FLAW ASSESSMENT IN STRUCTURES OF STRAIN HARDENING MATERIAL

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Energy release rate of central cracked plate of non-linear elastic material under tension is presented by two terms, describing a crack caused change of energy along edge and central axis of a plate. The first term is a drop of energy along central normal to crack axis of plate caused by crack. The second term is an increased of energy, due to crack, along edge of plate. Relation between these components depending on crack size rate and strain hardening exponent is found. *J*-estimation scheme for elasto-plastic crack in term of these components was found to be more convenient. An analytical estimation of *J*-integral for any value of strain hardening exponent, which being in good agreement with finite element results, is carried out.

INTRODUCTION

A fracture avoidance criterion of structure containing flaw can be presented as a condition of avoidance of crack growth initiation (Ainsworth(1)) in the form $J \leq J^{c}_{mat}$, where J is a parameter offered by Cherepanov (2) and Rice (3), which is the basic measure of the stress-strain state in a vicinity of a crack tip under monotonous loading, J^{c}_{mat} is constraint dependent critical value J at crack growth initiation. For J determination the model of a non-linear elastic body is convenient, as then J is energy release rate with crack growth .The uniaxial tensile stress-strain relationship of such materials is $\varepsilon / \varepsilon_0 = \alpha(\sigma / \sigma_0)^n$, where ε_0, σ_0 and n are yield strain, yield stress and strain hardening exponent, respectively, and α is a material constant. Then for elastic-plastic bodies J-estimation is based on interpolation between linear and nonlinear meanings (Ainsworth (1), Kumar et al (4), Bloom (5) and Turner (6)). The aim of this paper is to develop J-integral estimation and methods of interpolation between linear and nonlinear cases for simple central cracked plate geometry.

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DECOMPOSITION OF ENERGY RELEASE RATE ON COMPONENTS

We consider a centre-cracked plate under tension with sufficiently large height *2H,* so that the disturb field of stress and strain about a crack does not reach ends (Fig.1). In a linear case it is possible by virtue of Saint-Venant principle, according to which height of a disturb zone is proportional to the width of plate *2c*. However, in a case, when the parameter of strain hardening exponent tends to infinity, this principle is invalid. On the basis of (7) it is possible to assume, that at $n \leq n_* < \infty$ with sufficiently large height ($H > \sqrt{nc}$) the disturb field will fade on height in a plate not reaching the ends. Stress and strain in a plate with a crack may be presented in the form:

$$\begin{cases} \varepsilon_{ij} = \varepsilon_{ij}^{(0)} + \Delta\varepsilon_{ij} \\ \sigma_{ij} = \sigma_{ij}^{(0)} + \Delta\sigma_{ij} \end{cases} \tag{1}$$

where $\sigma_{ij}{}^{(0)}$, $\varepsilon_{ij}{}^{(0)}$ are components in plate without crack ($\sigma_{yy}^{(0)} = \sigma_g$, $\sigma_{xx}^{(0)} = \sigma_{xy}^{(0)} = 0$, $\varepsilon_{yy}^{(0)} = \varepsilon_g = \Delta/H$, $\varepsilon_{xy}^{(0)} = \varepsilon_{zz}^{(0)} = 0$, $\varepsilon_{xx}^{(0)} = -\nu\varepsilon_{yy}^{(0)}$, $\nu = 0.5$); $\Delta\sigma_{ij}$, $\Delta\varepsilon_{ij}$ are components of disturbans.

Qualitative character of distribution of stress and strain in plate can be presented in form of several zones (Fig.1). Around of the crack there is a zone of unloading {1} covering its surface. Here the components of the disturb field compensate the components of the undisturb field ($\Delta\sigma_{yy} \approx -\sigma_g$, $\Delta\varepsilon_{yy} \approx -\varepsilon_g$). In addition, the high loaded zone {3} appears. It includes a zone of high stress and strain concentration in the vicinity of crack tip {2}, and reaches the edge of the plate. In this zone the disturb strain can considerably exceed uniform strain ($\Delta\varepsilon_{ij} >> \varepsilon_g$).

By choosing a contour of integration on edge of a body and on central line of symmetry (Fig.1), *J*-integral may be presented as:

$$J = \Delta W_{L_1} + \Delta W_{L_2}\,, \tag{2}$$

where $\Delta W_{L_1} = \int\limits_{L_1} \left(w - w_g\right) dy$; $\qquad \Delta W_{L_2} = \int\limits_{-L_2} \left\{\left(w_g - w\right) + \sigma_{xx}\varepsilon_{xx}\right\} dy$.

