CRACK GROWTH INITIATION OF ASYMMETRICALLY BRANCHED CRACK IN AN INFINITE ELASTIC MEDIUM

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In the present work the study of the plane elastostatic problem of crack growth initiation of an asymmetrically branched crack embedded into an infinite body, has been studied. The fracture characteristic quantities which are the critical load for crack extension and the angle at the initial crack propagation, were determined. Values for the stress intensity factors K_I and K_{II} were taken from the literature. Using the Strain Energy Density Criterion, the critical applied stress for crack growth and the angle of crack extension $è_{cr}$, were obtained. Important results concerning the dependence of the most vulnerable branch tip from which further fracture initiates, where derived as well as the effect of the crack geometry configuration on the above fracture characteristic variables.

INTRODUCTION

Experimentally the crack extension has often been observed to occur in different directions from the plane containing the initial crack. These extensions are usually called branches; more than one such branches may develop from the crack tip with various angles and lengths, during the crack propagation. Several publications so far ,were referred to the determination of the stress intensity factors K_I and K_{II} and the stress field near to branch tips. Andersson (1,2), attempted to solve the plane elastostatic problem of a star shaped crack in an infinite sheet. Vitek (3), developed a method of calculating stress intensity factors for branched and bent cracks embedded in an infinite body. Also, Theocaris (4), solved the problem of an asymmetrically branched crack for several geometries. On the other hand, the experimental method of caustics was successfully used by the same author (Theocaris (5), Theocaris and Blonzou (6)) for the determination of stress-intensity factors at the tips of branched cracks. All the above referred cases were limited to the determination of the stress intensity factors and were not extended to fracture phenomena. In the present work the study of the plane elastostatic problem of the symmetrically and asymmetrically branched crack in an infinite body is oriented to the determination of the relevant fracture characteristic quantities. For this reason the strain energy density criterion proposed by Sih (7), is used.

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SPECIMEN GEOMETRY AND MATERIAL PROPERTIES

Let us consider an asymmetrically branched crack composed of the main crack OA=2a=5 cm with two unequal and non symmetrically oriented branches OB and OC inside an infinite isotropic elastic plate as in Figure 1. The branch OB is assumed of constant length b=1 cm, but the length c of the other branch OC varied between c=0.5 and c=4 cm. Also the angles θ_c varied between 15° and 90°. Two values of the inclination angle of the other branch θ_B =15° and 90° are considered. Thus the length ratio c/b varied between 0.5 and 4.0.

The whole plate is under generalized plane stress conditions and the only existing loading is a tensile loading σ applied at infinity and normal to the main crack axis OA.

The material of the examined specimen is an aluminium alloy with the following technical properties:

Young modulus	$E=7.2 \times 10^6$ N/cm^2
Poisson ratio	í=0.33
Critical value of the strain energy density factor	S_{cr}=5.765 N/cm

RESULTS OF THE PARAMETRIC INVESTIGATION

For all the above crack geometry configurations the opening mode K_{IB} and K_{IC} and sliding mode, K_{IIB} and K_{IIC} stress intensity factors at the tips B and C were taken from the diagrams of the above mentioned publication (Theocaris (4)). All these stress intensity factors for both tips B and C are lower than the opening mode K_{IA} for crack tip A.

According to SED theory (S-criterion) the direction of crack initiation, in a general loading, is given by the maximum, of the local minima, of the strain energy density factor (S) or the quantity $(dw/dv)_{min}^{max}$, around the crack tip.. Fracture initiation occurs when the strain energy density factor (S) reaches a critical value S_{cr}, in the direction of crack growth.

Following the Sih' s strain energy density criterion, the critical applied stress σ_{crB} for crack extension and the corresponding angle θ_{crB} were determined for the tip B. The same procedure was then used for the determination of the σ_{crC} and θ_{crC} for the branch tip C. Thus, for a given geometry of the branched crack we obtain for each branch tip B and C, definite value of the critical applied stress $\sigma_{crB,C}$ required for crack propagation. It is evident that the smaller σ_{cr} of these two values $\sigma_{crB,C}$ would represent the critical applied stress of failure. The crack propagation will take place from the tip which corresponds to σ_{cr}

In order to establish the dependence of the fracture characteristic quantities of the plate on the inclination angle θ_B and θ_C and the length ratio c/b of the two branches we considered two cases corresponding to the values of θ_B=30°and 60° and for each case three values of the angle θ_C and length of the branch C between 0.5 and 4 cm (b=1 cm and c/b between 0.5 and 4). Thus our results are presented in the form of variation of the quantities σ_{cr} and θ_{cr} vs the length of the branch C, for various values of θ_c .

