THE INNER CUT-OFF RADIUS IN THE RITCHIE-KNOTT-RICE MODEL OF CLEAVAGE FRACTURE

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The stability of micro cracks in front of plastically deformed macro cracks is investigated. The plane strain plasticity is modelled as growth of smeared out dislocation bands on inclined slip planes. We assume that micro cracks form in the highly stressed zone directly ahead of the macro crack tip. It is shown that very small micro cracks near the macro crack tip are stable. For that reason the propabilistic theory of cleavage cracks has to be modified to exclude the zone of stable micro cracks in the integration over the region of all possible micro cracks. This provides a physical explanation of the threshold of cleavage fracture.

INTRODUCTION

Cleavage fracture in metals and intermetallics is a process in two steps. First, the fracture is initiated. This can occur at a weak particle or a lattice inhomogenity in front of the tip or it might be the sudden burst of the material near the highly stressed macro crack tip. Generally it is believed that the former mechanisms apply to technological materials like ferritic steels while the latter one are expected in rather clean metals e.g. in pure Armco iron. The second step of the brittle fracture event is the instable crack propagation.

In the original version of the Ritchie-Knott-Rice (RKR) model (1) of cleavage fracture it is assumed that in order to initiate the brittle fracture a critical stress σ_{cl} has to be overcome in a certain region of size l^* at the crack tip. This σ_{cl} can be interpreted as the cleavage stress of the fracture triggering particle or in case of a burst of clean metals as their theoretical tensile strength. RKR assume that once the fracture is initiated it immediately leads to an instable behaviour of the entire crack system. This simplification does not take into account the importance of the second step of the cleavage fracture. For instance Lin et al. (2) report that in a spherodized 1008 steel some micro cracks in front of the crack tip were stable.

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The conclusion of this experimental result is that not each micro crack initiation automatically causes an unstable behaviour. The reason for this is that during the formation of the micro crack too less energy is supplied to overcome the resistance against a further crack propagation. Stable micro cracking can occur in both cases when the micro crack initiates at a weak particle in front of the tip and when it initiates at or near the macro crack tip.

In this paper we investigate the second scenario. The crack tip plasticity is described by an inclined strip yield model. In the following this method for describing plasticity is explained. Subsequently we analyse the stability of micro cracks formed directly ahead of the macro crack tip. Anticipating the result we shall find that the micro crack formation is unstable for cracks larger than some critical size. For smaller cracks, however, the micro cracks are stable. For that reason an inner cut-off radius should be incorporated into Anderson's (3) statistical interpretation of the RKR model. In doing so one would find a threshold stress intensity for cleavage cracking. This is in agreement with experiments in Armco iron performed by Grundner (4).

THE INCLINED STRIP YIELD MODEL

The inclined strip yield model of crack tip plasticity is explained in detail by Riemelmoser and Pippan (5). It is similar to the Bilby-Cottrell-Swinden (BCS) theory (6) of cracks. The difference is that the singular integral equation is solved numerically. Therefore situations more complex than the BCS arrangement can be analysed. In particular, plasticity can be described on inclined slip planes and in the wake of the crack. It is also possible to incorporate the strain hardening of the material.

In this method for describing plane strain (inclined) plasticity the plastic zone is discretized into uniform displacements elements. These elements are constructed in such a way that they displace the elastic body above the element in respect to the body below the element by a certain length measured in Burgers vectors of the crystal dislocations. An element gives rise to stresses in the total body and in particular in the midpoints of the other elements. We have called the shear stress induced by an element j with the "charge" $d_j = 1$ Burgers vector in the midpoint of the element i the influence function g_{ij}. In (5) this influence function is derived and it is shown that the induced stress in element i for general charges d_j is simply the product $d_j g_{ij}$. The total resolved stress in the midpoint of an element i, τ_i, is then the resolved shear stress of the remote loading, τ_i^a, plus the stress contribution of the elements. It is given by Eq. 1.

$$\tau_i = \sum_j d_j g_{ij} + \tau_i^a \,, \tag{1}$$

The procedure of finding the displacements and of the dislocation density in the inclined slip planes is as follows. First, in a pre-processing, the influence functions g_{ij} for every two elements are calculated. In the main part of the program the crack is loaded in steps. In each step the resolved shear stress in the midpoints of the elements is computed according to Eq. 1. If this stress is larger than a given critical resolved shear stress τ_i^y the charge in the corresponding element is increased. These computation steps are repeated until maximum load. An appropriate way for taking into account the strain hardening is also described in (5).

