A CRACKS CLOSURE IN COMBINED TENSION AND BENDING OF PLATES

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The phenomenon of the cracks closure in thin plates under the simultaneous influence of the membrane and bending loads is studied in a two-dimensional statement on the basis of classical theories of plane stress state and bending of plates. Analytical and numerical solutions of the contact problems for the infinite plate with an isolated crack and periodical system of cuts are obtained. There have been given the diagrams of the cracked plates' limit equilibrium, which show the influence of the crack closure effect on the load carrying capacity of the plate in combined loading.

INTRODUCTION

The well-known analytical solutions of the problems of bending thin elastic plates with slits of zero width as a rule do not take into account the edges of the defects. Therefore the solutions are valid for the case in which tensile membrane forces which prevent closure of the edges are applied to the plate, in addition to the bending load. The influence of the bending within the low tension level leads to the partial crack closure at the compressed edges. The strict description of this phenomenon from the point of view of three-dimensional theory of elasticity faces considerable calculating difficulties. Therefore the idea of the closure effect study within the traditional applied theories seems to be rather attractive.

Analytical research on the crack closure in the bending plate has been made by Shatsky (1), Young and Sun (2) on the basis of the classical Kirchhoff's theory and theory of plane stress state. In the present paper the ideas are generalized for the case of combined loading with an arbitrary ratio of tension and bending values.

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FORMULATION OF THE PROBLEM

We examine an infinite isotropic plate with thickness $2h$ weakened with a rectilinear continuous crack with the length $2l$ which is not loaded. At the infinity the plate is subjected to the effect of uniformly distributed normal forces n and bending moments m (Fig.1). We examined the effect of the crack and of possible contact of its edges on the stress state of the plate.

The following configurations with a high degree of symmetry were also used to investigate the present problem, i.e. an infinite plate with the periodical system of cracks situated in line and an infinite plate with periodical system of undisplaced parallel cracks.

Equations

The elastic equilibrium of the plate outside the crack is described by a pair of biharmonic equations of the generalized plane stress state and of classical bending theory of the plates:

$$\Delta\Delta\varphi = 0, \quad \Delta\Delta w = 0, \quad (x,\ y) \in R^2 \setminus L, \tag{1}$$

where φ is stress function, w is the deflection of the plate, Δ is the Laplace operator, L is the line of crack .

Boundary Conditions

For the membrane forces and bending moments at infinity it is decided to satisfy the conditions:

$$N_x = N_{xy} = 0, \quad N_y = n, \quad M_x = M_{xy} = 0, \quad M_y = m, \quad (x, y) \to \infty. \tag{2}$$

Taking into account the symmetry of the problem in relation to the x-axis we examine the boundary conditions in the crack. Three cases of interaction of the edges of the crack can occur depending on the ratio of values of tension and bending in the cut L or in its parts L_i .

1.The edges of the crack do not interact (Fig. 2a). The opening of the crack at the point $(x, 0, z)$ is characterized by the opening $[v](x)$ in the median plane and by a jump of the angle of rotation of the normal $[\vartheta_y](x)$. Here and on the rest $\vartheta_y = \partial w / \partial y$. The conditions of the free edge and of impenetrability of faces are satisfied in the slit:

$$N_y = 0, \quad M_y = 0, \quad [v] \geq h|[\vartheta_y]|, \quad x \in L_1. \tag{3}$$

2. The edges of the crack are in partial contact (along the height). Taking into

account Kirchhoff's hypothesis on the rigid normal element, this contact is interpreted as closure of the sharp edges of the slit $z=-h$ (Fig. 2b) or $z=h$. Such approach results in the following boundary conditions in the place of the contact (see paper (1)):

$$[v]=h|[\vartheta_y]|\geq 0, \quad M_y = hN_y \operatorname{sgn}[\vartheta_y], \quad N_y \leq 0, \quad x\in L_2. \tag{4}$$

3. There is a complete contact of the edges of the crack (Fig. 2c). It is evident that in this case:

$$[v]=0, \quad [\vartheta_y]=0, \quad x\in L_3. \tag{5}$$

In addition to this, we mention, that $L=L_1\cup L_2\cup L_3$; the points contour division L_i are obtained from the conditions of the smooth closure of the crack edges in the change of the type of boundary conditions.

The formulated boundary-value problem (1)-(5) is solved using the singular integral equations method.

