THE SOLUTIONS OF ELASTICITY PROBLEMS FOR CRACKED PLATES STIFFENED BY PATCHES

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Stress-strain analysis and determination of the limiting equilibrium of the thin plate stiffened by wide patches and weakened by the arbitrarily located curvilinear crack are given. Discrete joint of the patch with cracked plate and a continuous joint along the boundary of the patch are considered. The problems are reduced to the systems of singular integral and integral-differential equations of the first and the second kinds along the open and closed contours. Numerical investigations are carried out for the stress intensity factors and critical loads of the stiffened cracked plate for various structural and physical parameters of plate, patch and fasteners.

INTRODUCTION

The stress distributions near the tips of a rectilinear cracks in the stiffened plates was studied earlier by Dowrick et al (1), Chen and Hans-Georg (2) for symmetrical plate loading and crack location and by Savruk and Kravets (3, 4) for the arbitrarily cracks location in the plates stiffened by wide patches which are joined continuously along their contours. The integral equations for the cracked plate stiffened by wide patch using the rigid discrete joints was constructed by Savruk and Kravets (5). The purpose of the work reported here, was to apply approach proposed in references (4, 5) to solution of the boundary value problems of elasticity theory and fracture mechanics for the cracked plate stiffened by wide patch using no ideal discrete or continuous attachments in the case of arbitrarily crack location and biaxial plate loading.

FORMULATION OF THE PROBLEMS

Consider the elastic equilibrium of an infinite plate with constant thickness h which is weakened by curvilinear crack L and stiffened by a wide patch S with contour Γ and thickness h_s. The position of the crack in the plate is arbitrary, but $L \cap \Gamma = \varnothing$. Introduce the Cartesian coordinate system xOy related to the crack contour, as shown in Figure 1.

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It is assumed that both the main cracked plate and the wide patch are under the conditions of generalized plane stress. A crack surfaces are not in contact and they are subjected to the self-equilibrating forces

$$h\left\{N^{\pm}(t)+iT^{\pm}(t)\right\}=f(t), \quad t\in L, \tag{1}$$

where $N^{\pm}(t)$ and $T^{\pm}(t)$ are the normal and tangential stresses in the plate as we move to L from the left (+) or right (-). The stresses p and q are specified at infinity of the plate.

Discrete joint. The discrete fasteners (rivet, point welding, bolt or pin) are modeled by a penny-shaped elastic inclusion of small radius r. Their influence on a plate is simulated by arbitrarily directed point forces applied to the centers of circular inclusions with radius r

$$P_k(z_k^0)=X_k(z_k^0)+iY_k(z_k^0), \quad k=\overline{1,N_z}, \tag{2}$$

where N_z is a total number of the fasteners, z_k^0 - centers of inclusions. It is assumed that the moments do not transmit from the fasteners. The contour of the patch is free of loads

$$N_s(t)+iT_s(t)=0, \quad t\in\Gamma, \tag{3}$$

where $N_s(t)$ and $T_s(t)$ are the normal and tangential stresses in the patch. For the discrete fastening the compatibility conditions include the equilibrium of forces

$$P_k(z_k^0)=-P_k^s(z_k^0), \quad k=\overline{1,N_z} \tag{4}$$

and compatibility conditions for the elongation of the N_z-1 intervals in plate and patch

$$\Delta_j\left\{u(z_j^1)+iv(z_j^1)\right\}-\Delta_j\left\{u_s(z_j^1)+iv_s(z_j^1)\right\}=\frac{c_r}{Eh}\left\{P_j^s-P_1^s\right\}, \quad j=\overline{2,N_z}, \tag{5}$$

where $P_k^s(z_k^0)$ are concentrated forces acting upon the patch at the its circular inclusions; $\Delta_j\left\{f(z_j^1)\right\}=f(z_j^1)-f(z_1^j)$, $z_j^1=z_j^0+r\exp(i\beta_j)$, $z_1^j=z_j^1-b_j\exp(i\beta_j)$. The relative flexibility of fasteners $c_r=5Eh/(2rE_r)+0{,}8\{1+Eh/(E_sh_s)\}$ is defined by Creager and Liu (6) for elastic fasteners or $c_r=0$ in the case of rigid discrete fastening. Here E, E_s, E_r are the elasticity modulus of plate, patch and fasteners respectively.

