RELATION BETWEEN THE CRACK TIP DISPLACEMENT VECTOR AND THE J-INTEGRAL IN MIXED MODE DUCTILE FRACTURE

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The relationship between the crack tip displacement vector δ_v and the J-integral is derived within the HRR-theory and verified with FE-calculations and experimental results. The implications for fracture initiation are discussed on the basis of available data. It is shown that the δ_v = constant assumption predicts the decrease of initiation-J with increasing mode II components of ductile steels. A correlation between δ_v and the magnitude of maximum equivalent plastic strain ahead of the crack was not found. At high mode II loading a critical combination of strain and triaxiality is attained at the crack tip, which causes a void-sheeting fracture at relatively low J-values.

INTRODUCTION

In previous papers it has been shown by the authors (1-3) that the crack tip displacement vector

$$\delta_v = \sqrt{\delta_I^2 + \delta_{II}^2} \quad (1)$$

is more appropriate to characterise ductile mixed mode I/II crack initiation and stable crack growth than the J-integral. Experiments carried out on biaxially loaded cruciform specimens with slanted cracks and small compact-tension-shear (CTS) specimens, Figure 1, of the structural steel StE 550 (yield plateau σ_y=580, ultimate strength σ_{uts}=650 MPa) showed approximately constant technical (Δa = 0.2mm) δ_v-initiation values over almost the entire investigated mixed mode range. The shear crack resistance curves in terms of δ_v were independent of specimen geometry and mixed mode ratio. On the other hand increasing mode II load components led to decreasing J-initiation values and lowered the level of the J-R-curves, which fell even under the mode I J-Δa-curve of C(T) specimens in case of near mode II loading. Also other authors found this decrease of the mode II J-integral tearing resistance in ductile ferritic steels with low work hardening capability or yield plateau (4-6).

In this paper the δ_v-J-relationship of the HRR-field will be derived under the

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δ_v = constant assumption. The implications for fracture initiation are discussed on the basis of available data. To get further insight to the δ_v-J-relation plane stress and plane strain finite element calculations were performed and some preliminary results are presented.

FINITE ELEMENT CALCULATIONS

Within the framework of mixed-mode fracture characterisation of StE 550 two-dimensional, plane stress and plane strain finite element (FE) analyses of CTS specimens were performed using conventional small strain theory. In Figure 1 the FE model of a CTS specimen loaded at ϕ=75° is represented. Details on the fixture modelling and load introduction are found in (2). A core of square-shaped elements with a length of about 1/32000 the crack length surrounded the crack tip. Suitable scaling of element dimension was obtained by biasing nodes towards the border of the focused region. The angular discretisation consisted of 24 elements per ring. Though this choice allowed to generate regularly shaped elements, it resulted posteriorly to be much too rough for an accurate estimation of stresses and strains in crack propagation direction.

Alike the experiments (2), the FE-models were loaded until a δ_v = 0.19 mm at a position 0.5 mm behind the undeformed crack tip was reached. This value corresponded to the physical initiation value obtained from back-extrapolation of the available experimental data and metallographic investigations of sections of the crack tip region (7). The drop in technical δ_v of pure mode II loaded specimens, see below, was not considered.

J-δ_v-RELATIONSHIP IN THE HRR-FIELD

As proposed by Amstutz (8) a relationship of the J-integral and the crack tip displacement vector δ_v is obtained from the plane strain mixed-mode HRR-fields (9) by evaluating δ_v at the distance

$$r = \sqrt{u_r^2 + u_\theta^2} \tag{2}$$

behind the crack tip. The polar displacement components u_r and u_θ are evaluated at θ = 180° and -180°. Therefore two different reference points are obtained for the upper and lower flank of the crack under mixed-mode loading. The final result is however conform to the well known expression (10):

$$\delta_v = (\alpha\varepsilon_0)^{\frac{1}{N}} D_n \frac{J}{\sigma_0} \tag{3}$$

where σ_0, ε_0, α and N are the Ramberg-Osgood yield stress, yield strain, constant and strain hardening exponent respectively. For typical metallic materials the dependence of δ_v on $(\alpha\varepsilon_0)^{1/N}$ is weak. Figure 2 depicts the variation of D_n with mode mixity for medium to low work-hardening. The J-δ_v-relationship obtained from plane strain and plane stress finite element (FE) calculations described in previous section are also displayed in Figure 2 together with experimentally measured values of the 4 mm thick specimens (2). The $(\alpha\varepsilon_0)^{1/N}$-factor for StE 550 was assumed as 0.81.

Even if different δ_v measurement positions were used, the general trend and the actual D_n-values in Figure 2 are quite similar. The increase of the crack-tip displacement vector with increasing mode II components suggests a decrease of constraint which was also found in other finite element calculations (11, 12).

