DEVELOPMENT OF METHODS FOR INCLUDING CONSTRAINT EFFECTS WITHIN THE SINTAP PROCEDURES

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To enable constraint based approaches to be applied to the practical assessment of defective components, development work within SINTAP is addressing a number of areas: constraint parameters for surface defects, constraint parameters for secondary stresses, simplified weight function methods for calculation of T-stress and relevant fracture toughness data. This paper describes these developments and indicates how the methods will be included in the overall SINTAP procedures.

INTRODUCTION

Constraint based approaches to fracture assessment are now becoming well established and have, for example, been included in the R6 procedures, Ainsworth and O'Dowd (1). However, while the principles of these methods are established, practical issues such as the calculation of relevant constraint parameters and collection of appropriate constraint-dependent fracture toughness data remain.

Within SINTAP (Structural **Int**egrity **A**ssessment **P**rocedures for European Industry), these practical issues are being addressed with the aim of including constraint based approaches within an overall defect assessment procedure. In this paper, constraint approaches are first described. Then progress within SINTAP on producing advice to enable practical application of the approaches is briefly summarised.

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CONSTRAINT BASED APPROACHES

Within SINTAP, an approach is being developed which allows results to be presented either as a crack driving force or on a failure assessment diagram. A fully consistent approach is being ensured so that the assessment result is independent of the way in which the results are presented, Ainsworth, Kim, Zerbst, Gutierrez-Solana and Ocejo (2). The crack driving force, in terms of the parameter J, is expressed for pure mechanical loading as a function of the load, F, by

$$J/J_e = f(L_r) \qquad (1)$$

where

$$L_r = F/F_Y \qquad (2)$$

where J_e is the elastically calculated value of J and F_Y is the yield load of the cracked structure which is proportional to the material yield stress σ_Y.

Failure is conceded when J reaches a critical material toughness, J^c_{mat}, which depends on a constraint parameter. This parameter depends on load magnitude and is here written as βL_r. When the elastic T-stress quantifies constraint, β depends on geometry, crack size a and loading type but not on load magnitude. Other parameters such as Q additionally depend on material and are non-linear functions of load so that β is no longer a constant.

In the failure assessment diagram representation, two parameters are calculated. One is L_r defined by eqn(2). The other is

$$K_r = K(a,F)/K_{mat} \qquad (3)$$

where K_{mat} is the fracture toughness measured under high constraint conditions using standard deeply cracked bend specimens. Having calculated L_r and K_r, the point (L_r, K_r) is plotted on a failure assessment diagram and failure is avoided provided

$$K_r \leq f(L_r) \qquad (4)$$

where the function f is that in eqn(1). Various forms of f are being developed within SINTAP (2). A limit is also applied to avoid plastic collapse. To incorporate constraint effects, two methods are possible (1). In the first, K_{mat} in eqn(3) is replaced by K^c_{mat}, the fracture toughness including a constraint dependence, the K equivalent of J^c_{mat}. In the second, the definition of eqn(3) is retained but eqn(4) is modified to

$$K_r \leq f(L_r)(K^c_{mat}/K_{mat}) \qquad (5)$$

Developments within SINTAP for estimating constraint parameters or for obtaining constraint dependent toughness values are briefly described below.

CONSTRAINT PARAMETERS FOR SURFACE DEFECTS

Three constraint parameters have been evaluated for plates under uniaxial and biaxial tension. Two biaxial ratios, 1:0.5 and 1:1, have been considered for a shallow (a/t = 0.10) and a deep (a/t = 0.50) semi-elliptical surface crack for power law materials with n = 3, 5 and 10. The constraint parameters are defined by

$$Q = (\sigma_{\theta\theta} - \sigma_{\theta\theta}^{SSY}) / \sigma_Y \quad \ldots \quad (6)$$

$$Q_m = (\sigma_h - \sigma_h^{SSY}) / \sigma_Y \quad \ldots \quad (7)$$

$$H = \sigma_m/\sigma_e - (\sigma_h/\sigma_e)^{SSY} = h - h^{SSY} \quad \ldots \quad (8)$$

where, $\sigma_{\theta\theta}$ is the opening stress, σ_h the hydrostatic stress and σ_e the von Mises effective stress. These stresses are evaluated for the actual geometry. The quantities denoted by superscript SSY are corresponding values from the small scale yielding solution of the material under plane strain conditions. Fig.1 shows sample results. Detailed information for different cases can be found in Sattari-Far (3) and from these results it can be concluded that:

