A CTOA DESCRIPTION OF SIZE EFFECTS IN R-CURVES FOR LARGE GROWTH

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A conventional 3D finite element model has been made of two different sizes of bend pieces that show a strong effect of size for the J-R-curve when taken to large amounts of growth. The model data has been analysed in terms of a particular definition of the crack tip opening angle (CTOA) for a steady state regime of large growth, by using the plastic component of mouth opening and the position of the instantaneous centre of rotation. The computed and experimental data are shown to be consistent. Taking the CTOA of one size as a datum and using an already proposed rule, CTOA$\sqrt{G}$ = H ≈ constant, allows the size dependant behaviour of the other size to be predicted as approximately inversely proportional to square root of size. Computations can be mede in terms of the combined elastic plus plastic CTOA, without recourse to special elements. The transient regime of small growth is not included in this analysis.

INTRODUCTION

Stable ductile crack growth is usually described by a J-R-curve obtained from a test on a small piece taken into extensive plasticity. It is generally agreed that an R-curve is a function of configuration, but there is no consensus on transferability or on whether R-curves are a function of size. Two early studies in which opposite views appear, both based on a high strength low hardening (hslh) steel, HY130, in the plane strain regime, are cited as examples. In Etemad and Turner (1), a 'wider-lower' trend was shown, in which wider pieces gave R-curves of lower slope. In (2), Davis et al showed 'no-trend'. Differences exist between the two studies. The wider-lower data in (1) were for plain deep notch bend (DNB) pieces, width 12 < W < 66mm, thickness B = 50mm, $a_o/W \approx 0.5$, taken to 60% growth. J_{TPT}, a term similar to the deformation theory J_d, was used for J. In (2), the no-trend data were for 20% side grooved compact tension (CT) pieces, $0.55 \leq a_o/W \leq 0.85$, $W \approx 100$mm, 12 < B < 50mm, taken to about 20% growth, using the then standard ASTM term J_{i+1} for J. Several other papers could be cited to support either view but no clear picture emerges from a review of such work. Recent numerical work by Hutchinson (3), using the softening Gurson-Tvergaard model and A533B properties showed no-trend for small growth but a wider-lower trend for large growth.

The present work uses data for a geometrically similar pair of DNB pieces of HY130, Dagbasi and Turner (4), which were there analysed in terms of a particular crack tip opening angle (CTOA). Turner and Kolednik (5), suggested a law for

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transferability with both size and configuration of the form CTOA$\sqrt{G}$ = H, where G is the usual lefm term and H is a known function of CTOA that, for modest changes, can be taken as near constant. This paper gives a numerical analysis in support of those results, using a 3D finite element suite modofied from WHAMSE, (6). That employs brick elements with 'hour-glass' control for a conventional elastic-plastic continuum formulation of real elastic-plastic (rep) behaviour, i.e. incremental material with linear elastic unloading. Complementary studies have been made using a 2D finite element program with 8-node iso-parametric elements, although that work is not reported here.

TRANSFERABILITY THROUGH A PARTICULAR MODEL FOR CTOA

CTOA has been seen as a description of crack advance ever since initiation was described by crack opening displacement (COD). Here, a specific definition allows it to be measured simply in a standard type DNB test, (5). Only the plastic component is used in the analysis, although an elastic component is also needed in the computational modelling. This definition is $\alpha_{g\cdot pl}$, where sub g shows that the CTOA, α, is taken from a global measurement. Sub pl denotes the plastic component, which is inferred via the distance ahead of the crack tip of the instantaneous centre of rotation, r*b, where b is current ligament size, $b_o - \Delta a$. For DNB, equations are given by Braga and Turner, (7),

$$r^* = [(dV_{pl}/dq_{pl}) - 4a_o/S] / [4b_o/S] \qquad \alpha_{g\cdot pl}/r^* = (4b/S)(dq_{pl}/da) \qquad (1a,b)$$

where S is span. An example of this analysis is given later; r* is found from the plastic components of mouth opening, V_{pl}, and beam displacement, q_{pl}, in an unloading compliance test. The difference from the usual COD term rb, where $r \approx 0.4$, is that r* defines the instantaneous centre, rather than the initial centre. These equations apply to the quasi-steady state of large amounts of growth that develop under full plasticity, where the load is reducing with crack growth; they exclude the small growth regime.