Continuous joint. The continuous hairline joint along the contours of the patch (welding or adhesive joint along a narrow strip) is simulated by joining via a narrow adhesive layer with constant small thickness h_0 and width d_0. It is assumed that the material of this layer is absolutely rigid or operates under the conditions of pure shear. Boundary condition is definite by relation (1) and compatibility conditions have a following forms

$$-h\left\{\left[X_n(t)+iY_n(t)\right]^+-\left[X_n(t)+iY_n(t)\right]^-\right\}=h_s\left\{X_n^s(t)+Y_n^s(t)\right\}=p_0(t), \tag{6}$$

$$\frac{d}{dt}\left\{\left[u(t)+iv(t)\right]-\left[u_s(t)+iv_s(t)\right]\right\}=\frac{h_0}{d_0G_0}\frac{d}{dt}\left\{p_0(t)\right\}, \quad t\in\Gamma, \tag{7}$$

where superscript + (-) denote the limiting value as we move to Γ from inside (outside); $h_0/(d_0G_0)=0$ in the case of rigid fastening; G_0 is the shear modulus of adhesive layer.

SYSTEMS OF THE INTEGRAL EQUATIONS

Discrete joint. Displacement fields of cracked plate and wide patch with system of loading circular inclusions was defined using the method of analytical functions. Satisfying the compatibility conditions (4), (5) with the aid of integral representations of the displacement components $u(z)$ and $v(z)$ we obtain the following equations

$$\Delta_j\left[\sum_{k=1}^{N_z}\left\{\left(F_3(z_j^1,z_k^0,\kappa,\kappa_s,\lambda_s)+\frac{\pi c_r\lambda_s}{2(1+\nu)}(\delta_{jk}-\delta_{1k})\right)P_k+F_4(z_j^1,z_k^0,\kappa,\kappa_s,\lambda_s,r)\overline{P_k}\right\}-\right.$$
$$-\int_\Gamma\left\{F_1(t,z_j^1,\kappa_s)g_s'(t)dt+F_2(t,z_j^1)\overline{g_s'(t)dt}\right\}+2\pi i z_j^1 A+$$
$$\left.+\lambda_s\int_L\left\{F_1(t,z_j^1,\kappa)g'(t)dt+F_2(t,z_j^1)\overline{g'(t)dt}\right\}\right]=2\pi\lambda_s\Delta_j\left\{f_1(z_j^1,p,q,\gamma)\right\},\ j=\overline{2,N_z}\,; \quad (8)$$

where the kernels F_1, F_2 and function F_3, F_4, f_1 can be obtained from reference (5); $\lambda_s=G_sh_s/(Gh)$ is a relative stiffness of the patch, A - unknown real constant; δ_{jk} - Cronnecer's symbol; $\kappa=(3-\nu)/(1+\nu)$, $\kappa_s=(3-\nu_s)/(1+\nu_s)$, where ν and G (ν_s and G_s) are the Poisson's ratio and shear modulus of the plate (patch). The unknown functions $g'(t)$, $t\in L$ and $g_s'(t)$, $t\in\Gamma$ are definite at the boundaries of crack and patch respectively

$$g'(t)=\frac{2Gh}{i(1+\kappa)}\frac{d}{dt}\left\{[u(t)+iv(t)]^+-[u(t)+iv(t)]^-\right\},\quad g_s'(t)=\frac{2G_sh_s}{i(1+\kappa_s)}\frac{d}{dt}[u_s(t)+iv_s(t)]^+\,. \quad (9)$$

