STRESS INTENSITY FACTORS FOR OBLIQUE EDGE CRACK UNDER TRAVELLING LOAD

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The complete linear elastic fracture mechanics solution for an inclined edge crack in a semiplane under a concentrated load applied on the boundary is proposed. The superposition principle was applied by using a complete set of Weight Functions determined for the specific geometry.

Complex Mode I and Mode II SIF distributions were obtained which include discontinuities when the applied load approaches the crack mouth. Even though contact stresses have been neglected in the present solution, some interesting indications can be obtained for the fatigue crack propagation in components with tensile stresses near the surface due to external loads or residual stresses.

INTRODUCTION

The presence of an edge crack inclined with respect to the normal at the external surface, is a quite usual situation in several mechanical components. These kinds of cracks can be a consequence of surface fatigue or wear and can be observed for instance in gears or rolling contact bearings. For these components, the presence of a travelling load which cyclically passes over the crack mouth is an usual loading condition (for instance a ball of a rolling contact bearing moving along the external or internal ring). In order to predict the fatigue life of these components, on the basis of a fracture mechanics approach, a predictive procedure should appear quite attractive which is capable of evaluating with satisfactory accuracy and efficiency the Stress Intensity Factor (SIF) for the inclined crack versus the position of the travelling load. This evaluation is made complicated by the simultaneous occurrence, for the great majority of crack geometry and loading conditions, of both Fracture Modes I and II, due to the lack of symmetry of the problem.

The Weight Function (WF) method appears to be particularly suitable for solving this problem, as it allows the SIF being evaluated by a simple integration, over the crack length, of the nominal stress that is the stress due to the load in the uncracked body.

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In the present paper, a complete specific WF recently developed by the authors was applied to analyse the case of a travelling concentrated force acting on the external surface of a semi-infinite body carrying an inclined edge crack. The problem was simplified by neglecting non-linear effects, such as those produced by the contact of the crack edges, and therefore it represents only a first contribution to the solution of the more complex problem of the fatigue crack growth.

WEIGHT FUNCTIONS FOR THE INCLINED EDGE CRACK

The analysis of the inclined edge crack problem (Fig. 1) by the WF approach leads to the following general relationships:

$$K_I(a) = \int_0^a \left[h_{1\sigma}(x,a)\cdot\sigma(x) + h_{1\tau}(x,a)\cdot\tau(x)\right]\cdot dx \tag{1a}$$

$$K_{II}(a) = \int_0^a \left[h_{2\sigma}(x,a)\cdot\sigma(x) + h_{2\tau}(x,a)\cdot\tau(x)\right]\cdot dx \tag{1b}$$

where $\sigma(x)$ and $\tau(x)$ indicate the normal nominal (σ_{xx}) and shear nominal (σ_{xy}) stress components respectively acting on the crack line in the equivalent uncracked body and $h_{M\mu}(x,a)$ are the WFs giving the contribution on Mode I SIF (M=1) and Mode II SIF (M=2) of normal (μ=σ) and shear (μ=τ) nominal stress. It is interesting to observe that the solution of the problem actually requires the knowledge of four distinct WFs, two of which ($h_{1\sigma}$ and $h_{2\tau}$) represent direct effects, while the two others ($h_{1\tau}$ and $h_{2\sigma}$) represent coupling effects (K_I due to shear stress and K_{II} due to normal stress). For an edge crack, coupling effects appear as soon as the crack is inclined ($\theta \neq 0$ in Fig. 1) due to the loss of the symmetry.

These four WFs were determined by the authors (1) for θ in the range (0-80°) making use of the technique discussed in ref. (2) and briefly resumed hereafter. A series of FE analyses were employed to determine the SIFs (both I and II) and the stress acting ahead of the crack tip for some reference loading conditions. A parametric analytical representation for the WFs was assumed which fulfilled the asymptotic properties. By imposing an enough large number of integral relationships relating the WFs to the obtained SIFs, the coefficients of the parametric representations were evaluated by a least square method. The general structure chosen for the WFs was the following:

$$h_{M\mu}(x,a,\theta) = \sqrt{\frac{2}{\pi a}}\sum_i \alpha_i^{(M\mu)}(\theta)\cdot\left(1-\frac{x}{a}\right)^{i-\frac{3}{2}} \tag{2}$$

where the dependence on the angle θ is explicited. The angular functions α were approximated by five term Fourier's expansions of cosines for direct WFs ($M\mu = 1\sigma$ and 2τ), as in the following expression:

$$\alpha_i^{(1\sigma)}(\theta) = \lambda_{i0}^{(1\sigma)}\cdot\tan^2(\theta) + \sum_j \lambda_{ij}^{(1\sigma)}\cdot\cos((j-1)\theta) \tag{3}$$

and of sines for coupling WFs ($M\mu = 1\tau$ and 2σ):

$$\alpha_i^{(1\tau)}(\theta) = \lambda_{i0}^{(1\pi)} \cdot \tan^2(\theta) \cdot \sin(\theta) + \sum_j \lambda_{ij}^{(1\tau)} \cdot \sin(j\theta) \tag{4}$$

being $\lambda_{ij}^{(M\mu)}$ suitable constant coefficients.