ANALYSIS OF THE RESULTS

An Infinite Plate Containing a Crack

In this case the boundary-value problem has analytical solution. It can be written as (Shatsky (3)):

$$[v](x)=4a(n,m)(l^2-x^2)^{1/2}/B, \quad [\vartheta_y](x)=-4b(n,m)(l^2-x^2)^{1/2}/(Da),$$

$$N_y(x,0)=n-a(n,m), \quad M_y(x,0)=m-b(n,m), \quad x\in L.$$

where

$$a(n,\ m)=\left\{\begin{array}{ll} n, & \eta\geq\kappa \\ (n+|m|/h)\kappa/(1+\kappa), & -1\leq\eta\leq\kappa \\ 0, & \eta\leq -1 \end{array}\right\},$$

$$b(n,\ m)=\left\{\begin{array}{ll} n, & \eta\geq\kappa \\ \operatorname{sgn} m(hn+|m|)/(1+\kappa), & -1\leq\eta\leq\kappa \\ 0, & \eta\leq -1 \end{array}\right\},$$

$B = 2Eh$, $D = 2Eh^3/(3(1-\nu)^2)$, $a = 3 - 2\nu - \nu^2$, E and ν are Young's modulus and Poisson's ratio of the material of the plate, $\kappa = (3+3\nu)/(3+\nu)$, $\eta = n/(h|m|)$ - parameter, representing ratio of loads.

At $\eta \geq \kappa$ there is no contact between the edges $(L_1 = L, L_2 = L_3 = \varnothing)$, and the solution can be obtained by superposing the independent solutions of the problems of tension and bending. At $-1 \leq \eta \leq \kappa$ the crack is closed along the entire length as shown in Fig. 2b $(L_2 = L,\ L_1 = L_3 = \varnothing)$. At $\eta \leq -1$ within the limits of the accepted model crack has no effect on the stress-strain state of the plate.

<u>Fracture criterion.</u> For the intensity factors of the forces and moments in the vicinity of the crack tips we obtain the following expressions:

$$K_N = -(B/4\sqrt{l})\lim_{x\to l}(l^2 - x^2)^{1/2}[v]'(x) = a(n,m)\sqrt{l},$$

$$K_M = -(Da/4\sqrt{l})\lim_{x\to l}(l^2 - x^2)^{1/2}[\vartheta_y]'(x) = b(n,m)\sqrt{l}, \tag{6}$$

To estimate the limit equilibrium of the cracked plate we use the energy criterion of linear mechanics of fracture in combined tension and bending :

$$\left(\pi/(4h^2E)\right)\left\{K_N^2 + \kappa\left(K_M/h\right)^2\right\} = 2\gamma_*$$

($2\gamma_*$ is the density of the material effective surface energy).

In our case using the result (6) we obtain the following criterial equation:

$$\left\{\begin{matrix} n^2 + \kappa(m/h)^2, & \eta \geq \kappa \\ (n + |m|/h)^2\kappa/(1+\kappa), & \eta \leq \kappa \end{matrix}\right\} = n_0^2,$$

where $n_0 = 2h\sqrt{2E\gamma_*/(\pi l)}$ is the fracture force for the cracked plate, determined from the experiment on tension. The respective diagram for $\nu = 0.25$ is shown in Fig. 3: it contains a classical ellipse in the range of the absence of contact and a straight line in the area of the crack closure.

<u>Periodical System of Collinear Cracks</u>

The rapprochement of defects at the orientation mentioned above equally influences the shape of the opening in tension and bending. Given all the points, for this system cracks closure effect will be identical to that of the one containing a isolated crack. We should just reduce the coordinates of all the points of the limiting diagram in k times ($k = \sqrt{(1/\lambda)\tan(\lambda)}$, $\lambda = \pi l/d$, d - period of cuts disposition). For detailed information we refer the reader to the article (Shatsky (4)).

Periodical System of Parallel Cracks

We assume that we have a number of defects parallel to x-axis with their centers situated on the y-axis with the period d. In this case the rapprochement of the slits tells on the shape of their opening in tension and bending unequally. It means, that in case of the change of the η parameter the boundary conditions cannot be changed simultaneously along the entire length of the crack. In other words we have the mixed problem.

Thorough analysis of the integral equations system of this problem is made by asymptotic small parameter method and by numerical method of mechanic quadratures with different values of the parameter $\lambda = 2l/d$. The following situation is obtained.

