ANALYSIS OF FAILURE LOADINGS AT CONTACT OF ANISOTROPIC AND ISOTROPIC HALF-PLANES WITH LOCAL SURFACE RECESS-TYPE FLAWS

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A two-dimensional elastic contact problem on the frictionless interaction of anisotropic and isotropic half-planes is considered under the assumption that the bodies boundaries have local recesses. Using complex representations of stresses and strains the problem is reduced to a singular integral equation in the gap height function. The unknown limits of integration intervals (the co-ordinates of gaps tips) are derived from the additional conditions to be satisfied by the sought function. A mechanical system with an interface gap due to one symmetrical surface recess is considered as a particular example. The expressions for stresses inside the half-planes are obtained analytically. The strength analysis for the mechanical system components is carried out on the base of Fisher's failure criterion. The recommendations on optimal material orientation in the anisotropic half-plane are made.

INTRODUCTION

Not so long ago we (1) have considered imperfect interaction of contact assembly components provided the rigidity of one of them is essentially greater than the rigidity of another one. We have simulated such a contact as a precompression of an elastic half-plane and a rigid base with locally uneven surface. The contact model developed allows elastic body anisotropy and geometric surface disturbances of the base. It was shown that stresses in the under-surface regions of elastic half-plane rise greatly due to contact imperfectness caused by interface gaps. Grounding on these results quite recently Kryshtafovych (2) has estimated a strength of the anisotropic body being in contact. There were established the most weak for failure zones inside a deformable body dependently on the orientation of elastic symmetry axis of the anisotropic material with respect to the interface.

The purpose of the work reported here was to investigate an interaction of two precompressed elastic bodies, namely isotropic and anisotropic half-planes with surface recess-type flaws and to estimate a strength of such mechanical system.

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TREATMENT OF THE PROBLEM

We are considering two elastic half-planes D_a (anisotropic) and D_i (isotropic), which boundaries have at a certain finite interval small local deviations from a straight line. Let these deviations in the Cartesian co-ordinate system xOy be described by the functions $r_k(x)$ (k=a,i) such that $r_k(\infty) = 0$ and $r_k'(x)$ satisfy Helder's condition. If the half-planes are brought into contact by the external pressure σ_y^∞ then their mechanical contact will be imperfect, i.e. there will be gaps at the interface L=Ox.. We assume the half-planes touch without friction and their boundaries are free from external traction along the unknown hitherto zones of gaps $L' = \bigcup_n [a_n, b_n]$.

Boundary conditions of the problem are the following:

$$\tau_{xy_a}(x,0) = \tau_{xy_i}(x,0) = 0, \quad x \in L; \qquad \sigma_{y_a}(x,0) = \sigma_{y_i}(x,0) = 0, \quad x \in L';$$

$$v_a(x,0) - v_i(x,0) = r_a(x) - r_i(x) = r(x), \quad x \in L \setminus L'; \qquad \sigma_y(x,\infty) = -p \ (p > 0).$$

In order to solve the problem we use Muskhelishvili's (3) and Lekhmitskii's (4) complex presentations of σ_y, τ_{xy} and v through the arbitrary piece-wise analytical functions Φ_k (k=i,a) such that $\Phi_k(\infty) = 0$:

$$\begin{aligned}
\sigma_{y_a}(x,y) &= 2\,\mathrm{Re}\left[\Phi_a(z_1) + s_3\Phi_a(z_2) + s_4\overline{\Phi_a}(z_2)\right] - p,\\
\tau_{xy_a}(x,y) &= -2\,\mathrm{Re}\left[\mu_1\Phi_a(z_1) + \mu_2 s_3\Phi_a(z_2) + \mu_2 s_4\overline{\Phi_a}(z_2)\right],\\
v_a'(x,y) &= 2\,\mathrm{Re}\left[q_1\Phi_a(z_1) + q_2 s_3\Phi_a(z_2) + q_2 s_4\overline{\Phi_a}(z_2)\right],
\end{aligned} \tag{1}$$

$$\begin{aligned}
\sigma_{y_i}(x,y) &= \mathrm{Re}\left[\Phi_i(z) - \Phi_i(\bar{z}) + (z - \bar{z})\overline{\Phi_i'(z)}\right] - p,\\
\tau_{xy_i}(x,y) &= -\,\mathrm{Im}\left[\Phi_i(z) - \Phi_i(\bar{z}) + (z - \bar{z})\overline{\Phi_i'(z)}\right],\\
v_i'(x,y) &= \mathrm{Im}\left[\kappa\Phi_i(z) + \Phi_i(\bar{z}) - (z - \bar{z})\overline{\Phi_i'(z)}\right] / (2G_i).
\end{aligned} \tag{2}$$

A certain form of complex potentials Φ_k is to be determined using the boundary conditions of the problem. Satisfying them by representations (1), (2) gives a series of linear connection problems for the boundary values of Φ_k functions. Having solved them, we express the complex potentials in terms of intercontact gaps height h(x), that is to be determined from a singular integral equation

$$\int_{L'} \frac{h'(t)\,dt}{t - x} = R(x) \equiv \int_L \frac{r'(t)\,dt}{t - x} - \frac{2\pi p}{A_2}\left(1 - \frac{A_2(\kappa + 1)}{G}\right), \quad x \in L'. \tag{3}$$

The sought function should satisfy the conditions $h(a_n) = h(b_n) = h'(a_n) = h'(b_n) = 0$, which have a natural physical sense and are employed for determination of the gap tips.