Figure 3 highlights the decrease of the mixed-mode J-initiation values J_c of ductile steels (2, 5, 6, 13, 14) and a finite element calculation based on damage mechanics (12) with increasing mode II load components. The mixed-mode fracture data was normalised by the mode I J_{Ic}, obtained from mode I loaded mixed-mode specimens (if available). The lines in Figure 3 show the J_c/J_{Ic}-M_e-relationship of the plane strain, mixed-mode HRR-fields derived under the δ_v = constant assumption for two work hardening exponents N. As may be expected from the uniform slopes of the curves in Figure 2, the normalised curves in Figure 3 display little effects of the strain hardening exponent. The results of the FE-calculations are also shown in Figure 3. At least the trend of the initiation-J-decrease with increasing mode II components is predicted correctly with the δ_v = constant assumption and the HRR fields. Even for the special case of StE 550, the FE calculations apparently did not match the experimental results more accurately. With respect to the very low J_c/J_{Ic}-ratios for pure mode II loading of StE 550 and A-503-8, it should be noted here, that also the δ_v-initiation values of StE 550 evaluated at $\Delta a = 0.2$ mm were reduced by 30% compared to the mode I crack tip openings (2).

PRELIMINARY RESULTS OF THE RELATION OF δ_v WITH STRESS/STRAIN DISTRIBUTION AT THE CRACK TIP

Systematic experimental investigations and FE-calculations (for example in (15)) have demonstrated that ductile cracks initiate when a critical combination of triaxiality of stress and equivalent plastic strain $\varepsilon_{v,pl}$, is reached in a certain region ahead of the crack tip. The stress triaxiality is usually expressed by the ratio of the hydrostatic stress to the v. Mises equivalent stress σ_m/σ_v. In this section the implications of the δ_v = constant assumption on the damage parameters are presented. Again the values are taken for a load yielding the initiation δ_v of 0.19 mm at a distance of $r = \delta_v = 0.19$ mm ahead of the crack tip. The HRR-results were calculated with N = 20.

Figure 4 shows the maximum triaxiality and equivalent strain for plane strain. The values had to be taken at different positions ahead of the crack tip. For increasing mode II the constraint or stress triaxiality decreased and the effective plastic strain increased. In contrast to what was stated by (14) a constant δ_v was not equivalent to a constant maximum $\varepsilon_{v,pl}$ for any mode mixity. For plane stress (not included in Figure 4) the triaxiality varied slightly with M_e ($\sigma_m/\sigma_v \approx 0.58$), whereas the equivalent strain showed no consistent trends. Probably the localised regions with intense straining were not matched by the coarse angular mesh.

More than the maximum stresses and strains, the values in crack propagation direction are of interest. The stable crack deflection angles α (positive for tensile crack growth) in Figure 5 were measured on the broken specimen halves. The stable crack extended in the maximum shear or maximum equivalent strain direction at crack sliding components of $M_e < 0.68$. Relatively high crack tip opening displacements caused a crack path deviation in the direction normal to the maximum tensile stress. The constant offset between the HRR and the measured shear deflection angles together with the dimples on the shear crack surfaces detected in (1) suggested that the so called „shear crack“ was in fact a combination of sliding off and void coalesce. The crack deviated from the zero or negative hydrostatic stresses in the shear band, to allow the growth of some microvoids. Final rupture then occurred by a microvoid sheeting mechanism (16). Therefore the combinations of triaxiality and strain taken in crack growth direction displayed some amount of positive triaxiality also in case of shear crack growth. This data is plotted in Figure 6 together with damage curves measured on notched specimens of similar steels (17, 18). Even if the absolute values of σ_m/σ_v and $\varepsilon_{v,pl}$ are sensitive to the microstructural critical distance at which the data is extracted, some kind of more or less horizontal[1] cut-off for a maximum strain is insinuated from the behaviour of the shear crack values in Figure 6. In the ductile steels the critical crack tip displacement can not reach the high values implied a by mixed-mode independent J-integral. Under predominant mode II loading a critical strain, which (together with a small amount of positive triaxiality) causes a void sheeting mechanism, is attained before the J-integral reaches the high mode I J_{Ic}-values.

CONCLUDING REMARKS

The relationship between the crack tip displacement vector δ_v and the J-integral was derived within the HRR-theory and verified with FE-calculations and experimental results. It was shown that the decrease of initiation-J with increasing mode II components of own results and literature data is qualitatively predicted under the δ_v = constant assumption.

On the basis of plane strain FE-calculations and the HRR-field estimates of triaxiality σ_m/σ_v and equivalent strain $\varepsilon_{v,pl}$ at incipient crack propagation were made. The results clearly showed that there is no straightforward relationship between δ_v and $\varepsilon_{v,pl}$ as stated by (14), even though the FE-models had a rather coarse angular meshing and the plane strain assumption is too strict for thin specimens (19).

As proposed by Teirlink et al. (20) in their fracture maps, the change in fracture mechanism from tensile fracture to shear fracture adds an additional line in the diagram of critical σ_m/σ_v-$\varepsilon_{v,pl}$ combinations.

AKNOWLEDGEMENTS

Helpful discussions with Dr. H. Amstutz of the University of Darmstadt are gratefully acknowledged.