Figures 2 and 3 present the variation of the critical applied stress σ_{cr} vs the length of the branch OC for two values of the angle $\theta_B = 30^0$ and 60^0 respectively for crack tip B and C . From Figure 2 we observe that for $\theta_C=15^0$, the crack starts to grow from the tip B for length of the branch c < 0.870 cm and from the tip C for length c > 0.870 cm. For $\theta_C = 45^0$, the crack start to grow from the tip B for c < 1.318 cm and from tip C for c > 1.318 cm. Finally , for $\theta_C = 90^0$, fracture take place from crack tip B for all values of length of branch C. Also, it is observed that, when the crack starts to grow from the tip B the σ_{cr} decreases as the θ_c increases, whereas when the initiation of the growth starts from the tip C the σ_{cr} increases as the θ_c decreases.

From Figure 3 we observe that for $\theta_C=15^0$ the fracture take place from tip C, whereas for $\theta_C = 90^0$ from tip B for all values of length c. For $\theta_C=45^0$ the crack starts to grow from tip B for length c < 0.833 cm and from tip C.

Figures 4 and 5 give the values of the crack extension angle θ_{cr} for the cases of figures 2 and 3 respectively. The discontinuities of the curves are drawn by dotted lines. The reason of discontinuities is the change of the tip from which the ctack extension starts. All the above figures give a good picture of the dependence of the angle θ_{cr} on the crack geometry configurations. Thus, further discussion of the figures is avoided.

CONCLUSIONS

The main results of this study may be summarises as follows:

a) For a wide range of combinations of values for θ_B and θ_C and up to the limiting value of c/b, the fracture of the branched crack starts from the crack tip B. Otherwise the fracture starts from the other branch tip C. Generally, the limiting value of c/b decreases with increasing value of the inclination angle θ_B and with decreasing value of the inclination angle θ_C .

b) When the crack starts to grow from tip B then the critical applied stress σ_{cr} decreases with increasing value of θ_c . The reverse is true for crack initiation from tip C.

c) The newly formed branches after extension either of tip B or C, tend to follow direction parallel to the main crack for values of c/b near to the limiting value for which the fracture change branch tip. Furthermore, as the ratio c/b deviate from this limiting value, the crack extension angle θ_{cr} decreases algebraically .

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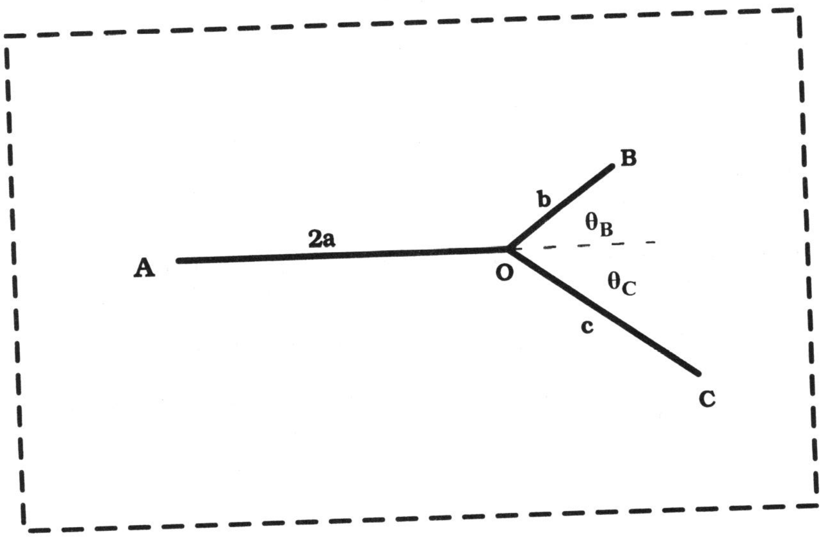

Figure 1. Geometry of the symmetrically branched crack.

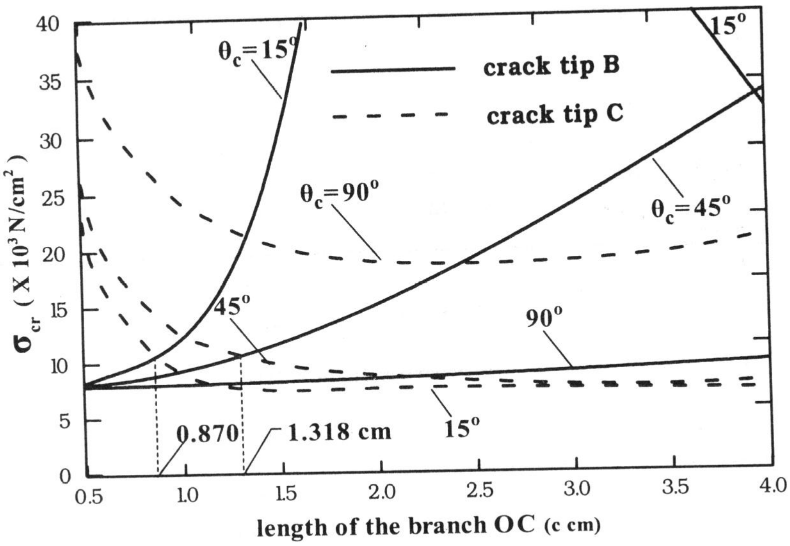

Figure 2. Variation of the critical applied stress (σ_{cr}) versus length of the branch OC for θ_C =30° and for length of the branch OB=b=1 cm.