In (5), ductile crack growth by micro-void coalescence is seen as a series of micro-instabilities in which the micro-ligament between the crack tip and a void growing ahead of it, is stretched until it fails by a local instability. This process was modelled uniaxially because the local constraint would have been relaxed by the free faces of the crack tip and void. The driving force was the load on the micro-ligament due to the interruption by the void of the (unloading) G field. That led to:-

$$\alpha_{g\cdot pl}\sqrt{G} = H(\alpha_{g\cdot pl}) \qquad (2)$$

where H() is a known function, near constant in the range $0.05 < \alpha_{g\cdot pl} < 0.25$rad but reducing by about 50% if $\alpha_{g\cdot pl}$ increases to 0.45rad., as it might for a ductile material. The CTOA value derived from the DNB test is used as a datum. For a component, or other size of piece, $\alpha_{g\cdot pl}$(unknown) can be evaluated from Eq.3

$$\{\alpha_{g\cdot pl}\sqrt{G}\}_{unknown} = \{\alpha_{g\cdot pl}\sqrt{G}\}_{datum}\{H()_{unknown}/H()_{datum}\} \qquad (3)$$

If the H() values are similar, iteration or just trial and error suffices but Eq.3 could be programmed as the criterion for crack advance. The value of G depends, of course, on the load and configuration. In general, G is known for any applied load, Q, via the lefm shape factor, Y, but for an actual test in the plastic state the load is not known, a priori, since even a modest amount of hardening allows Q to increase well above the limit load, with either L or σ_o rising correspondingly, (4), during growth.

TEST DATA

The data being modelled are for two geometrically similar DNB pieces of HY130,

$a_o/W \approx 0.54$, $S/W = 4$, $B = 20$, $W = 37$mm and $B = 50$, $W = 95$mm (pieces 37 and 95TT from (4)). Initiation was not closely studied. The properties are 0.2% proof stress, 930MN/m^2, tensile strength, 1000MN/m^2, $J_{Ic} \approx 0.15$MN/m. After an initial transient stage, a steady state regime was found up to about 60% growth. Shear-lips extended $0.2b_o$on each side, constant with growth, but causing an appreciably curved crack front that made it difficult to define a suitable value of Δa. All the data used here are for a 9-point average length, deduced in (4) from the compliance measurements. The J-R-curves in (4) use the increment of work, dU, normalised by initial area, Bb_o and $\eta = 2$

$$J_o = J_i + \Sigma dJ_o; \quad dJ_o = \eta dU/Bb_o \tag{4}$$

The curves are shown, Fig.1, to be strongly wider-lower, such that if plotted (not shown) verus $\Delta a/b_o$ they are practically coincident. In view of a possible dependence on the choice for J, the same data are also shown in terms of J_{i+1}, as used for the cases from (2). Neither formulation is specifically advocated here. Values of r*, CTOA and G, for the steady state regime, as given in (4) for the 20 and 50mm thick pieces are:-

r* = 0.89 and 0.82; $\alpha_{g,pl} = 0.15$ and 0.09rad; G = 0.17(0.22) and 0.31(0.35) MN/m. The open values of G refer to a plot of $\eta_{el}w_{el}/B$ versus b; the bracketed values refer to the mean of values obtained from load and the Y factor. The two formulae give different values because the definition of Δa for a curved crack is not unique.

