PROBABILISTIC STUDY ON FATIGUE LIFE OF PROOF TESTED CERAMICS SPRING

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A new method was proposed to evaluate the probabilistic distribution of residual flaw size after proof test. The theory based on both process zone size failure criterion and two parameter Weibull distribution of σ_f and K_{IC}. A numerical crack growth analysis and fatigue test on Si_3N_4 model spring were conducted to predict the fatigue life. The calculated result showed good agreement with the experimental probabilistic fatigue life.

INTRODUCTION

Ceramics has excellent resistivities to heat, corrosion and wear, and ceramics coil spring has been developed[1]. However, generally speaking, ceramics is less reliable compared with metals. Because, their fracture toughness are not so high and they are sensitive to flaws. Their allowable flaw size is so small that it is almost impossible to detect flaws by NDI and repair them. To overcome this problem with reality, proof test is developed. Proof test is very useful for static load and it is verified. However, it is useful or not to cyclic load is not well verified. Because fatigue life dominantly depends on initial flaw size. Then it is very important to determine the distribution of residual flaw size after proof test. However, most structural ceramics show non-linear fracture behaviour[2]. Then, to determine the distribution of residual flaw size, non-linear fracture criterion[3] should be used. In this paper, new theory based on process zone size failure criterion[3] is proposed, to determine the residual flaw size distribution after proof test. Fatigue test has been made on proof tested ceramics model spring, and it is verified that the theory is useful to evaluate the probabilistic fatigue life.

THEORY

Correlation between Proof Stress and Probability of Residual Crack Size

Fig.1 shows a correlation between fracture stress(σ_C) and equivalent crack length(a_e) in structural ceramics. Solid line shows a average correlation between σ_C and a_e. This

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line is given by using process zone size failure criterion for ceramics[3]. It is already reported that this criterion showed very good agreement with experimental results for many kinds of structural ceramics[3].

$$\frac{\pi}{8}\left(\frac{\overline{K_{IC}}}{\overline{\sigma_f}}\right)^2 = a_e\left\{\sec\left(\frac{\pi\sigma_e}{2\overline{\sigma_f}}\right) - 1\right\} \qquad (1)$$

where, $\overline{\sigma_f}$ is average fracture stress of plain specimen and $\overline{K_{IC}}$ is average plane strain fracture toughness.

By using Eq(1), average residual crack size a_p after proof stress of σ_p is given.

$$a_p = \frac{\pi}{8}\left(\frac{\overline{K_{IC}}}{\overline{\sigma_f}}\right)^2\left\{\sec\left(\frac{\pi\sigma_p}{2\overline{\sigma_f}}\right) - 1\right\}^{-1} \qquad (2)$$

If σ_f and K_{IC} are constant, the residual crack size a_P is a constant and given by Eq(2). However, both σ_f and K_{IC} show scatter[3], generally, then the residual crack size is not constant but probabilistic one. Now we assume that the distribution of both σ_f and K_{IC} are given by 2-parameter Weibull distribution function. Firstly, we assume the case where fracture stress of plain specimen σ_f is the constant value of σ_0 and only K_{IC} is probabilistic one as shown in Fig.1. The probability $H(a_{PX})$ is defined that larger crack a_{PX} than a_P will reside. The probability $H(a_{PX})$ is equal to the probability that K_{IC} is greater than K_P as shown in Fig.1 by alternate long and short dash line and chain line. Then it can be given by following equation, easily.

$$H(a_{px}) = 1 - F(K_p) \qquad (3)$$

where, F(K) is two parameter Weibull distribution function of K_{IC}, and K_P is easily given from Eq(1) by substituting a_p, σ_p, σ_0 and K_p for a_e, σ_e, σ_f and K_{IC}, respectively.

Generally, both σ_f and K_{IC} are probabilistic. Then, probability $dG(a_{PX})$ is defined that larger crack a_{PX} than a_P will reside for the range from σ_0 to $\sigma_0 + d\sigma_0$. The probability $dG(a_{PX})$ is given by using probabilistic density function $f(\sigma_f)$ of σ_f, easily.

$$dG(a_{px}) = H(a_{px})\, f(\sigma_0)\, d(\sigma_0) \qquad (4)$$

where $f(\sigma_0)$ was given by substituting σ_0 for σ_f of $f(\sigma_f)$.