By satisfying boundary conditions (1), (3) with the aid of integral representations of the stresses in cracked plate and wide patch we obtain the two singular integral equations

$$\sum_{k=1}^{N_z}\left\{M(z_k^0,t',\kappa)P_k+N(z_k^0,t',r)\overline{P_k}\right\}+\int_L\left\{K(t,t')g'(t)dt+L(t,t')\overline{g'(t)dt}\right\}=\pi f_2(t')\,,\ t'\in L; \quad (10)$$

$$\sum_{k=1}^{N_z}\left\{M(z_k^0,t',\kappa_s)P_k+N(z_k^0,t',r)\overline{P_k}\right\}-\int_\Gamma\left\{K(t,t')g_s'(t)dt+L(t,t')\overline{g_s'(t)dt}\right\}=0,\ t'\in\Gamma\,; \quad (11)$$

where the kernels K, L, M, N and function $f_2(t')$ are defined in reference (5). In order to close the equations system (9) - (11) it is necessary to satisfy the additional conditions

$$\int_L g'(t)dt=0\,;\qquad \sum_{k=1}^{N_z}P_k(z_k^0)=0\,;\qquad \sum_{k=1}^{N_z}\Im\left\{z_k^0P_k(z_k^0)\right\}=0\,. \quad (12)$$

Continuous joint. By satisfying the boundary (1) and compatibility conditions (6), (7) with the aid of integral representations of a stress and a displacement components in the plate and the patch we obtain the system of three integral and integral-differential equations

$$\int_L\left\{K_{11}(t,t')g'(t)dt+L_{11}(t,t')\overline{g'(t)dt}\right\}+$$
$$+\int_\Gamma\left\{K_{13}(t,t')p_0(t)+L_{13}(t,t')\overline{p_0(t)}\right\}ds=\pi f_3(t'),\quad t'\in L\,; \quad (13)$$

$$\frac{\pi(1+\kappa_s)}{2i}g_s'(t')+\int_\Gamma\left\{K_{22}(t,t')g_s'(t)dt+L_{22}(t,t')\overline{g_s'(t)dt}\right\}+$$
$$+\int_\Gamma\left\{K_{23}(t,t')p_0(t)+L_{23}(t,t')\overline{p_0(t)}\right\}ds=0,\quad t'\in\Gamma\,; \quad (14)$$

$$\lambda_s\int_L\left\{K_{31}(t,t')g'(t)dt+L_{32}(t,t')\overline{g'(t)dt}\right\}-\int_\Gamma\left\{K_{32}(t,t')g_s'(t)dt+L_{32}(t,t')\overline{g_s'(t)dt}\right\}+$$
$$+\int_\Gamma\left\{K_{33}(t,t')p_0(t)+L_{33}(t,t')\overline{p_0(t)}\right\}ds+2\pi c_z\frac{d}{dt}\left\{p_0(t')\right\}=0,\quad t'\in\Gamma\,; \quad (15)$$

where the kernels K_{ij}, L_{ij} $(i,j=1,2,3)$ and function $f_3(t')$ can be obtained from reference (4); $c_z = G_s h_s h_0/(aG_0 d_0)$ is a relative attachment flexibility; s is the arc abscissa of the point $t \in \Gamma$. In this case the complement additionally conditions include the relationship

$$\int_{\Gamma} p_0(t)ds = 0 \tag{16}$$

and the first one in the additionally conditions (12).

This approach allows generalizations of the problems for the plate with system of cracks and patches. Fuller details of this technique can be found in references (3)-(5).