The obtained WFs proved to be able to calculate both Mode I and II SIFs with an accuracy better than 1 % for crack inclination angles ranging from 0 to 60° and not worse than 3% for the extreme angular values.

HYPOTHESES AND PROBLEM DEFINITION

The problem of a semi-infinite plane body carrying an inclined edge crack was considered (Fig. 1). The body was loaded with a force uniformly distributed through thickness having intensity P (force per unit thickness) and applied inward the body normally to the surface at a generic distance L from the crack mouth. Material was assumed linear elastic and no contact were considered between the crack edges. Under these assumptions material compenetration is permitted and no effect of friction was taken into account.

For the stress distribution produced by P in the uncracked body (nominal stress) the Boussinesq solution as presented by Timoshenko and Goodier (3) was assumed which gives the analytical expression of the nominal stress components at any position x along the crack, for any crack inclination θ and load position L:

$$\sigma = -\frac{2P}{\pi} \frac{L^2 \cdot \cos^2(\theta) \cdot x}{\left(x^2 + L^2 - 2 \cdot L \cdot x \cdot \sin(\theta)\right)^2} \tag{5a}$$

$$\tau = -\frac{2P}{\pi} \frac{\left(x - L \cdot \sin(\theta)\right) \cdot L \cdot \cos(\theta) \cdot x}{\left(x^2 + L^2 - 2 \cdot L \cdot x \cdot \sin(\theta)\right)^2} \tag{5b}$$

The nominal stress components (5) are analytic and bounded functions of the position x when L is not zero. Particular attention has to be paid when the force approaches the crack mouth. In this case a strong singularity arises at the crack mouth while both stress components tend to zero for any not null value of x. In order to investigate the loading condition for L approaching zero, it is interesting to integrate the nominal stress components along the crack face thus obtaining the resultant forces applied perpendicularly and parallel to the crack respectively. An analytical expression can be obtained for these forces as a function of the parameters P, L, a and θ.

It can be simply verified that the following relationships hold:

$$\lim_{L\to 0^+} \int_0^a \sigma \cdot dx - \lim_{L\to 0^-} \int_0^a \sigma \cdot dx = -P \cdot \sin(\theta) \tag{6a}$$

$$\lim_{L \to 0^+} \int_0^a \tau \cdot dx - \lim_{L \to 0^-} \int_0^a \tau \cdot dx = -P \tag{6b}$$

Relationships (6) demonstrate that, as the applied load crosses the crack mouth, a discontinuity is produced in the global forces inducing the crack loading. The discontinuity is independent of the crack length thus suggesting that this effect is due to the stress properties at the singularity ($x=L=0$). It is worth noting that resultant normal force eqn. (6a) appears discontinuous only for inclined cracks $\theta \neq 0$ while the discontinuity in the global sliding force eqn. (6b) is independent of the crack inclination.

The SIFs were obtained by substituting Eqns (5) into Eqns. (1). Integrals were evaluated numerically with a Gaussian quadrature in the domain and analytically near the points of singularity by using the asymptotic expressions of the functions where necessary.

The following characteristic value:

$$K_0 = P \cdot \sqrt{\frac{\pi}{a}} \tag{7}$$

was used to normalize the obtained SIFs (both Mode I and II). Therefore the proposed results can be considered independent of the scale, as the normalized SIFs:

$$\frac{K_I}{K_0} = K_I^*(\frac{L}{a}, \theta) \tag{8a}$$

$$\frac{K_{II}}{K_0} = K_{II}^*(\frac{L}{a}, \theta) \tag{8b}$$

are functions of the dimensionless distance L/a and the angle θ.

RESULTS AND DISCUSSIONS

Normalized values of K^*_I and K^*_{II} are reported in Figs. 2 and 3 respectively as a function of load position and for different crack inclination angles. From Fig. 2 it can be observed that, excluding θ=0°, a discontinuity occurs in the K_I functions at L/a=0 (i.e. when the force approaches the crack mouth), with different finite left and right limits. For any position of the applied load K^*_I is negative thus indicating the tendency of the crack to be closed under this kind of loading. Negative K^*_I were observed for every crack inclination (up to 80°) and position of the applied load. As it could be expected, the absolute value of the SIF is larger when L>0 (the force is applied to the most deformable part of the body) and the maximum absolute value increases for more inclined cracks.