Then the probability $G(a_{PX})$ where larger crack a_{PX} than a_P will reside for the proof stress σ_p is given by integrating Eq(4) from σ_p to infinite. Subsequently, fatigue life reliability $R(a_{PX})$ is given by Eq(5) as a function of proof stress σ_p

$$R(a_{px}) = \{1 - G(a_{px})\} \times 100\% \qquad (5)$$

where, $G(a_{px})$ is given by the following equation.

$$G \quad a_{px} \quad \int H \quad a_{px} \quad f \quad \sigma_0 \quad d\sigma_0 \tag{6}$$

Probabilistic Fatigue Life Evaluation of Proof Tested Sample

By above Eq(5), the correlation between residual equivalent crack size a_{px} and residual probability can be obtained. By using the equivalent crack size a_{px}, stress intensity factor was calculated by $K=\sigma\sqrt{\pi a}$ equation for infinite plate.

For probabilistic fatigue life analysis, semi-elliptical surface crack of arbitrary aspect ratio was assumed as an initial crack. The initial crack size was determined by the following way: (1)Assume aspect ratio. (2)Determine crack size which maximum stress intensity factor at σ_{max} is equal to the K_{max} by using Newman-Raju equation[4], where K_{max} is given by $K_{max}=\sigma_{max}\sqrt{\pi a_{px}}$.

Subsequently fatigue life is predicted by using Paris power law and final failure condition was given by $K_{max} \geqq K_{IC}$.

SPECIMENS AND EXPERIMENTAL PROCEDURE

Specimen and Experimental Procedure

Sample is Si_3N_4 sintered at 1850°C, in 1 atm N_2 gas. This sample is not hot pressed, then it has considerably many flaws such as small pore. The sintered batch were cut into test pieces(0.8mm x 10mm x 100mm), because this size is sometime used as a plate spring. After cutting, surfaces of the test pieces were ground and polished before testing in accordance with the Japan Industrial Standard(JIS)[5], and final specimen's thickness was made to 0.8mm accurately.

The fracture strength was measured by a three-point bending test following the JIS method[5]. The span length and cross head speed were 30mm and 0.5mm/min, respectively. Fracture toughness was measured by the indentation method(load=49N) using Niihara's equation[6] for convenience. Proof test and fatigue test were carried out at room temperature in an air environment using mechanical fatigue testing machine. The both tests were carried out under deflection control mode. Fatigue test conditions were as follows: test frequency 10Hz, stress ratio R=0, and uniform moment zone is 10mm(wide)x20mm(length). Fatigue test was stopped when specimen didn't fracture up to 10^7 number of cycles. Proof test was carried out by stressing up to 880MPa as quick as possible to avoid a crack growth during proof test.

RESULTS AND DISCUSSION

Weibull Properties of Material Tested

Fracture stress σ_f is a material's constant and should be measured from the sample which has no obvious flaws on fracture surface. After bending test, fracture surfaces were investigated in detail using SEM. Twelve specimens were obtained by the test and Weibull properties of σ_f were listed in Table 1. Twenty K_{IC} data were obtained by indentation method and their Weibull properties were also listed in Table 1.

Fatigue Test Results

After cyclic fatigue test, the fracture surfaces were investigated in detail by using SEM. On fourteen specimens, fatigue crack was recognized to start from initial flaw, and the flaw shape could be defined clearly. From these specimens, initial flaws shape and aspect ratio were determined. From these data, it can be seen that most flaws were embedded one and their aspect ratio were almost about 1.0. However, their flaws shape are neither elliptic nor semi-elliptic and are very complex. It is very difficult or almost impossible to determine the K value for such flaws, exactly. In this paper, following simple method was adopted to determine the K value, for convenience. Semi-circular surface crack was assumed, which area is equal to real flaw area. Subsequently, initial stress intensity factor K_i of the semi-circular crack was calculated by using Newman-Raju Eq[4] and σ_{max} . K_i versus number of cycle to failure N_f was plotted in Fig.2. The relation between K_i and N_f largely depends on material's constant C and m in Paris's power law{$da/dN=C(\triangle K)^m$}. Appropriate values were searched by try and error method, and finally, $C= 1\times10^{-23}$ and m=23 were determined as the best values as shown in Fig.2.

In Fig.3, fatigue test results on Si_3N_4 spring model was shown. Solid symbols show the test results on no proof tested(virgin) specimen. Open symbols show the test results on proof tested specimen. Proof test has been made at the stress σ_p of 880MPa. This sample is not hot pressed and they have many flaws. Then their fatigue life showed very wide scatter. However, proof tested specimen show narrower scatter than virgin specimen. Lower band of σ_{max} - N_f curve for proof tested specimen is higher than that of virgin specimen by about 100MPa. From this figure, it can be concluded that proof test is useful for fatigue life. If proof stress σ_p is higher, the usefulness of the test will become more remarkable. However, if σ_p is settled at high level, few specimen will survives and cost will become high.

