+\frac{0.97969\,b}{a}-\frac{0.52601\,b^2}{a^2}+\frac{0.10557\,b^3}{a^3} \quad \text{.......(6)}$$

$$C_4\left(\frac{b}{a}\right)=0.11569-\frac{0.18205\,b}{a}+\frac{0.09851\,b^2}{a^2}-\frac{0.02134\,b^3}{a^3} \quad \text{.......(7)}$$

And :

$$Gm(\theta, b/a)=Nm+Pm\,\mathrm{Cos}[\theta]^m \quad \text{.......(8)}$$

With :.......(9)

$$Pm(m, b/a)=0.493-0.096\,m+0.009\,m^2+\frac{b\left(0.105+0.0645\,m-0.0105\,m^2\right)}{a}-\frac{b^2\left(0.0204+0.0039\,m-0.00069\,m^2\right)}{a^2}$$

$$Nm(m)=0.3-0.165\,m+0.025\,m^2 \quad \text{.......(10)}$$

For all combinations of parameters investigated, the equation (3) is at less than 1.7 % of the results given by the body force method. (Herein, « percent error » is defined as the least square error normalized by the maximum value of the function).

A PRACTICAL EXAMPLE.

Double notched specimens of nickel base superalloy N18 have been tested under creep fatigue loading conditions at 650°C (5,6). Finite element analysis have been conducted, employing the elastoviscoplastic constitutive equations established for this alloy. The notch root is highly plastified at the first cycle. Then the computation of the further cycles shows a strong stress redistribution behind the notch within a few cycles. After, about 30 cycles the tensile stresses and strains become steady and the stress-strain hysteresis vanishes. The results of four computations are presented on Fig. 4. The first plot (A), represents the stress-strain curve obtained for a point at 0.5 mm from the surface, for a maximum applied stress of 1000 Mpa. The second plot (B), presents the steady stress fields obtained for applied stresses of 700, 800, 900 or 1000 MPa, which are obviously very far from the elastic solution. These stress fields haven been taken into account, by employing the set of formulae proposed above. A polynomial has been fitted on each stress field. Initial crack length and shape are assumed *($b=100$ μm and $a=90$ μm)*. From these data the stress intensity factor can be determined. Then by using a Paris law, established from experiment conducted on smooth specimen, the crack extent along the surface and at 90° of the surface is determined. From these computations the following results can be presented : Fig. 5(A), the crack aspect ratio as a function of the crack length is plotted for each stress field. Fig. 5 (B), the stress intensity factor as a function of crack length is plotted for each case. The obtained stress intensity factors are very similar for all the viscoplastic computations. As a matter of fact, within the first millimeter, from the notch root, the local tensile stress fields obtained for applied stresses of 1000, 900, 800 or 700 MPa are very similar, due relaxation of the material during the first cycle.

CONCLUSIONS

Stress-intensity factors obtained from computations using the body-force method were used to develop an empirical set of equations for the stress intensity factor for a semi-elliptical crack in a semi-infinite body within a complex tensile stress field. The form of the stress field is a polynomial of the third order. The equation applies for any parametric angle and ratios of crack depth to crack length ranging from *0.5 to 2.0*, which allows the determination of stress intensity factors for tunneling cracks. For all configurations, the fitted set of equations is at less than *1.7* % of the body force results. The set of equations was used to determine the stress intensity factor for a crack growing from a plastified notch, with prediction of the crack shape and stress intensity factor during the growth of the crack. In this particular case the crack can present alternatively a b/a ratio less than *1* and larger than *1* during its growth. The stress intensity factor equations presented herein should be useful for correlating fatigue crack growth rates of 'mechanically' short cracks.

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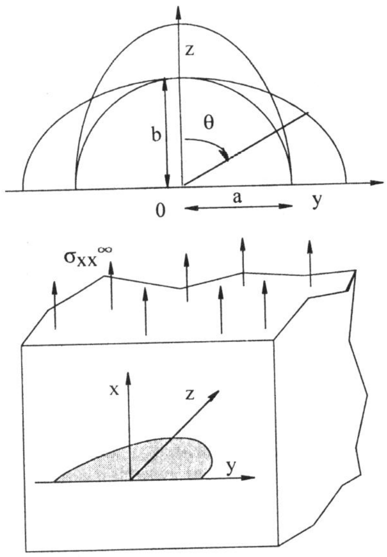

Figure 1 Axis and coordinate system of the semi-elliptical crack.,

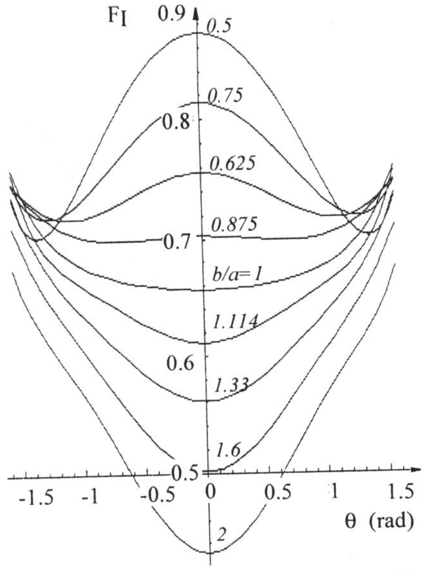

Figure 2 Stress intensity magnification F_I, along the crack front. ($m=0, \nu=0.3$)

