METHOD FOR INVESTIGATION OF THE LONG-TERM FRACTURE AND RELIABILITY OF BRITTLE MATERIALS

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From practice we know the forms of fracture of brittle structures when the flaws and cracks are not visually observed up to the moment of fracture. In these cases the statistical test-model method is developed to study the process of long-term fracture (and, hence, the reliability) of brittle materials (ceramics, glass, compounds etc.). The method is based on the thermofluctuation theory of strength, the linear fracture mechanics, and the reliability theory.

INTRODUCTION

A new statistical test-model method for solution of the estimation problem of structure reliability (made of brittle materials) at their long-term operation is proposed where a dominating influence of the material surface and environment on strength is considered. The method is based on the thermofluctuation (dilatation) theory of strength, the linear fracture mechanics, and the reliability theory. The method rests upon the following conceptions and hypotheses.

Fracture progresses with time t and reaches its critical stage in a surface layer of the material with small thickness (from units up to hundred mkm for various brittle materials), where stress changes slightly along the normal to the surface of a structure (Margolin et al. (1), Margolin and Osadchuk (2)).

In the macroscopically isotropic surface layer there are microdefects with uniform distribution and arbitrary orientation. They are modelled by the planar surface

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microcracks with typical initial sizes l^0 (Batdorf and Heinisch (3)).

Microdefects exert no influence on the macroscopic stresses determined employing the theory of elasticity ((3)).

A microcrack starts to grow when a local tensile stress, normal to its face, exceeds a safe level ((1), (2)).

During crack growth deep into the bulk the self-similarity principle is realized, while a crack form Y is assumed to be constant (Boehm and Lewis (4)).

Microcrack growth is initiated when the local tensile stress normal to a crack plane exceeds the criterion value σ^* ((1), (2)).

LONG-TERM FRACTURE OF BRITTLE MATERIALS

The durability model. Durability τ of material is determined by the minimal growth period of the local normal stresses σ_i from their initial value up to their critical value σ^*, or a corresponding growth period of a depth crack l_i up to its critical value l^*

$$\tau = \min\left\{ \int_{\sigma_i^0}^{\sigma^*} \left[\dot{\sigma}_i(t)\right]^{-1} d\sigma_i \right\}, \qquad (1)$$

$$\tau = \min\left\{ \int_{l_i^0}^{l^*} \left[\dot{l}_i(t)\right]^{-1} dl_i \right\}, \quad i = \overline{1, m} \qquad (1a)$$

$\dot{\sigma}_i = d\sigma_i / dt$, $\sigma_i = K_{Ii} / (2\pi r_{0i})^{1/2}$, $K_{Ii} = Y_i K_i$, $K_i = \sigma(l_i)^{1/2}$, $\dot{l}_i = dl_i / dt$, σ is the macrostress normal to the i-th crack, m is a number of defects on a specimen surface under the stress σ, r_{0i} denotes a distance from a base of the i-th crack to a bond break zone.

The method proposed uses thin disk specimens under stationary concentric-ring bending (Fig 1). In this case, the uniform equibiaxial plane tension is realized on a part of specimen's surface with a base area s_0. In such a stressed state within an area s_0 the same stationary normal tension σ acts on all surface defects (all microcracks are normal to the surface) with arbitrary orientations. In Fig. 1 $\sigma = BF / H^2$, $B = (3/2\pi)\{(1-\nu)[(r_2^2 - r_1^2)/2r_2^2](r_2^2/r_3^2) + (1+\nu)\ln(r_2/r_1)\}$, ν is Puasson's ratio.

The kinetics model. For description of a crack growth kinetics under stationary stresses it is proposed to use the following equation:

$$\frac{dK_i}{dt} = Q_i \exp(q_i K_i), \qquad (2)$$

$$\frac{dl_i}{dt} = C_i K_{Ii}^{P_i}, \tag{2a}$$

where Q_i, q_i are the constant parameters in the i-th crack zone, C_i, P_i are the analogous parameters of material from the Perris equation.