- the different crack-tip constraint parameters steadily decrease with increased remote loading both for uniaxial and biaxial loading conditions;
- substantial loss of constraint is observed at the crack front positions near the free surface relative to positions at the deepest point of the crack front;
- the elliptical surface crack geometries indicate significant loss of constraint, especially the H-values, at the deepest point under uniaxial loading compared with the corresponding values in SEN(T) specimens with similar depths;
- at the deepest point, biaxial loading has a minor influence in changing Q, but considerable influence on Q_m and H, suggesting these latter parameters are better at incorporating biaxial effects;
- in general, biaxial loading has more influence on Q_m than on Q, but in some cases Q_m can become higher than Q due to biaxial loading.

TREATMENT OF SECONDARY STRESSES

The above approaches have been developed for primary loading. However, extension to combined primary and secondary loading is straightforward, Ainsworth, Sanderson, Hooton and Sherry (4). Using the FAD, eqn(3) is modified to

$$K_r = K/K_{mat} + \rho\,(K_{mat}^c/K_{mat}) \quad \ldots \quad (9)$$

where K is the total stress intensity factor for the combined loading and ρ is a plasticity correction parameter defined in R6 (5). If the definition of K_r is modified for constraint by replacing K_{mat} by K_{mat}^c then K_r becomes simply $(K/K_{mat}^c) + \rho$.

Having extended the approach to secondary loading, an important input is a value of a constraint parameter such as T or one of the parameters in eqns (6-8).

Weight function methods have been developed for evaluation of T for the non-linear stress distributions typical of secondary stresses and these are described in the next section. For Q, inelastic finite element calculations of some plane geometries have been performed and these suggest the approximation

$$
\begin{aligned}
Q &= Q^p + Q^s && Q^s > 0 \\
Q &= Q^p + Q^s\,(1-L_r) && Q^s < 0 \text{ and } L_r \leq 1 \\
Q &= Q^p && Q^s < 0 \text{ and } L_r > 1 \quad \ldots \quad (10)
\end{aligned}
$$

where Q^p is the value under the primary stresses alone as estimated from known solutions of the type described above. The value of Q^s may be estimated as $Q^s = T^s/\sigma_Y$ where T^s, the value of T-stress for the secondary stresses, is obtained from weight functions. Some finite-element results (4) for a single-edge cracked plate with a shallow crack, $a/t = 0.1$, are compared with eqn(10) in Fig.2. Eqn(10) is conservative in over estimating constraint, and the results confirm that the influence of secondary stresses becomes negligible once plasticity is widespread ($L_r > 1$).

WEIGHT FUNCTION METHODS FOR T-STRESS

It has been proposed, Hooton, Sherry, Sanderson and Ainsworth (6), that T-stress solutions may be derived from

$$T = T_{uc} + \int_0^a \sigma(x)t(x,a)dx \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

where T_{uc} is the value of T-stress in the uncracked body and $t(x,a)$ is a weight function which, for simplicity, may be represented by the second order polynomial

$$t(x,a) = (a^2 - x^2)^{-1/2}\{A_o + A_1\,(x/a) + A_2(x/a)^2\} \quad \ldots\ldots \quad (12)$$

If the normal stress distribution is represented by a second order polynomial

$$\sigma(x)/\sigma_0 = S_0 + S_1\,(x/t) + S_2(x/t)^2 \quad \ldots\ldots\ldots\ldots \quad (13)$$

where t is plate thickness, σ_0 is a normalising stress, and it is assumed that T may also be represented by a second order polynomial such as

$$T/\sigma_0 = T_0 + T_1\,(a/t) + T_2\,(a/t)^2 \quad \ldots\ldots\ldots\ldots \quad (14)$$

then combining eqns(11-13) gives an equation whose coefficients of $(a/t)^0$, $(a/t)^1$ and $(a/t)^2$ may be compared with those of eqn(14) to give:

$$
\begin{aligned}
T_0 &= S_0(\pi A_0/2 + A_1 + \pi A_2/4) \\
T_1 &= S_1(A_0 + \pi A_1/4 + 2A_2/3) \\
T_2 &= S_2(\pi A_0/4 + 2A_1/3 + 3\pi A_2/16) \quad \ldots\ldots\ldots\ldots \quad (15)
\end{aligned}
$$

A general T-stress solution for varying crack depth may then be obtained from finite element results for varying crack depth in a fixed geometry and a fixed second order stress distribution. Values of the constants T_0, T_1, T_2 of eqn(14) are obtained from a curve fit of T-stress results versus crack depth. Substitution of these values into eqn(15) then gives values of the constants A_0, A_1, A_2, and back substitution of these into eqns(11, 12) gives an equation for T-stress, as a function of the constants S_0, S_1, S_2 of any second order stress distribution and the crack depth ratio (a/t).