EXAMPLES AND NUMERICAL RESULTS

As an example we consider the interaction of isotropic and anisotropic half-planes if there is a symmetrical recess

$$r(x) = k_0\left(b^2 - x^2\right)^{\alpha} / b^2, \quad \alpha \geq 1, \quad k_0 << 1 \tag{4}$$

on the boundary of one of the half-planes. In this case there will be one gap between the bodies and the solution to equation (3) can be obtained explicitly. Let us perform the numerical analysis of the problem if the power index in Eq. (4) is $\alpha=3/2$. In this case the following formulae give the problem solution:

$$h(x) = k_0\left(a^2 - x^2\right)^{3/2} / b^2, \quad x \in L', \qquad a = b\sqrt{1 + 4p/(3k_0 A)},$$

$$\Phi_i(z) = (\mu_1 - \mu_2)\,\Phi_a(z)/\mu_2, \qquad z \in D_a \cup D_i,$$

$$\Phi_a(z) = (-1)^s \frac{1.5\,k_0\,A\,\mu_2}{b^2\left(\mu_2 - \mu_1\right)}\left(z\left(\sqrt{z^2 - a^2} - \sqrt{z^2 - b^2}\right) + \frac{a^2 - b^2}{2}\right), \quad z \in D_k,$$

where $A = A_2 / \left(1 - A_2(\kappa + 1)/G\right)$, $s=1$ if $k=i$ and $s=2$ if $k=a$.

Now we are able to calculate the stresses σ_y, σ_x, τ_{xy} inside the half-planes by using Eqs. (1) and (2) and then to estimate the system components strength using a certain criterion. Here we employ the Fisher criterion for anisotropic bodies and the Mises criterion for isotropic ones. It is worth to remind that in accordance with Fisher's criterion (see Fisher (5)) an anisotropic body fails if the parameter V

$$V = \frac{\sigma_1^2}{\sigma_{B1}^2} + \frac{\sigma_2^2}{\sigma_{B2}^2} + \frac{\tau_{12}^2}{\tau_B^2} - 0.5\frac{E_1(1+\nu_{12}) + E_2(1+\nu_{21})}{\sqrt{E_1E_2(1+\nu_{12})(1+\nu_{21})}}\frac{\sigma_1\sigma_2}{\sigma_{B1}\sigma_{B2}} \tag{5}$$

exceeds the value $V=1$.

Put for definiteness $k_0=0.01$, $b=1$ cm and choose the following materials as the materials of interacted half-planes: on the one hand - strongly anisotropic graphite reinforced plastic with relation $E_1/E_2 \approx 25$ ($E_1=149$ GPa, $E_2=6$ GPa, $G_{12}=4$ GPa, $\upsilon_{12}=0.31$, $\sigma_{B1}=1120$ MPa, $\sigma_{B2}=140$ MPa, $\tau_B=63$ MPa) or weakly anisotropic glass-reinforced plastic with relation $E_1/E_2 \approx 1.5$ ($E_1=36.8$ GPa, $E_2=26.8$ GPa, $G_{12}=4.14$ GPa, $\upsilon_{12}=0.431$, $\sigma_{B1}=369$ MPa, $\sigma_{B2}=300$ MPa, $\tau_B=69$ MPa), and on the other hand - isotropic strongly rigid steel ($E=210$ GPa, $G=81$ GPa, $\sigma_B=1000$ MPa) or less rigid brass ($E=100$ GPa, $G=37$ GPa, $\sigma_B=140$ MPa).

Pieces of isotropic and anisotropic half-planes with V-contours plotted for the contact couple brass - graphite-reinforced plastic ($\varphi=30°$) are shown in Fig.1. Below we present the table of maximum values of the parameter V for several couples of contacted bodies materials.

TABLE 1 - V_{max} for some Couples of the Half-planes' Materials.

Material couples	Angle φ						
	0°	15°	30°	45°	60°	75°	90°
graphite plastic rigid	0.361 -	0.571 -	0.903 -	1.03 -	0.792 -	0.558 -	0.024 -
graphite plastic steel	0.37 0.017	0.627 0.018	0.917 0.022	1.06 0.029	0.798 0.037	0.385 0.038	0.038 0.047
graphite plastic brass	0.379 0.678	0.618 0.717	0.931 0.848	1.09 1.1	0.87 1.47	0.42 1.37	0.057 1.63
glass plastic steel	0.121 0.023	0.115 0.024	0.112 0.025	0.113 0.028	0.115 0.03	0.112 0.033	0.109 0.034
glass plastic brass	0.124 0.898	0.118 0.919	0.114 0.981	0.117 1.08	0.118 1.19	0.116 1.28	0.114 1.31

Grounding on the results presented here in the table and illustrated in the figure and generalizing data obtained for other loads, recess shapes and other material couples, we can draw the following conclusions on the strength of the system.