For the K_{II} a discontinuity is present also when θ=0°. A positive K_{II} value indicates a positive (in the x direction) relative sliding of the upper crack face (y+) as regard the lower face (y-) (Fig. 1). Also in this case, the absolute value of the SIF is larger when the force is applied to the most deformable part and the maximum value increases for more inclined cracks.

For both Mode I and II, when the applied load approaches the crack mouth, the SIF discontinuity resembles the nominal stress discontinuity. However the coupling effects produces a proper contribution to the SIF discontinuity mainly for the most inclined cracks. These discontinuities are due to the idealized point-like force and disappear if a more realistic finite pressure is considered being applied in a region having not zero width (such as due to a Hertzian contact). However a steep SIF gradient versus the load position can be expected in those cases too, if the width of loaded region is small as compared to the crack length.

The K_I -K_{II} locus for a given crack inclination produced by the applied load travelling along the surface boundary from $L/a = -\infty$ to $L/a = +\infty$ is depicted in Fig. 4. The crack tip experiences a quite complex loading history: both SIFs tends to zero when the distance of the load from the crack mouth increases while an sudden jump occurs as the applied load passes across the crack mouth. For the considered crack, the maximum variation of the K_{II} is indeed produced when the applied load crosses the crack mouth.

The SIF cycles so obtained cannot be directly considered in a fatigue crack growth analysis when P is the only applied load. Indeed, the effective SIFs cycle at the crack tip is strongly influenced by the contact actions between the crack faces which are expected in this case. Contact stresses, which are necessarily positive, tend to increase (algebraically) the K_I and, due to the coupling effects, they produce an effect on the K_{II} too. Moreover contact stresses induce friction between the crack faces and, consequently, tangentiual stresses which affect both the SIFs. For a complete solution of this non linear problem, also the COD functions (opening and sliding) have to be determined (as shown by the authors in a more simple problem (4)).

It is interesting to observe that the cycles obtained in the present paper could be employed in a fatigue crack growth analysis if, besides the load P, a tensile stress is acting near the body surface and this stress is high enough to open the crack and to prevent the contact between the faces. An opening effect can also be produced by local tensile residual stresses which, for this reason, makes effective the ΔK due to the travelling load thus producing a strong indirect effect on the fatigue resistance of the component.

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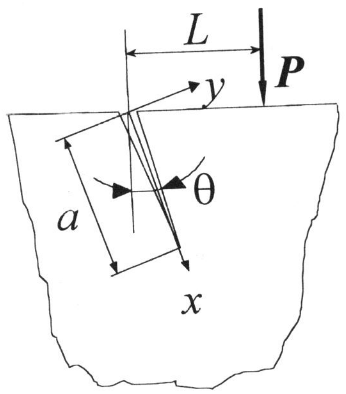

Figure 1. Scheme of the problem

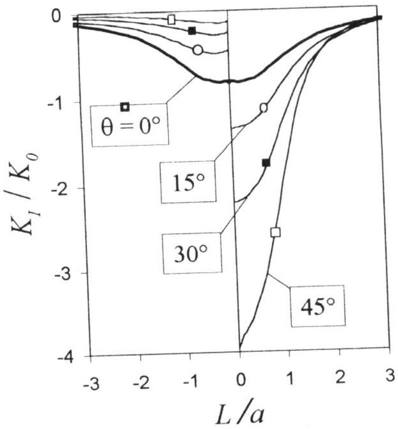

Figure 2. Dimensionless Mode I SIF for different crack inclinations

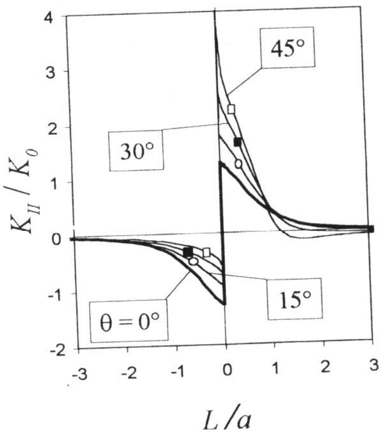

Figure 3. Dimensionless Mode II SIF for different crack inclinations

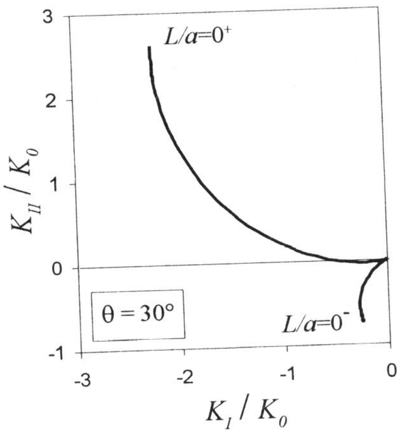

Figure 4. K_I-K_{II} locus for θ=30° produced by the movement of the load