T-stress solutions have been obtained in this way (6), for an unrestrained edge-cracked plate, subject to an equibiaxial quadratic thermal stress distribution, giving

$$T/\sigma_0 = -0.5209\, S_0 - 0.7654\, S_1\, (a/t) - 0.7426\, S_2\, (a/t)^2 \quad (16)$$

and for a fully circumferentially cracked cylinder, subject to an equibiaxial quadratic thermal stress distribution, which gives

$$T/\sigma_0 = -0.5333 S_0 - 0.05472 S_1 (a/t) - 0.003458 S_2 (a/t)^2 \quad (17)$$

CONSTRAINT DEPENDENT FRACTURE TOUGHNESS DATA

A review of cleavage and ductile toughness data within the SINTAP project, Ainsworth and Sharples (7), showed that the general trends were:

(a) the fracture toughness at cleavage tends to increase with reducing constraint;
(b) the toughness at ductile initiation is insensitive to constraint but after some ductile crack extension tends to increase with reducing constraint;
(c) in view of (a) and (b), for ferritic steels there tends to be a reduction in the brittle to ductile transition temperature with reducing constraint;
(d) data on the influence of biaxial loading do not show consistent effects.

Work in SINTAP has concentrated on item (d). As noted above, different parameters for surface defects are influenced differently by biaxial loading, and this may make it possible to rationalise results. Experimental work is addressing interpretation of tests on A533B steel where a strong effect of biaxiality on fracture of through-cracked wide plates in the lower transition regime has been observed. Data on bend specimens at -100°C with crack depth to specimen width ratios in the range 0.1 to 0.5 are shown in Fig.3. The cleavage toughness is elevated for $a_0/w = 0.1$ to the extent that fracture only occurs after some ductile tearing. These data interpreted in terms of constraint parameters will be used to provide validation from the biaxial tests for constraint based approaches being developed within SINTAP.

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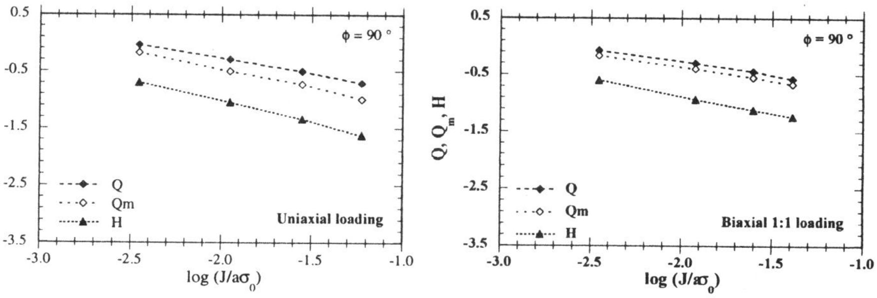

Figure 1. Constraint Under Uniaxial and Biaxial Loading at the Deepest Point of a Surface Crack, a/t = 0.5, n = 5

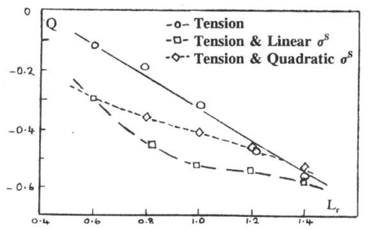

Figure 2. Constraint Q under combined primary and secondary loads

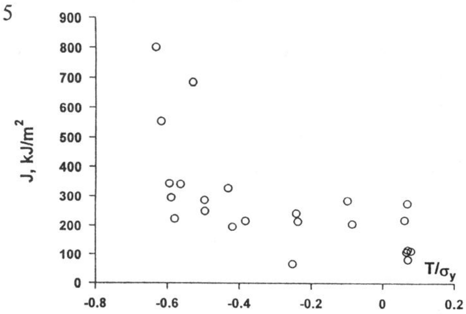

Figure 3. Cleavage toughness data as a Function of T-stress