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[1] It should be recalled, that the points for pure mode II loading (smallest σ_m/σ_v) are too high in Figure 6 since a smaller δ_v was measured in the experiments.

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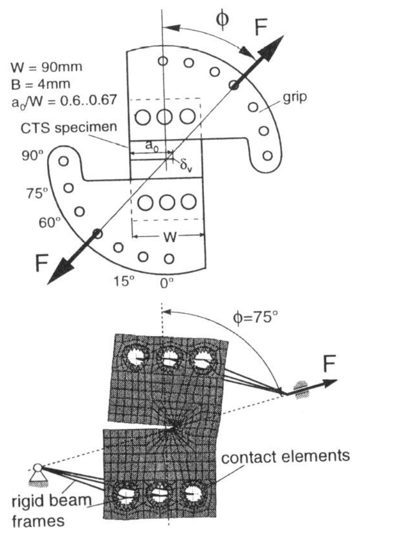

Figure 1: CTS-specimen, loading device and FE-model.

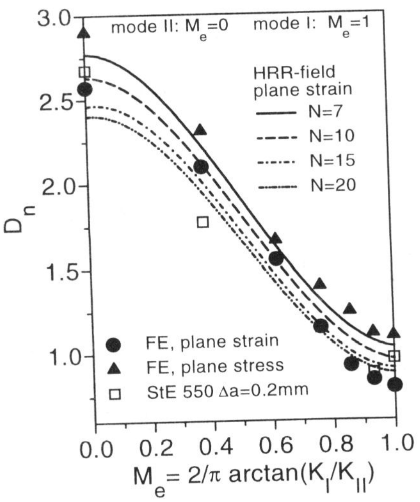

Figure 2: Variation of D_n with mode mixity (HRR-field, FE and expm. results).

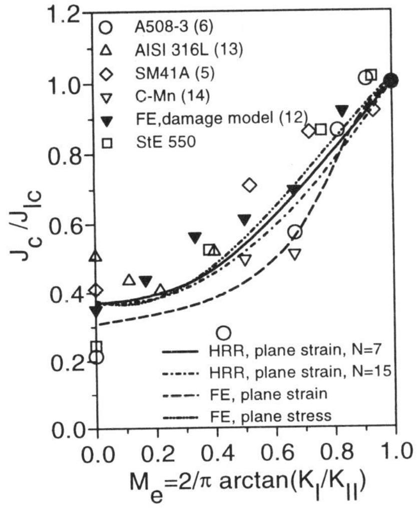

Figure 3: The decrease of J_c with increasing mode II is predicted by imposing a constant δ_v in the HRR-field.

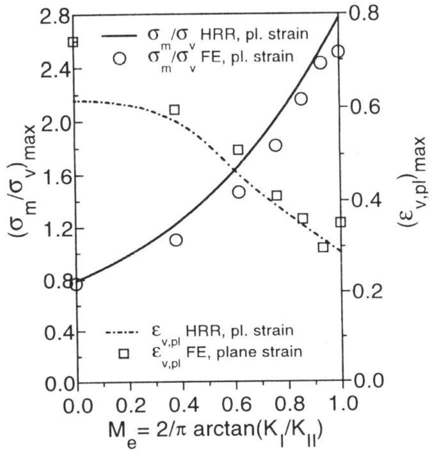

Figure 4: Maximum triaxiality and equivalent strain in function of mode mixity.

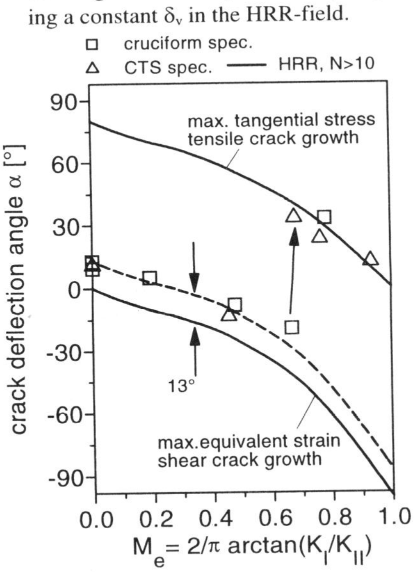

Figure 5: Experimental crack deflection angles of StE 550.

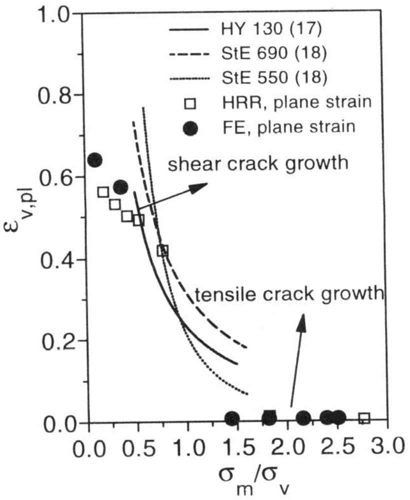

Figure 6: Combinations of plane strain triaxiality and equivalent strain at initiation.