THE PRESENT COMPUTATIONS

The input to the computations is the experimental data of dU versus Δa. For a given choice of J that gives the same R-curves as in Fig.1. The program operates in 3D, but the shear-lips and crack curvature have not been modelled since the object is to simulate directly the experimental analysis, based on the 9-point average crack length.

The first output of interest is the load-displacement diagrams, shown normalised, Fig.2, together with the experimental data. The input, of course, defines the increments dU = Qdq, but not the two components, load, Q, and increment of displacement, dq. The good agreement seen suggests that the use of the 9-point average for Δa is itself satisfactory, although the sensitivity to other measures of Δa has not been explored. For the smaller size, initiation occurs very close to the maximum load, such that in the experiments, initiation seemed to be at maximum load to within the accuracy of the data. A steady state was achieved within about 1 or 2mm of growth. For the larger size, initiation clearly occurred before maximum load, with some 4mm of growth made under rising load and a further 1 or 2 mm before steady state was achieved. The present analysis therefore covers a range from about 10 to 60% growth.

Once the computed loading diagrams have been shown to be realistic, the output of interest is the CTOA. Since the opening at the crack tip is always zero for the brick elements used (and also for the iso-parametric elements of the associated 2D study), the computed CTOA is defined as COD/h where COD is the crack opening at one element behind the crack tip and h is the element size, itself a constant value of 0.4mm with growth, for both sizes of piece. The values of CTOA are shown Fig.3. They decrease from initiation, over about 1.6mm (10%), to a mean steady state value of 0.219rad for the smaller size and over about 5.6mm (12%), to a mean of 0.198rad, for the larger. Whilst these are quite small growths in relation to the 60% now analysed, they are large in relation to the R-curve as usually measured for the purpose of either determining initiation or of giving an intial value of dJ/da. The experimental data used are not adequate (and were not intended) to define this transient regime closely.

The present interest is in the steady state CTOA which starts, Fig.3, at about 10%

growth and extends in both cases, to within the accuracy shown, to the 60% limit computed. The close similarity of value, about 0.2 rad, for the two sizes, is perhaps surprising since the R-curve measure of growth shows the strongly wider-lower trend. But the important point is that the CTOA of Fig.3, is a combined elastic plus plastic value and not that defined as $\alpha_{g,pl}$. Fig.4 shows the plot of the plastic components of the mouth opening, V_{pl}, versus beam displacement, q_{pl}, from the slope of which r* is obtained using Eq.1a. Fig.4 also shows a plot of ln b versus q_{pl}, of which the inverse slope gives $\alpha_{g,pl}/r^*$, using Eq.1b. In plotting this figure the transient behaviour just remarked on has been ignored and b has been treated as (b+h), although that has little effect unless b is not dominant. Both r* and $\alpha_{g,pl}/r^*$, and thus $\alpha_{g,pl}$, are constant with growth in this steady state regime. The values for the small and large sizes are:-

$$r^* = 0.831\ (0.89) \text{ and } 0.816\ (0.82),\ \alpha_{g,pl} = 0.143\ (0.15) \text{ and } 0.089\ (0.09)\text{rad},$$
$$G = 0.17\ (0.17\text{-}0.22) \text{ and } 0.39\ (0.31\text{-}0.35)\text{MN/m},$$

where the bracketed values are from the experiments of (4). The agreement is quite satisfactory. This measure of CTOA, $\alpha_{g,pl}$, differs for the two sizes. To complete the picture as so far developed, the plastic measure $\alpha_{g,pl}$ should be related to the combined term from the computations, Fig.3. That relationship is $\alpha_{comb} = \alpha_{el} + \alpha_{pl}$ where α_{el} is defined through the elastic opening $2v_{el}$ as $d(2v_{el})/da$. This definition of elastic CTOA is necessary to give the actual opening at one element behind the tip, consistent with the treatment of the dominant plastic term from Eq.1, (5). From lefm,

$$d(2v_{el})/da = \{2\sqrt{(2G/E'h\pi)}\}\{1 + (h/G)(dG/da)\} = \alpha_{el} \qquad (5)$$