Probabilistic Fatigue Life by Analysis

The correlation between probability $G(a_{px})$ and equivalent crack size a_{px} was calculated by using Eq(6) and material's constants in Table 1, where $G(a_{px})$ is a residual probability of equivalent crack larger than a_{PX}. By using the a_{PX} and Paris's power law, fatigue life can be calculated for the case of σ_P =880MPa as a function of σ_{max}, reliability and aspect ratio of initial surface crack. In Fig.4, calculated σ_{max} - N_f curves were shown as a function of reliability. This is a case of σ_P=880MPa and aspect ratio=1.0. These curves were made by the following ways: (a)Calculate a_{PX} as a function of probability as shown in Table 2. (b)Presume aspect ratio and determine the surface crack size which K_{max} is equal to that of equivalent crack a_{PX}. (c)Presume σ_{max} to some value. (d)Calculate N_f by using Paris's power law. From Fig.4, it can be seen that calculated probabilistic σ_{max} - N_f curves show very good agreement with experimental one.

CONCLUSIONS

Study has been made on the residual crack size distribution after proof test and usefulness of proof test on fatigue life of sintered Si_3N_4. The main results are as follows:

(1) A new method is proposed to evaluate the residual crack size distribution after proof test by using process zone size failure criterion and two parameter Weibull distribution function of both σ_f and K_{IC}.

(2) By fatigue test, it was well verified that proof test was very useful technology to guarantee fatigue life of ceramics members.

(3) Probabilistic fatigue life of proof tested specimen was evaluated by using the above method proposed in this paper, calculated probabilistic fatigue life showed good agreement with experimental one.

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TABLE 1- Two Parameter Weibull Coefficients of Fracture Stress σ_f and Plane Strain Fracture Toughness, K_{IC}.

	Mean Value	Scale Parameter	Shape Parameter
σ_f	1110 MPa	1150 MPa	17.2
K_{IC}	6.65 Mpa$\sqrt{m}$	6.78 MPa$\sqrt{m}$	18.4

TABLE 2- Relationship between Residual Equivalent Crack Size a_{PX} and Reliability of Cyclic Fatigue Life

$R(a_{PX})$	50 %	60 %	70 %	80 %	90 %
a_{PX} (μ m)	6.7104	7.1486	7.5935	8.0855	8.7183
a_{PX} a_P	1.0106	1.0766	1.1436	1.2177	1.3130

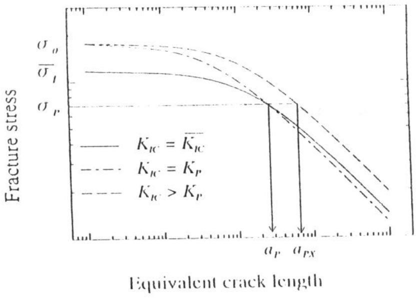

Figure 1 Correlation between fracture stress σ_C and equivalent crack length, and also proof stress σ_P and residual crack size a_P

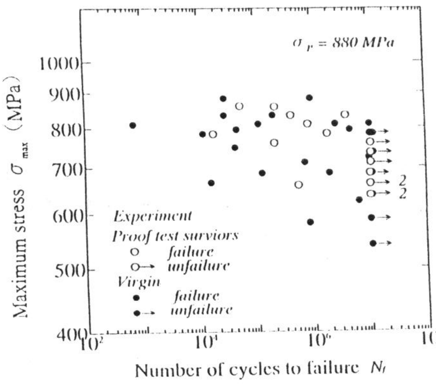

Figure 3 Relationship between cyclic maximum stress σ_{max} versus number of cycle to failure N_f

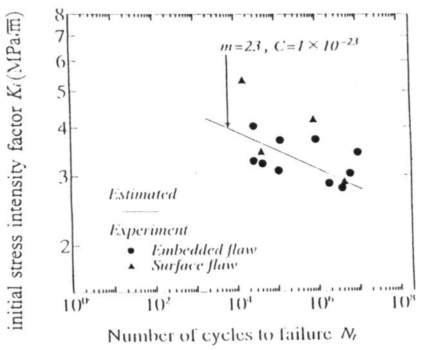

Figure 2 Correlation between initial stress intensity factor K_i and number of cycle to failure N_f

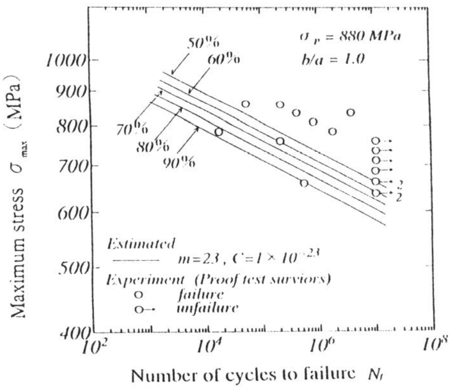

Figure 4 Comparison between calculated fatigue failure probability and experimental data