At $\eta \geq \eta_*(\lambda)$ the edges of the crack do not interact $(L_1 = L, L_2 = L_3 = \varnothing)$, and so traditional ellipses are found on the diagram of the limit equilibrium. At $\eta \leq \eta_{**}(\lambda)$ the contact line considers with the whole section L $(L_2 = L,\ L_1 = L_3 = \varnothing)$. So, on the diagram we obtain the straight lines system. In the intermediate sector $\eta_{**}(\lambda) \leq \eta \leq \eta_*(\lambda)$ we observe the mid-crack part of the contact and stress-free edges in the vacinity of the tips $(L_1 = (-l, -\beta l) \cup (\beta l, l),\ L_2 = [-\beta l, \beta l],\ L_3 = \varnothing)$. For this case the parametric plan of the solution of non-linear problem is used. The values $\beta \in (0,1)$ were successively set and η-parameter and jumps in the points of collocation were obtained using the system of linear algebraic equations. As a result the curves of the limit equilibrium for this range are represented in its parametric variant. It should be mentioned, that the expressions $\eta_*(\lambda)$, $\eta_{**}(\lambda)$ are obtained in the course of the numerical solution of the problem.

Analysis of diagram in Fig. 4 shows that rapprochement of parallel crack and their closure results in the growth of the bearing capacity of the plate.

REFERENCES

(1) Shatsky, I.P., Dopovidi Akademii Nauk UkrRSR. Ser. A, No 7, 1988, pp.49-51.

(2) Young, M.J. and Sun, C.T., Int. J. of Fract., Vol. 55, 1992, pp. 81-93.

(3) Shatsky, I.P., Fiz.-Khim. Mekh. Mater., Vol. 25, No 2, 1989, pp. 46-50.

(4) Shatsky, I.P., Dopovidi Nation. Akademii Nauk Ukraine, No 10, 1995, pp. 62-64.

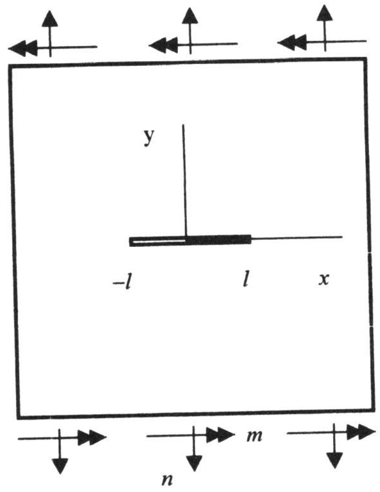

Fig. 1. The infinite cracked plate in combined loading.

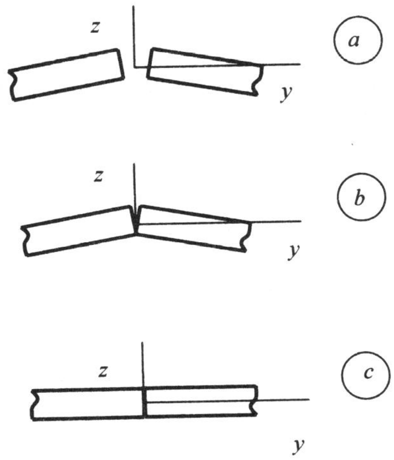

Fig. 2. The diagram of the interaction of the crack edges.

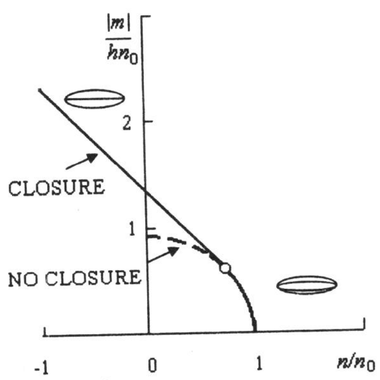

Fig. 3. Limit equilibrium of the plate whit the isolated crack.

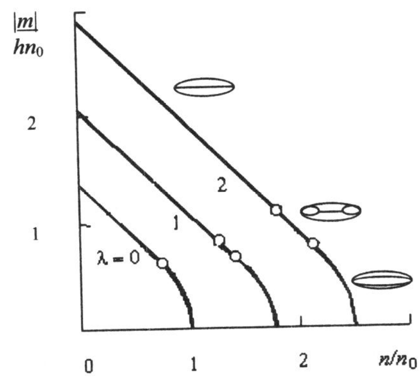

Fig. 4. Limiting diagram for the periodical system of parallel cracks.