For the steady state regime only the first term is relevant. Using the computed values of G just quoted gives respectively for the small size and then for the large:-

$$\alpha_{comb} = \alpha_{el} + \alpha_{pl} = 0.070 + 0.143 = 0.213\ (0.219) \text{ and } 0.106 + 0.089 = 0.195\ (0.198)$$

The open values are shown as lines on Fig.3 whilst the bracketed values are for the mean computed steady state values (not shown). This combination of the elastic and plastic components thus allows the computational modelling of crack growth using conventional formulations of both plasticity and elements, where the CTOA data, be it input or output is, as argued (5), the term α_{comb}.

Finally, taking the small size as the experimental datum and treating the larger size as a component for which data are required, the values of $\alpha_{g,pl}\sqrt{G}$ are examined. They are respectively 0.0570 and 0.0553, giving a ratio of $\alpha_{g,pl}\sqrt{G}$ for unknown/datum of 0.97. The ratio of H() for unknown/datum for these values of $\alpha_{g,pl}$, from the curve in (5), is 0.920/0.955 = 0.97. The CTOA$\sqrt{G}$ law is thus satisfied for the transferance of the $\alpha_{g,pl}$ measure of CTOA from the one size to the other.

DISCUSSION

An explicit relationship between local (CTOA) and global terms (dJ from work done) is possible because of the specific definition adopted for the datum CTOA in the fully plastic DNB test. For that case, Merkle et al (8) relates E'G to the normalised load L, as

$$E'G \approx L^2\sigma_o^2 b \quad \text{where } Q = L\sigma_o Bb^2/S, \qquad (6)$$

but either L or the yield stress σ_o must increase with hardening. In (5), CTOA is related to $dJ_{o,pl}/da$ by $(b/b_o)\eta L\sigma_o\alpha_{g,pl}/4r^*$. Combining that expression with Eq.6 and the transferability rule, $\alpha_{g,pl}\sqrt{G} = H() \approx$ constant, gives $dJ_{o,pl}/da$ = constant$(\sqrt{b})/r^*b_o \approx$ constant$(\sqrt{b})/b_o$, since r* is substanially independent of size within the fully plastic

DNB regime for a given material. That immediately shows a wider-lower picture for the steady state behaviour in terms of J, although it neglects the transient or 'elbow' of the conventional rising R-curve. It is therefore possible that the transient regime up to about 10% growth follows some other law, possibly less size dependent, as in (2),(3). As noted (5), the rising R-curve is not a measure of (local) toughness in the energetic sense since it is a normalisation of work including all plasticity even rather remote from the tip. The CTOA measure of (local) crack tip toughness, as used here, is constant with growth, but the strongly rising (global) R-curve relates directly to it.

This analysis was developed in (5) to give a datum value of CTOA from the steady state of crack growth under falling load in hslh materials. The concept of a centre of rotation applies only to rigid body analysis as plausible for the fully plastic state and cannot be used if elastic behaviour is approached. The analysis excludes the transient growth immediately following initiation. It is suspected that Eqs.1a and b may not hold for growth under rising load or G, a situation that may in essence greatly extend the transient regime. No data suitable for analysis are known to the writers. However, it was argued (5) that the rule for transferability, Eq.3, is applicable for any degree of deformation although the approximate relationship $(\sqrt{b})/r^*b_o$ just derived for the dependence of the slope of the J_o based R-curve, applies only to the fully plastic state.

CONCLUSIONS

1) 3D finite element studies of ductile crack growth, albeit without representation of crack front curvature, have confirmed an experimental analysis, whereby a particular measure of CTOA, $\alpha_{g,pl}$, can be derived from crack mouth opening in a fully plastic DNB test, (4), via the position of the instantaneous centre of rotation of the arms, (7).