NUMERICAL RESULTS

Systems of the integral equations (9) - (11) and (13) - (15) coupled with respectively additional conditions (12), (16) are solved numerically using a mechanical quadratures. Numerical investigations are carried out for the plate with parabolic or rectilinear crack $\omega_1(\xi) = a\left[\xi + i\varepsilon_1(\xi^2 - 1)\right]$, $\xi \in [-1;1]$ and elliptical or circular patch $\omega_2(\tau) = z_0 + R\left[\cos(\tau) + i\varepsilon_2 \sin(\tau)\right]$, $\tau \in [0;2\pi]$. Here z_0 is the complex coordinate of the patch center and a is the half-crack length.

In order to operate the structure economically and safely it is necessary to be able to determine the critical loads of the stiffened cracked plate or the critical length of crack beyond which fracture occurs. These are controlled by stress intensity factors (SIF) at the crack tips, which are affected by structural and physical parameters of the plate, patch and fasteners. Using σ_θ-criterion of fracture the critical loads are determined by relations

$$p_c^{\pm} / p_0 = 1 \Big/ \left\{ \left[\widetilde{K}_{\mathrm{I}}^{\pm} \cos(\theta_c^{\pm}/2) - 3\widetilde{K}_{\mathrm{II}}^{\pm} \sin(\theta_c^{\pm}/2) \right] \cos^2(\theta_c^{\pm}/2) \right\}, \tag{17}$$

where $\theta_c^{\pm} = 2\arctan\left[\left(1 - \mathrm{sgn}(K_{\mathrm{I}}^{\pm}) \sqrt{1 + 8\left(K_{\mathrm{II}}^{\pm} / K_{\mathrm{I}}^{\pm}\right)^2} \right) \Big/ \left(4K_{\mathrm{II}}^{\pm} / K_{\mathrm{I}}^{\pm} \right) \right]$ is the initial angle of crack growth (Panasyuk (7)); $\widetilde{K}_{\mathrm{I,II}}^{\pm} = K_{\mathrm{I,II}}^{\pm} \big/ p\sqrt{\pi a}$ are the mode I and mode II relative SIF in right (+) and left (-) crack tips; $p_0 = K_{\mathrm{IC}} \big/ \sqrt{\pi a}$ is the critical load for the unstiffened infinite plate with Griffits crack. The SIF and critical loads for the stiffened cracked plate are obtained for various geometrical and physical parameters of the plate, crack, patch, adhesive layer and rivets (Fig. 2, 3 for the discrete joined patch and Fig. 4, 5 for the continuous joint along the patch contours in the case $\nu_s = \nu = 0.3$; $N_z = 24$).

The all numerical investigations are carried out in terms of relative parameters of the plate, crack, patch and fasteners. On account of this the obtained results are true for many real engineering materials with various geometrical parameters. Relative patch stiffness, rivets flexibility and flexibility of adhesive layers as well as crack length, its shape and location appear to have a significant effect on the stress and limiting states of the stiffened cracked plate. The patch shape and size effect is not significant for the SIF and critical loads of the plate when the crack tips are not at relative short distance to the patch boundary.

Obtained results are compared with those in reference (1) in the case of uniaxial loading of the cracked plate stiffened by riveted rectangular patch and in (2) for rectilinear crack and circular patch with continuously rigid fastening along the patch contour.

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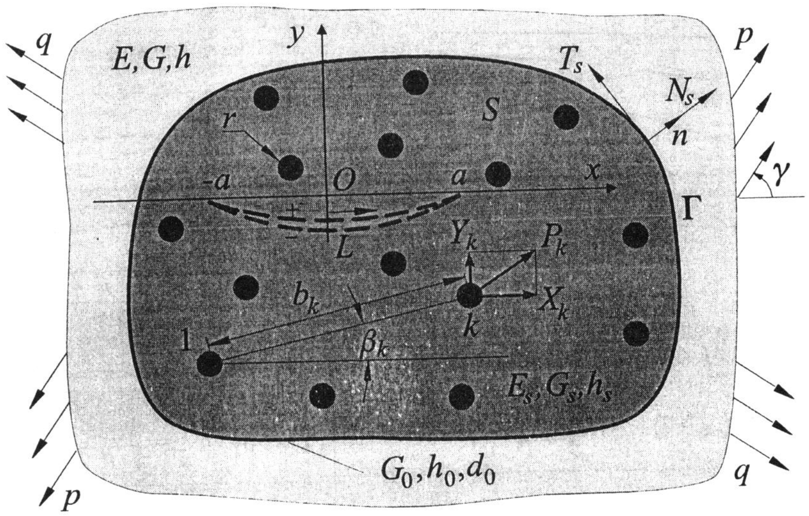

