A MECHANISM OF 3-D CRACK GROWTH AND SPLITTING IN BIAXIAL COMPRESSION

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Abstract: Experiments in biaxial compression on brittle materials are presented in this paper. Two of these were on resin samples each containing a single embedded disc-like crack. The results showed that unlike the experimental observations in uniaxial compression (Dyskin et al., (5)) where the crack growth was limited and did not cause failure, in these experiments the crack grew extensively parallel to the load directions and caused splitting. Thus, the presence of the intermediate principal compressive stress changes the mechanism of crack growth.

A model is proposed based on representing the growing crack as a disk-like crack oriented parallel to the loading direction and opened by a pair of concentrated forces at its centre. It is shown that the crack growth is stable until it reaches a size comparable to its distance from a free surface.

INTRODUCTION

There are many practical situations where rock undergoes biaxial loading, for example, at the wall of an unsupported excavation. At smaller scales, biaxial loading is the driving force of core disking (Obert, (11)) and thermal spallation (Germanovich,(8)). In all these cases fracture mechanisms are determined by the presence of a direction free of loading. However, very few experiments have been conducted to investigate failure in biaxial compression.

Kupfer et al. (9) studied failure of concrete samples of size 20 x 20 x 5 cm under uniaxial and biaxial compression using different stress ratios. They used brush bearing platens consisting of individual steel filaments to apply uniform loading. In both loading modes, numerous microcracks parallel to the applied load were formed. The complete collapse of the specimen was accompanied by the formation of a shear fracture induced at angles between of 18° - 30° to the loading plane.

Brown (2) carried out biaxial tests on 76 mm square samples of Wombeyan Marble of 25 mm thickness using both solid steel platens and brush platens made from

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3.2 mm square pins. When brush platens were used, vertical fracture planes were formed parallel to the direction of free load. These fractures were small and not apparent when the failure process was controlled. However, when the failure process was uncontrolled, the specimen were split into slabs parallel to the σ_1-σ_2 plane and extended to the loading faces of the specimens. When solid steel platens were used, cones of shear fractures were formed and then, fractures coalesced into an hour-glass shape in the σ_1-σ_2 plane.

Experiments by Papamichos et al. (12) on prismatic rock samples of height 90 mm, width 80 mm, and length 110 mm, confined sideways along its length by two rigid walls, and unconfined at the two ends, failed either by splitting parallel to the unconfined surface or by shearing, depending on the type of rock tested.

The above experimental observations are not consistent, and the mechanisms of fracture in biaxial compression remain to be understood. Therefore, in order to cast light on the mechanisms of 3D fracture development and failure in biaxial compression, it is necessary to undertake further experiments on a wide variety of materials, in particular, on samples with *pre-existing* internal cracks. This paper reports initial experiments in biaxial compression tests on transparent polyester resin, concrete, and sandstone.

EXPERIMENTAL SETUP

The testing apparatus consists of a true triaxial cubicle cell (of external dimension 25 cm x 25 cm x 25 cm) capable of applying the load independently through six pistons acting in three orthogonal directions by separate pumps, of 200 MPa capacity. For the present experiments the true biaxial loading condition was achieved by setting the load in one of the directions to zero.

Four cubic samples of side length 100 mm were tested: two solid sandstone and concrete samples, and two resin samples each containing a single disk-like crack of 10 mm diameter inclined at 30° to one of the loading axes (z-axis). The resin samples were prepared by embedding the initial crack in the resin prior to casting Dyskin (5). The samples were cut to size and polished; to ensure a brittle failure regime, they were kept in a freezer for at least 48 hours (at ≅-17 °C) until tested.

Inserts, made of 1 cm thick leather, chamfered along the four sides, were used for the first three tests in order to prevent the pistons from touching each other and interlocking during loading. For the fourth test, aluminium inserts were used instead of leather to check whether the results obtained were influenced by the lateral expansion of leather to cause the observed splitting.

The tests were conducted in the load controlled mode, the only regime available for this testing apparatus. Table 1 shows the loading rate and the maximum stress reached for each test and the biaxial load ratio.

TABLE 1 Biaxial tests

Sample	p_y^* (MPa)	p_z^* (MPa)	p_y/p_z	p_y (MPa/min)	p_z (MPa/min)
Sandstone	56	10	5.6	7	1.3
Concrete	43	40	1	10	10
Resin 1	45	41	1	10	10
Resin 2	28	28	1	7	7

*The numbers are approximate (within the accuracy of the control module).

RESULTS

Both sandstone and concrete samples failed by splitting parallel to the free surface along planes passing almost in the middle of the sample. The initial crack in each of the resin samples ejected wings from the upper and lower parts of its contours. The wings then propagated extensively eventually causing splitting of the sample, Figure 1.

The observed crack growth under biaxial compression was different from the one in uniaxial compression (5). In those tests the growth of wings emerging from a single inclined disk-like crack was limited; not exceeding double the size of the initial crack. This was mainly due to: (a) the inward curling (wrapping) of the wings around the initial crack, Figure 2, and (b) the inability of the crack to grow in the lateral direction because Mode III segments the crack front while the applied compression suppresses further development of the segments (12). In biaxial compression however, the wings grow extensively, without curling (wrapping) eventually causing the samples to split.