Some components here are neglected, as by virtue of symmetry of a field and equality of stress to zero at free edge of a body, and also the uniformity of the undisturb field, they are equal to zero. Here also to these components gross energy density w_g with opposite signs is added, that does not change expression for *J*. However now the integrands are not equal to zero, only while the contours of integration lay in a zone of disturbans of stress and strain. It is obvious, that the value of *J* is determined by the sizes of zones of disturbans and deviation of strain energy density in these zones from the gross value.

With monotonous increase of loading the plastic strains, which at first covered zone 2, spread in zone 3. If a crack is rather small, the plastic deformations can occur in the whole body except of zone 1, the size of which grows. It is obvious, that the nonlinear analytical expression for J and interpolation between linear and nonlinear cases should be considered separately for these components W_{L1} and W_{L2}

RELATION BETWEEN W_{L1} AND W_{L2} COMPONENTS

Let us consider a plate at fixed boundary displacement on the ends ($q=2\Delta=const$). Thus, J for nonlinear elastic body can be determine as a change of elastic energy in a body with change of crack length

$$J = -\frac{1}{2t}\frac{\partial W}{\partial a}\bigg|_{\Delta=const} . \tag{3}$$

Here t - is a thickness of a plate, which here after is put to 1. Instead of a change of crack length we can consider cutting off from the plate edges some strips of small thickness δb and inserting strip by thickness $2\delta a$ into centre of a plate. The change of energy with such transformations can be presented as (Rice and Druker(7)):

$$\Delta W = \int_{\Delta v} w dv + \int_{\Delta S} A ds , \tag{4}$$

where the integral over volume equals to the energy in a strip, which is separated or joined; A is a work of forces, on components of displacement on a surface of cutting off or joining on plate with transition from one condition to another. At a free surface, when thickness of cut-off strips tends to zero, the work tends to zero, as efforts on a free surface equal to zero. It is possible to show that the contribution of this work on center axis of the plate gives an additional component $\sigma_{xx}\varepsilon_{xx}$ in the equation (2). Then it is possible to consider a variation of energy of a plate with the fixed displacement on the ends in the form:

$$\delta W = 2\left(2Hw_g - \Delta W_{L_2}\right)\delta a + 2\left(2Hw_g + \Delta W_{L_1}\right)\delta b . \tag{5}$$

When $\delta b = -\delta a$ we have a crack growth, with fixed width $2c$, and we obtain the equation (2).

Let us consider the similar scheme for a plate, for which gross stress σ_g is fixed. Thus the load varies as $\delta P = 2\sigma_g \cdot \delta c$. On the base of Figure 2, it is possible to get relation :

$$(n+1)\delta A = -P\delta q + nq\delta P , \tag{6}$$

where $\delta A = \delta W\big|_{\Delta=const}$ is variation of energy with the fixed displacement at the ends, δq is a change of displacement at ends. On the basis of equations (5) and (6) it is possible to obtain a relation, which connects the components ΔW_{L1} and ΔW_{L2} with a component of displacement caused by crack $q_c = 2\Delta_c$ (Risnytchuk(9)):

$$-\Delta W_{L_2}\frac{a}{c} + \Delta W_{L_1}\frac{b}{c} = \frac{n-1}{n+1}\sigma_g q_c\,. \tag{7}$$

In a linear case (n=1) equations (2) and (7) lead to:

$$\Delta W_{L_1} = \frac{a}{c}J_e\,, \qquad \Delta W_{L_2} = \frac{b}{c}J_e\,. \tag{8}$$

Here J_e is related to the stress intensity factor K_I: $J_e = K_I^2 / E'$. The nonlinear case essentially differs from linear, as the exact relation between components is lost and depends on a parameter of strain hardening exponent n and load level. For the whole n range $(1 \le n < \infty)$ equations (2) and (7) lead to:

$$J_n = \frac{c}{b}\Delta W_{L_2} + \frac{n-1}{n+1}\sigma_g q_c \frac{c}{b}\,. \tag{9}$$

SOME APPROXIMATIONS FOR J

Besides equation (9) the values of q_c and J can be connected by equation:

$$J = \frac{1}{2t(n+1)}P\frac{\partial q_c}{\partial a}\bigg|_{P=const}\,. \tag{10}$$