Material Data

In the present paper we take the material data of Armco iron at $79K$. Experimentally it has been found (4) that here the strain hardening is only weak such that we assume a non-hardening behaviour. The critical resolved shear stress was found to be $\tau_i^y = 600MPa$ and the Young's modulus is $E = 208GPa$. The Poisson ratio is assumed as $\nu = 0.3$.

THE CRACK GROWTH MODEL

In Fig. 1a a loaded crack is sketched. Consider the inclined shear bands and the cohesive stresses ahead of the tip. These cohesive stresses are calculated by generating crack growth elements. The crack is advanced at the maximum stress intensity K_{max} by releasing the nodes of the crack growth mesh in the region, L_c, where the cohesive stresses are larger than the theoretical tensile strength $\sigma_{\text{theoretical}}$. Actually, the cohesive stresses overcome the $\sigma_{\text{theoretical}}$ within a small distance somewhat ahead of the macro crack tip but not directly at the tip where they tend to zero. However, the remaining ligament between micro and macro crack would only be on the order of a few atomic distances. For that reason we assume that the micro and the macro crack spontaneousely coalesce, which leads to a growth of the macro crack as sketched in Fig. 1b. The micro crack formation and its coalescence with the macro crack results in an opening of the crack, Fig. 1b, and in a release of energy. We assume that the real physical time for the crack advance is extremly small such that there is no time available for the crack tip to emit dislocations and to blunt during this critical phase. Then, after the crack propagation two scenarios are possible. First, the released energy is larger than the energy needed for a further crack propagation. In this case an unstable behaviour of the system would be the result. Alternatively, the released energy is less than the required one for further crack advance. We assume that in this case the new crack tip emits dislocations (blunts) whereby the stresses near the tip relax. The result is a stable behaviour of the crack system.

There is an important difference between the above described inclined plasticity model and the conventional continuum mechanics by the aid of the Finite Element Method. In our method the body is seen as a linear elastic continuum. Plasticity is modelled as the arrangement of dislocations (the result is the dislocation density along the inclined slip planes). The numerical advantage is that only the plastic zone has to be discretized in form of the uniform displacement elements. The remainder of the body keeps linear elastic where elastic potentials are well defined. In our case the elastic potentials are given in a complex form as discussed by Kolosov and Muskhelishvili (7). This allows to calculate a stress intensity at the new macro crack tip according to Eq. 2.

$$K = 2\sqrt{2\pi z}\lim_{z\to 0}(\Phi(z)) \qquad (2)$$

The complex potential $\Phi(z)$ has to be understood as the potential induced by the uniform displacement elements and the potential of the remote loading. The main advantage, now, of our method is that the stability analysis can be expressed in terms of the stress intensity factor rather than in an energy balance. The new macro crack is stable when the stress intensity at the tip is smaller than K_{Griffith}. Otherwise it is unstable.

RESULTS

Let us analyse the stability of the micro cracking in Armco iron at $79K$. The Griffith stress intensity is $K_{\text{Griffith}} = 1MPa\sqrt{m}$ and the theoretical tensile strength is assumed to be $\sigma_{\text{theoretical}} = 20000MPa$.

In Fig. 2 the length of the calculated micro crack, L_{cl}, is depicted as a function of the applied maximum stress intensity. As expected, this region grows with the stress intensity.

The corresponding increase of the local stress intensity at the new macro crack tip induced by the micro cracking is shown in Fig. 3. In our example the system becomes unstable when the increase is equal to or larger than $1MPa\sqrt{m}$. This critical limit is also shown in Fig. 3 (dashed line). It is seen that at small stress intensities the micro cracking is stable and it is unstable at large ones. The crossing over defines the critical stress intensity, K_{Ic}, for cleavage fracture. The calculated value of $29MPa\sqrt{m}$ agrees quite well with experimentally measured ones (4).