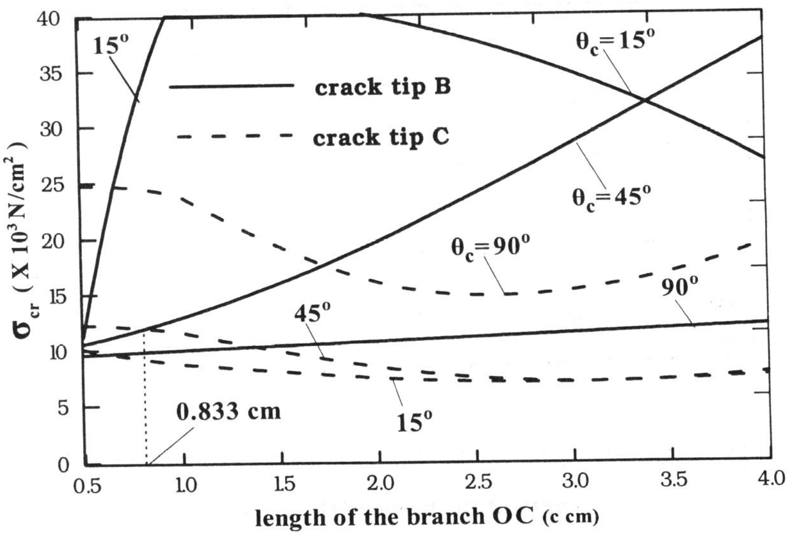

Figure 3. Variation of the critical applied stress (σ_{cr}) versus length of the branch OC for θ_C =60° and for length of the branch OB=b=1 cm.

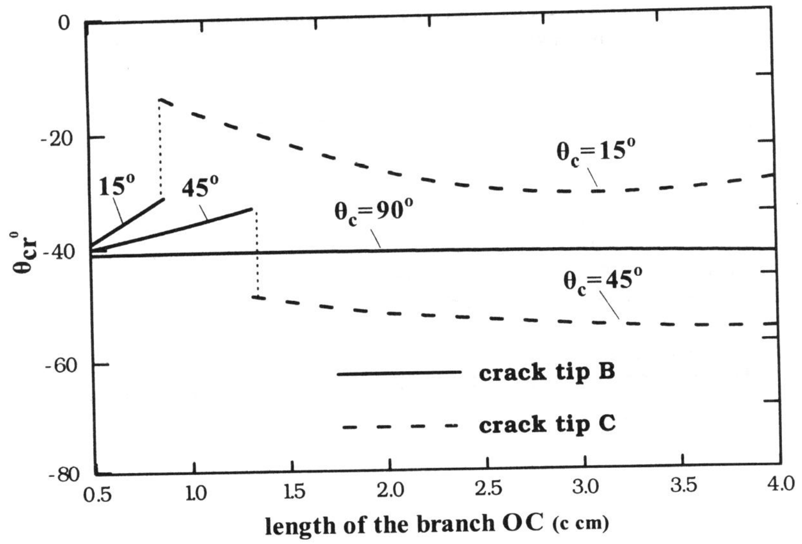

Figure 4. Variation of the initial crack extension angle (è$_{cr}$) for crack initialion versus length of the branch OC for θ_C =30° and for OB=b=1 cm.

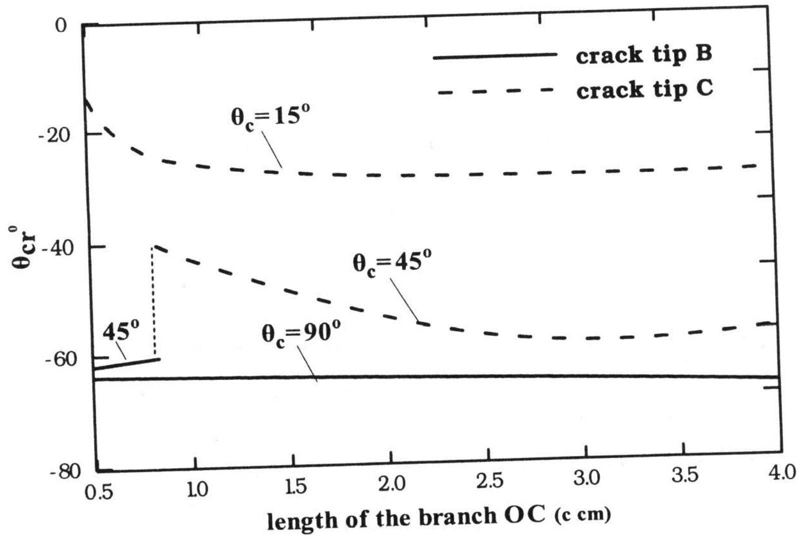

Figure 5. Variation of the initial crack extension angle (è$_{cr}$) for crack initialion versus length of the branch OC for θ_C =60° and for OB=b=1 cm.