On the basis of eqs. (9) and (10) it is easy to obtain (Risnytchuk(9)):

$$\Delta_c = \alpha\varepsilon_0 g(\xi,n)\left(\frac{P}{P_0}\right)^n b\,, \tag{11}$$

$$J = \alpha\varepsilon_0\sigma_0\sqrt{n}\frac{a}{b}f(\xi,n)\left(\frac{P}{P_g}\right)^{n+1} c + \alpha\varepsilon_0\sigma_0\frac{n-1}{n+1}g(\xi,n)\left(\frac{P}{P_0}\right)^{n+1} b\,, \tag{12}$$

$$g(\xi,n) = \sqrt{n}(n+1)\int_0^{\xi}\lambda(1-\lambda)^{n-2} f(\lambda,n)d\lambda\,, \quad \xi = a/c\,. \tag{13}$$

Here $f(\xi,n)$ is some slightly variable unknown function, which, from the condition of reducing to the known linear case $f(\xi,1)$ with $n \to 1$, we choose as:

$$f(\xi,n) = f\left(\frac{\sqrt{n}\xi}{\sqrt{n}\xi + (1-\xi)}, 1\right), \qquad f(\xi,1) = \pi\left(1 - 0.5\xi + 0.326\xi^2\right)^2. \tag{14}$$

The comparison of equation (12) with finite element solution (Kumar et al.(4)) is shown in Fig. 3 in terms of parameters (Miller and Ainsworth(10)):

$$F_P = \left(\frac{h(\xi,n)}{h(\xi,1)}\right)^{\frac{1}{n+1}}, \quad h(\xi,n) = \sqrt{n}\xi(1-\xi)^{n-1} f(\xi,n) + \frac{n-1}{n+1} g(\xi,n)$$

As it can be seen from Fig.3, we have a good agreement with the finite element solution.

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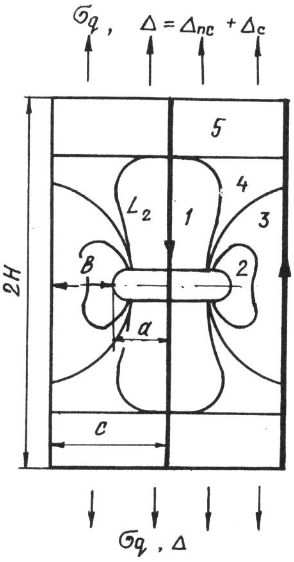

Figure 1 Central cracked plate under tension

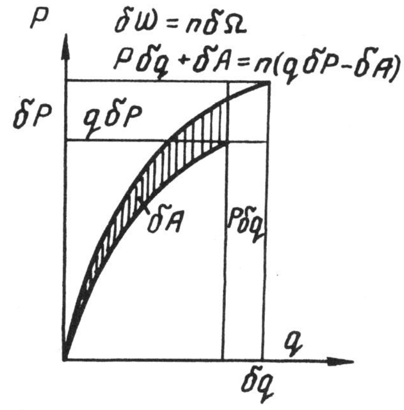

Figure 2 Load–displacement curves for two similar samples

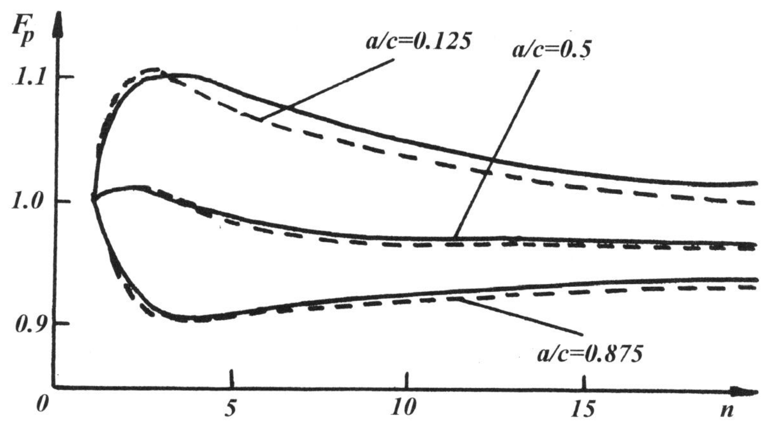

Figure 3 Dependence of F_p on strain hardening exponent n calculated by finite elements method (dashed lines) and by equation (12) (solid lines)