CONCLUSIONS ON THE SYSTEM STRENGTH

Presence of recess-type flaws on the boundaries of interacted bodies can diminish the system's strength several times.

For certain co-orientations of the interface and the elastic axis of anisotropic material the change of recess shape (i.e. the change of power index α in Eq. (4)) does not effect significantly the system's strength.

As it follows from the strength criterion expressions, the zones inside both anisotropic and isotropic bodies, where stresses σ_y and σ_x are relatively large and have the opposite signs are dangerous for failure due to the last term in Eq. (5). In the case of the half-planes interaction in the presence of interface gap, the parameter V reaches its extreme value in the neighbourhood of the initial recess tips and therefore these regions inside the bodies are the most weak for failure.

The picture of V-contours is symmetric with respect to the y-axis in the isotropic body but asymmetric in the anisotropic one if $\varphi \neq 0, 90^{\circ}$.

Reliability of the precompressed bodies system depends greatly on the material orientation inside the anisotropic half-plane, i.e. the same external load can either lead or not lead to the system's failure dependently on the φ value. For example, if the brass and graphite plastic half-planes with a surface recess (4) (α=3/2) are in contact due to the external precompression p=35 MPa, then the object of damage will be the isotropic body if φ=60°, both the half-planes if φ=45° and the angle φ=30° is more dangerous for an anisotropic body damage than for isotropic one.

Mechanical properties of a contact counter-body influence slightly the strength of anisotropic half-plane.

An orientation, at which the direction of the greater Young's modulus E_1 coincides with the interface (x-axis) is more dangerous for failure than the perpendicular orientation. The greater is the relationship E_1 / E_2 the greater is a difference of the anisotropic half-planes strengths for these two orientations.

It is impossible to establish theoretically for different anisotropic materials the unique tendency of V_{max} change with the change of angle φ. Moreover, a large number of elastic parameters of interacted bodies makes it difficult to determine, which component of a stress tensor effects most of all the strength of the body in general.

Strength of the isotropic body decreases with increasing the anisotropic body rigidity in the perpendicular to the interface direction, i.e. the strength diminishes if the angle φ increases in the range 0°→90°.

SYMBOLS USED*

$\bar{1},\bar{2}$ = directions of elastic symmetry axis in an anisotropic material

$_a$ and $_i$ (subscripts) = refers the quantity to anisotropic and isotropic body, respectively

a = gap half-length (m)

b = half-length of the recess (m)

E = Young's modulus of isotropic material (Pa)

$E_1>E_2$ = Young's modulus of anisotropic material in the directions $\bar{1},\bar{2}$ respectively (Pa)

G, G_{12} = shear moduli of isotropic and anisotropic materials, respectively (Pa)

v = normal to the interface component of a displacement vector (m)

υ, υ_{12} = Poisson's coefficients of isotropic and anisotropic materials, respectively

φ = angle between the interface and the direction $\bar{1}$

σ_1, σ_2, τ_{12} = components of the stress tensor in the directions $\bar{1},\bar{2}$ (Pa)

* Other notations used in this paper and not listed here are the same as in (1) and (3).

σ_y, σ_x, τ_{xy} = stresses in the directions of the Cartesian co-ordinate axis (Pa)

σ_{B1}, σ_{B2}, τ_B = strength limits of anisotropic material for compression and shear (Pa)

σ_B = strength limit of isotropic material for compression (Pa)

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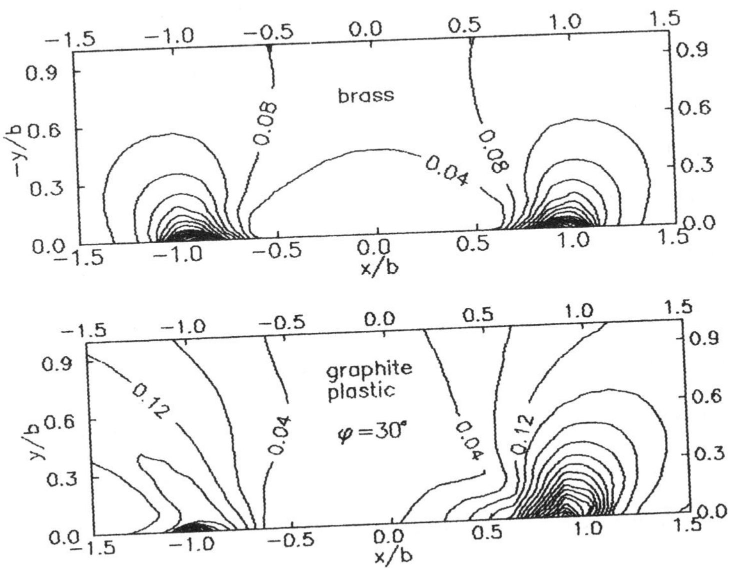

Figure 1 V-contours in the half-planes mated by external pressure p=35 MPa.