Figure 1. Model of cracked plate stiffened by wide patch

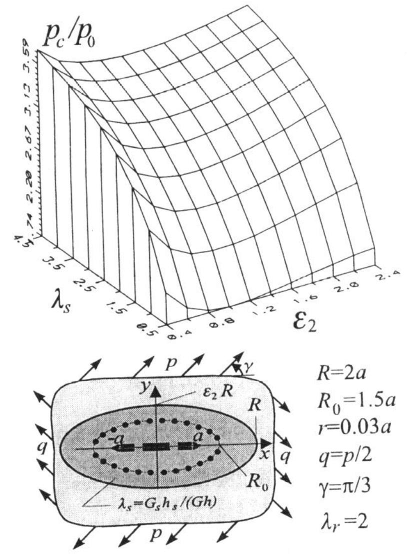

Figure 2. Effect of patch shape and stiffness one on the plate critical load

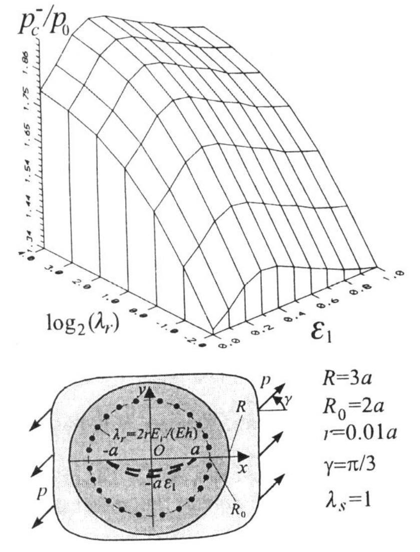

Figure 3. Rivet stiffness and crack shape effect on the critical load for the left crack tip

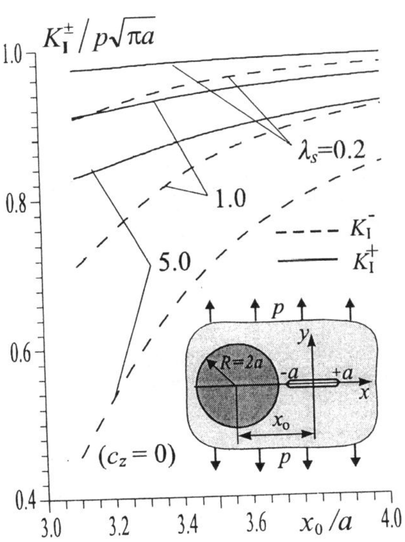

Figure 4. Influence of the external position of circular patch on the relative SIF

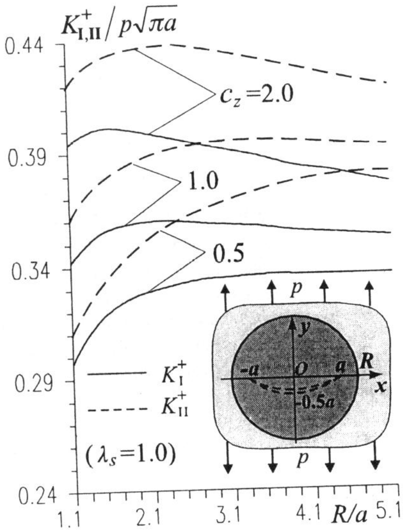

Figure 5. Attachment flexibility and patch size effect on the Mode I and Mode II SIF