The longevity distributions. Introducing the expression (2) into (1) and having integrated the last, we have

$$\tau_j = \left(Q_e q_e\right)_j^{-1} \exp\left\{\left[-q_e Y_e \left(2\pi r_{oe}\right)^{-1/2}\left(l_e^0\right)^{1/2}\right]_j \sigma\right\}, \quad j = \overline{1, n}. \tag{3}$$

Taking the logarithm of the obtained expression (3), we have

$$\lg \tau_j = \Pi_j - \Psi_j \sigma, \quad j = \overline{1, n}. \tag{4}$$

Here $\Pi_j = -\left(\lg Q_e + \lg q_e\right)_j$ and $\Psi_j = q_e Y_e \left(2\pi r_{0e}\right)^{-1/2}\left(l_e^0\right)^{1/2} \lg e$, n is a number of specimens, the subscript «e» corresponds to the extreme fracture situations.

The expression (3) is analogous to the exponential longevity Zhurkov model. Unlike this model, the expression (3) makes the basis of log-normal longevity distribution for the specimens τ, where the value randomness Q_e, q_e, Y_e, r_{oe}, l_e^0 and their numerical values are considered. This is of great importance. Respectively, for (1a) and (2a) we have:

$$\lg \tau_j = B_j - b_j \lg \sigma, \tag{4a}$$

where B_j, b_j are the parameters of the j-th specimen in the extremum situation described. The distribution function τ in this case will be log-normal one as well.

The statistical test-model method. Testing of k sets of n specimens is carried out for circular disks under the concentric-ring bending. Tests are carried out under the stationary stress $\sigma_v \left(v = \overline{1, k}\right)$ over the time $t = T_n$. The times up to fracture (durability) τ_j are determined.

The fracture probability of specimens is determined by the formula

$$W_j = n_j / (n+1), \tag{5}$$

where n_j is a number of fractured specimens for the time $t \le \tau_j$.

The test results for 5 sets of $n \ge 100$ specimens are given in Fig.2 for disks of glass type C52-1 with $s_0 = 182\ \text{mm}^2$, at the temperature $T° = 25° \pm 2°\,\text{C}$ and environment humidity $H_e = 96 - 98\,\%$.

Statistical processing was carried out in the logarithmically normal coordinate system. The test results confirm the hypothesis of logarithmically normal distribution of specimens durability with degree of certainty $P^* > 0.93$-0.96 for different sets of specimens according to the Pirson criterion. The corresponding statistical processing of experimental data according to the Weybull distribution has given a lower degree of certainty P*>0.67-0.72.

The confidence intervals (Fig.2) of dependences $W_v(\lg t)$ are determined in the median zone (for quantile $g_0 = 0$ at $W_v = 0.5$) and in a range of quantile $g_0 = -1$ ($W_v = 0.1587$), $v = \overline{1,5}$.

The results of such corrected statistical processing (including the confidence intervals in Fig.2) are presented in Fig.3. The expressions for these characteristics have the following form

$$S_{M,0} = S^0_{M,0} - \Psi^{-1}_{M,0} \lg t , \tag{6}$$

where $\Psi_{M,0} = \Pi_{M,0} / S^0_{M,0}$, subscripts M and 0 correspond to the median and strength of $g_0 = -1$, respectively. The corresponding inverse relations have the form

$$\lg \tau_{M,0} = \Pi_{M,0} - \Psi_{M,0}\sigma , \tag{7}$$

$$\Pi_{M,0} = \lg t_{M,0}, \quad \sigma = 0.$$

According to the logarithmically normal distribution of durability the reliability of an area element s_0 is defined by the expression

$$R_0(\sigma,t) = \begin{cases} 1 - \Phi(g_0), & \sigma > S_c,\ t > t^*_M ; \\ 1, & \sigma \le S_c ; \end{cases} \tag{8}$$

$$\Phi(g_0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{g_0} e^{-x^2/2} dx , \qquad g_0 = (\lg t - \Pi_M + \Psi_M \sigma) / (\Delta\Pi + \Delta\Psi\sigma) ,$$

$$\Delta\Pi = \Pi_M - \Pi_0, \qquad \Delta\Psi = \Psi_0 - \Psi_M .$$

RELIABILITY OF STRUCTURES MADE OF BRITTLE MATERIALS

We determine the stressed state of a structure. On the basis of the hypothesis of uniform defect distribution, the surface of a structure is divided into parts s_ξ with a quasi-uniform stressed state. Then the angular subdivision epure of normal stresses into angles $\Delta\varphi_r$ with quasi-constant stresses σ_r is carried out within the area element s_ξ.