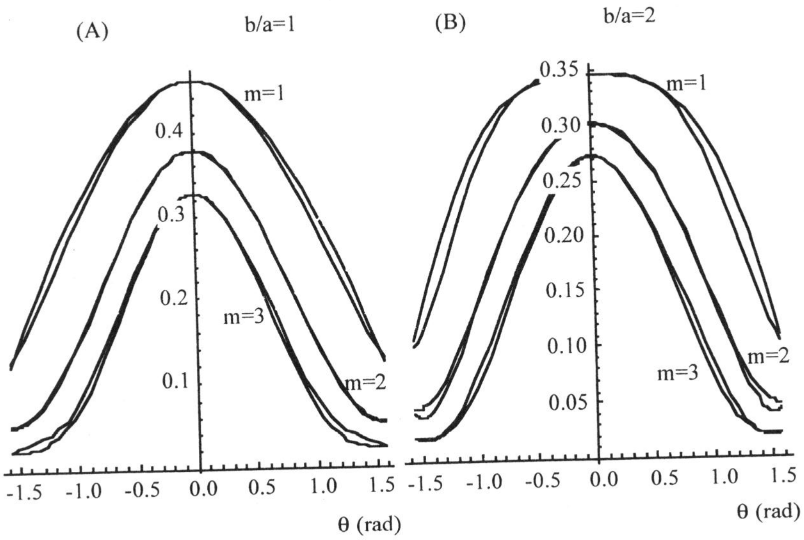

Figure 3 Stress intensity magnification factor F_I, for m = 1, 2 or 3, ($\nu=0.3$). (A) : $b/a=1$, (B) : $b/a=2$. Comparison between the analytical fit and the computations.

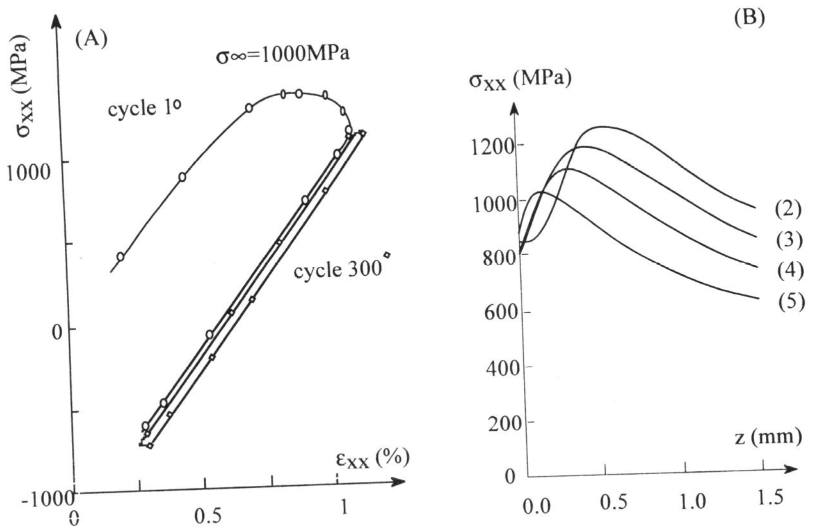

Figure 4 Complex stress fields at the root of a plastified notch, in a Nickel base superalloy subjected to creep-fatigue at 650°C.[5,6]. (2) 1000 MPa, (3) 900MPa, (4) 800 MPa, (5) 700 MPa.

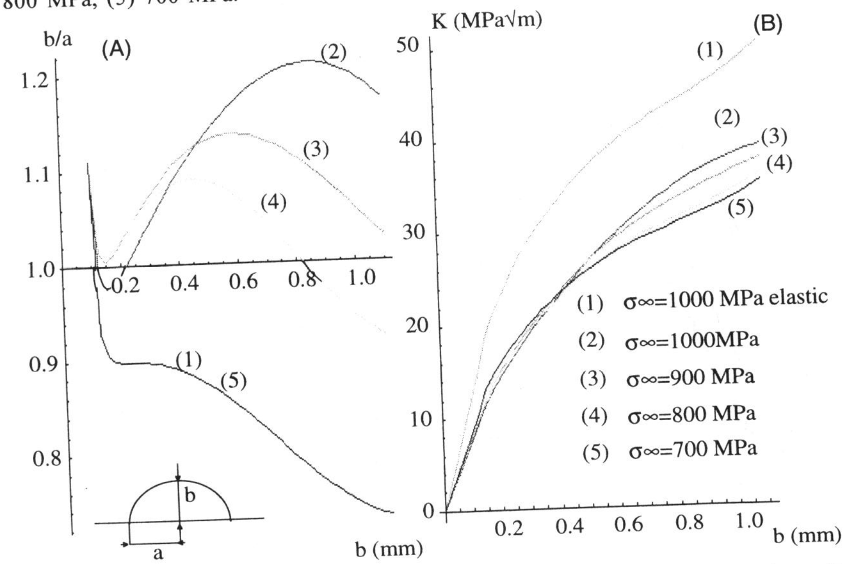

Figure 5 (A) Crack aspect ratio as a function of the crack length. (B), stress intensity factor as a function of the crack length for various applied stresses : (1) 1000 MPa (elastic), (2) 1000 MPa, (3) 900MPa, (4) 800 MPa, (5) 700 MPa (viscoplastic).