DISCUSSION

Such numerical analyses are necessarily simplifications of the complex behaviour of the material near crack tips. Maybe the most severe approximation is that we limit our considerations to 2 dimensions while a real micro crack formation is a 3D phenomenon. It is hardly predictable how this influences the stability analysis. However, we are more interested in trends rather than in absolute numbers. An interesting result is that the predicted K_{Ic} is very sensitive to variations in K_{Griffith}. Let us again consider Fig. 3. Here we see that a variation in K_{Griffith} of only $0.2MPa\sqrt{m}$ would lead to a change of K_{Ic} of about $6MPa\sqrt{m}$. At a real crack front such small variations of K_{Griffith} are very likely. For instance, the elastic anisotropy induces mismatch stresses near grain boundaries which in turn influences the K_{Griffith} value[1]. Another source for a fluctuation in K_{Griffith} would be an inhomogeneous dislocation distribution near the crack tip. For that reason it is clear that in experiments K_{Ic}-values

[1]Here intrinsic stresses which are effective only very locally are seen as an additional term in the crack growth resistance rather than as a contribution to the driving force. This is the ususal point of view of material scientists.

often have a large scatter.

In this paper we consider a deterministic system of micro and macro cracks. In the analyses we have found that very small micro cracks near the macro crack tip cannot cause final cleavage fracture. The RKR theory for technological material has been modified by Anderson (3) to include the propability of finding fracture triggering particles in front of the crack tip. Here possible locations of critical particles are integrated from the crack tip to the length l^* which is given by RKR. Our analyses, however, show, that critical particel at or very near the crack tip may cleave without causing an instable behaviour of the entire system. For that reason an inner cut-off radius has to be introduced in the propabilistic formulation of the RKR theory. This modified theory predicts a threshold of cleavage fracture in agreement with experiments.

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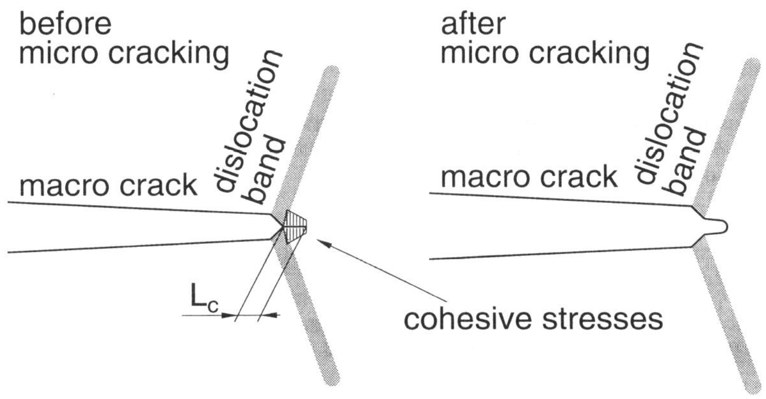

Figure 1: A crack tip before and after the micro cracking.

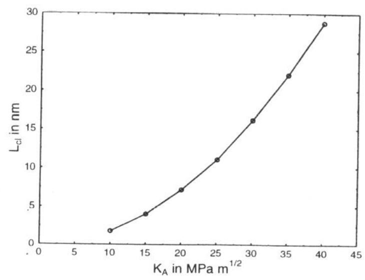

Figure 2: Size of the micro crack in dependence on the applied stress intensity.

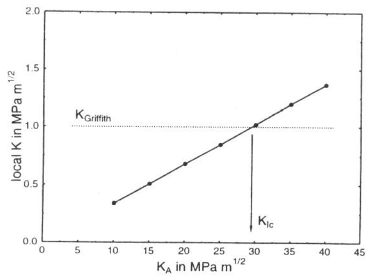

Figure 3: The stress intensity at the new macro crack tip after the crack advance.