2) Using a larger size of piece as an 'unknown' case, the computations confirm that this value of CTOA, if treated as a datum value, can be applied to the other case, as proposed (5), via the rule $\alpha_{g,pl}\sqrt{G} = H \approx$ constant.

3) The slope of the fully plastic J_o-R-curve in DNB tests of hslh material is shown to depend on $(\sqrt{b})/r^*b_o$ for large growth. Small growth is not treated.

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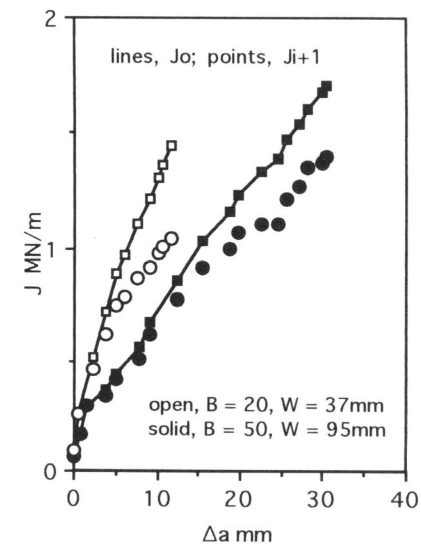

Fig.1 R curves for HY130, showing 'wider-lower' behaviour.

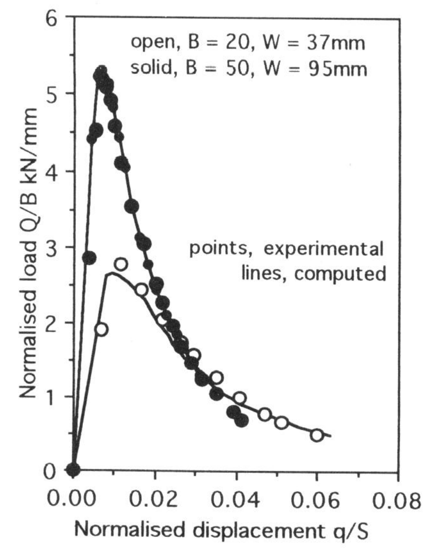

Fig. 2 Normalised experimental and computed loading diagrams.

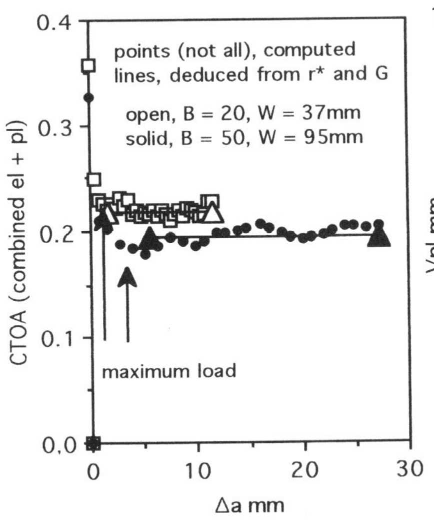

Fig. 3 Comparison of computed (el + pl) CTOA with steady state estimates.

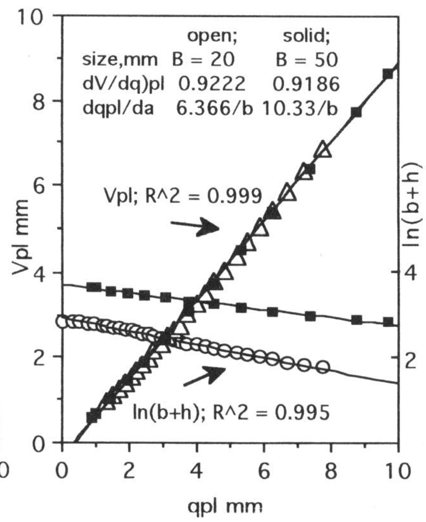

Fig. 4 Computed plot of V_{pl} v q_{pl} and ln(b+h) v q_{pl} to show linear slopes.