MODEL

Because of the extensive growth of the wings in biaxial compression and their almost circular shape it is proposed to model such a crack as a vertical disk-like crack of radius *R* being opened by a pair of concentrated forces applied at the centre, Figure 3. These forces model the action of the initial crack (eg, Germanovich, et al. (9)), their magnitude, *F*, being proportional to the magnitude of the compression component P_z acting on the initial inclined crack, and the initial crack area:

$$F = \pi a^2 P_z \sin^2 \alpha \cos \alpha \qquad (1)$$

where a is the radius of the initial crack, α is the angle of inclination of the crack surface to the direction of P_z.

For this crack the stress intensity factor is (eg, Cherepanov (4)): $K_I = F(\pi R)^{-3/2}$, which implies that the crack should grow in a stable manner. However, the presence of a free surface has a profound effect on the crack growth: 2-D calculations (Dyskin and Germanovich (5)) show that when a crack that is opened by a pair of concentrated forces grows to a length equal to its distance from the free surface, it becomes unstable. Similar calculations can be conducted in 3-D for a disk-like crack in half-space parallel to the free boundary. Using the solution by Srivastava and Singh (13) and retaining only the second asymptotic term (the first term corresponds to the crack in space) one has

$$K_I(R) = \frac{F}{(\pi R)^{3/2}}\left[1+\frac{5}{\pi}\left(\frac{R}{h}\right)^3\right] \tag{2}$$

where h is the distance of the crack from the free surface.

When the radius of the crack reaches a critical value, $R_{cr}=h(\pi/5)^{1/3}\approx 0.8565$, the crack growth becomes unstable.

This result can now be applied to analyse the above experiments. It should be noted though that in these experiments the crack was in the centre of the sample so that both free surfaces influenced the crack growth. Applying the proposed model means that the critical radius is overestimated. For the distance from the free surface of 5 cm the critical radius should not exceed 3.9 cm. An examination of the fracture surface of one of the samples, Figure 4, showed that it consisted of two parts: a smooth central part of maximum radius of 3 cm, region I, and an opaque external part, region II. This is a clear indication that at the boundary between these two parts the crack growth has changed from stable to unstable, since the dynamic crack propagation produces wavy surface (eg, Fineberg et al., (7)).

CONCLUSION

The conducted tests in true biaxial compression showed that all samples failed by splitting parallel to the free surface.

The wings emerging from the initial inclined crack in resin grew extensively towards both loading axes. One wing, the lower branch, extended its growth in a helical direction to the upper surface of the crack.

This behaviour is different to that observed for a similar crack under uniaxial compression where the growth of the wings (branches) is limited with wing wrapping (curling) taking place. The reason for this difference is in the action of the second component of compression that prevents wing wrapping thereby enabling the extensive growth of the wings. *This may be indicative of a different failure mechanism in biaxial compression compared to uniaxial loading.*

This type of wing crack growth can be modelled by a disk-like crack opened by a pair of concentrated forced. Growth of such a crack is initially stable until its dimensions become comparable with its distance from the free surface. Then the growth becomes unstable causing near-surface failure. This explains some phenomena like rock burst near excavation walls, core disking and thermal spallation.

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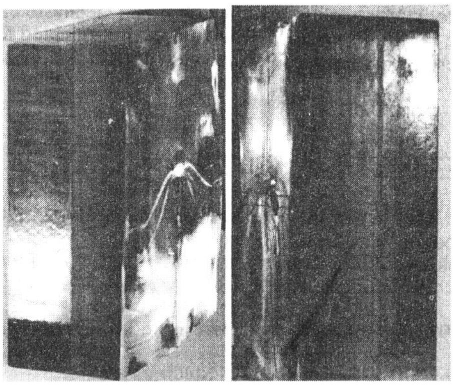


Figure 1. Extensive wing growth in resin causing splitting under biaxial compression

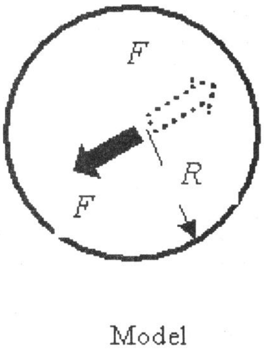
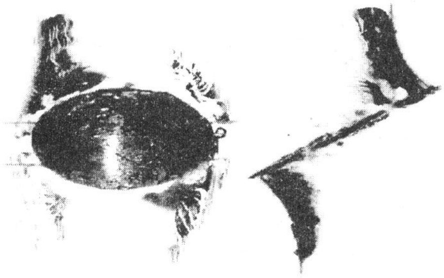


Figure 3. The proposed crack model



Front View Side View

Figure 2 Wrapping of the wing crack

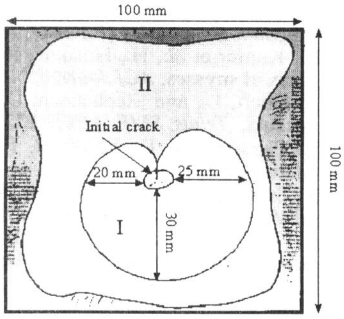


Figure 4. The change in the smoothness of the crack surface