The equivalent (with respect to defect presence) area element s_r corresponds to each of these angles. Now, using the assumption of the weakest link in this case, the shear stresses during the defect propagation can be neglected, the reliability of structure is determined by

$$R_c(t) = \prod_{\xi=1}^{m} \prod_{r=0}^{k} \left[R_0(\sigma_r, t)\right]^{M_r} , \tag{9}$$

where $R_0(\sigma_r, t)$ is described by the expression (8), $M_r = s_r / s_0$, $\sigma_r = [(\sigma_1 + \sigma_2) + (\sigma_1 - \sigma_2)\cos 2\varphi_r]/2$, $\sigma_{1,2}$ are the principal stresses, $s_r = s_\xi \Delta\varphi_r / \pi$, $s_\xi = \Delta x_\xi \Delta y_\xi$, φ_r is an angle between the stress vectors σ_r and σ_1.

For taking into account the shear stresses τ_r we introduce the equivalent (with respect to reliability) stress

$$\sigma_{er} = \left(\sigma_r^2 + \lambda\tau_r^2\right)^{1/2}, \tag{10}$$

$$\sigma_r > 0, \qquad \tau_r = [(\sigma_1 - \sigma_2)\sin 2\varphi_r]/2.$$

The factor λ is determined as follows. Testing of n rectangular bars is carried out over the time T_n using a 4-point bending technique. In this case the uniform uniaxial tension is realized on a surface s_e. The experimental reliability of specimens is determined

$$R_e = 1 - n_{Tn}/(n + 1). \tag{11}$$

Setting $R_c(T_n)$ equal to the experimental value of reliability R_e, we obtain the relation for determining λ.

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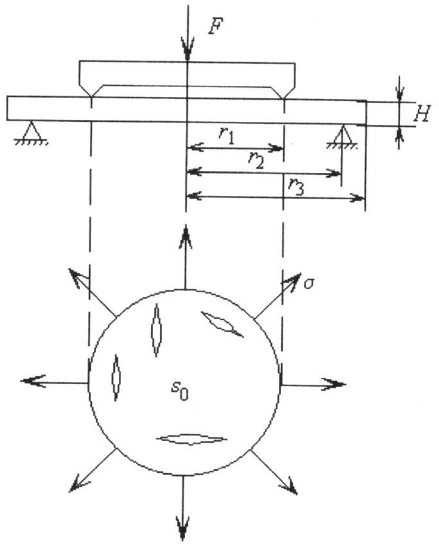

Figure 1 The test-method of concentric-ring bending ($s_o = \pi r_1^2$, $r_2 = 2r_1$, $r_3 \geq 10r_1$, $H \geq 0.6r_1$)

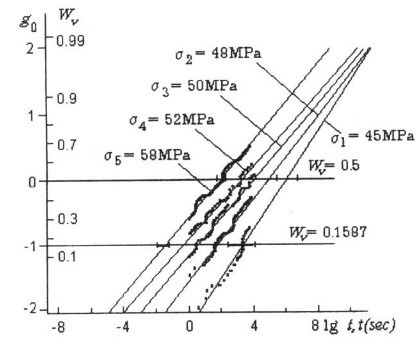

Figure 2 The statistical method

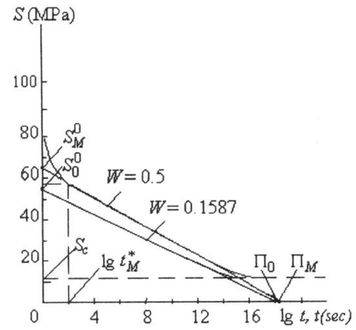

Figure 3 The statistical test-model method