MODELING OF CRACK SYSTEM FORMATION IN A CERAMIC DISK UNDER A THERMO-SHOCK

R.V.Goldstein* and N.M.Osipenko*

An asymptotic approach is suggested for modeling of multiple brittle fracture of a disk under a thermal shock at the stage when the characteristic crack scales are essentially larger as compared with their spacing. Analytical dependences of the crack length and beam width (i.e. distance between the cracks) were derived. On the basis of these formulae the relations between the relative crack density and their relative length were obtained. These relations are universal for the given loading regime. The obtained relations enabled to treat the observed difference of fracture processes at shock cooling when the cracks grow from the disk surface to its center and shock heating when cracking starts at the disk center.

INTRODUCTION

Multiple cracking is one of the characteristic mechanisms of brittle fracture under thermal loading (see, e.g. Hasselman (1), Nemat-Nasser et al (2), Bahr et al (3), Egorov et al (4), Lanin et al (5)). In particular, multiple cracking is often observed in ceramics under the thermal shock. A hierarchical structure of cracks occurs in the ceramic element (or at its surface). The crack structure has the form of a net or a system of subparallel cracks of different sizes and causes unloading in a certain region of the element from the redundaut thermal stresses. The cracking process is accompanied by the reduction of the element bearing capacity and/or violation of its operation regimes.

Modeling of the thermal cracking processes implies searching for the conditions of the cracking initiation, growth and stability, as well as evaluation of the residual strength and lifetime of the damaged structural elements.

We performed an asymptotic analysis of the multiple cracking taking in mind the experimental results of fracture of ceramic disks at heating and cooling (4), (5). The analysis is based on the evident assumption that during the crack system formation the growth of the largest cracks is adjusted by the stress state of the whole structural element with cracks while the behavior of other cracks in the hierarchical system is influenced by the local stress fields. Moreover, the less the crack size and the crack placing the more significant are the local stress effects. In other words, the advanced process of multiple fracture is characterized by the relatively independent thermomechanical reaction of the separate strips (beams) formed by the initial cracks. Practically the heat transfer at the crack surfaces is low since the crack opening is small in case of brittle fracture. The temperature distribution in the body is assumed to be independent on the presence of cracks.

* Institute for Problems in Mechanics, Russian Academy of Sciences, Moscow, Russia

BEAM MODEL OF THERMAL FRACTURE

Let us consider a stress state of a thin beam-halfstrip under the cooling shock at the free end with no heat transfer at its lateral sides. According to aforementioned features of the multiple fracture process this situation can be associated with a thermal shock at the boundary of a halfplane separated on a series of beams by the parallel cracks transvers to the boundary.

Note, that a simple asymptotics of the stress state exists in case of a thin beam. One can show that

$$\sigma_y(x) \approx (1/2)[EH/(1+\nu)](d\varepsilon/dx), (d\varepsilon/dx) = (dT/dx)\alpha \tag{1}$$

where x, y are the coordinates along and transverse the beam, 2H is the beam thickness, E, ν are the Young modulus and Poisson ratio of the material, α is the thermal expansion coefficient, T is the temperature.

As an example consider the thermal shock at the edge of the halfplane separated on the beams of width 2H. Taking into account the temperature distribution transverse to the boundary of the halfplane (Kovalenko (6)) $T = (\Delta T)\mathrm{erfc}(x/2\sqrt{at})$, where (ΔT) is the temperature jump at the boundary, a is the thermal conductivity coefficient, we obtain from formula (1) the stresses in the beams

$$\sigma_y(x,t) = -AH\sqrt{t}\exp[-x^2/4at], \quad A = (1/2)E\alpha(\Delta T)/(1+\nu)\sqrt{\pi a} \tag{2}$$

The lavel of the maximum stress decreases with time while the size of the stressed region along the beams increases.

In our asymptotic analysis of multiple fracture we choose a beam with the lateral sides along the surfaces of the subparallel cracks as an elementary cell of the crack system at the advanced stage of the process. A crack of a smaller size grows along the median of the beam. Then the growth of a crack system can be represented a suquence of the acts of single cracks propagation along the median of the appropriate effective beam.

Consider the conditions of the crack growth in such a beam (halfstrip) under the action of the thermal stresses (2) (Fig. 1). Note, that the transverse displacements at the lateral sides of the beam are forbidden.

Using the compliance method (Rice (7)) we obtain for the stress intensity factor $K_I = \sigma_y(x,t)\sqrt{H}$ where $\sigma_y(x,t)$ should be substituted from formula (2). The resulting formula has the following form

$$K_I = AH^{3/2}t^{-1/2}\exp[-x^2/4at] \tag{3}$$

The dependence (3) is nonmonotone function of time which tends to zero at $t \to 0$ and $t \to \infty$.

Assume that the crack grows in the regime which provides the maximum value of the stress intensity factor, i.e., in the regime most favorable for the dissipation of the beam deformation energy. The maximum value K_I is attained at $t=(x^2/2a)$ and is equal to

$$K_{Imax} = AH^{3/2} e^{-1/2} (2a)^{1/2} x^{-1} \qquad (4)$$

Denote by $x=\ell$ the crack length in the state of the limit equilibrium when $K_{Imax}=K_{Ic}$, K_{Ic} is the critical stress intensity factor. Then from formula (4) we obtain

$$\ell = AH^{3/2} K_{Ic}^{-1} e^{-1/2} (2a)^{1/2} \qquad (5)$$

This simple relation enables us to evaluate some characteristic parameters of the whole crack ensemble within the assumption that the beam of the larger effective width can be combined from several beams of smaller size. Note, that by this way we will obtain an upper estimate of the crack density since a part of the deformation energy in the vicinity is consumed on the growth of smaller ones.

Let us consider relative crack density (the ratio of the amount of the cracks of length larger than ℓ, $n(x>\ell)$, to the total amount of the cracks, n_Σ), as the integral characteristic of the crack system (3).

The relative crack density can be represented as follows

$$n_r = (n(x > \ell) / n_\Sigma) = \left(\int_{\ell}^{\ell_{max}} H(\ell)^{-1} d\ell \right) / \left(\int_{o}^{\ell_{max}} H(\ell)^{-1} d\ell \right) \qquad (6)$$

Then using formula (5) we obtain

$$n_r \approx 1 - (\ell / \ell_{max})^{1/3} \qquad (7)$$

This dependence is given in Fig. 2 along with the experimental data (Bahr et al (8)) associated with the similar variant of loading and the results of the numerical solution of the problem on a system of edge cracks in the halfplane (3). One can see that formula (7) gives a good estimate of the relative crack density within the almost whole range of the crack lengthes. Further, this relation is rather universal and only determined by the function of the temperature distribution.

Note, that the transformations of the function of two variables $\sigma_y(x,t)$ performed in formulae (2)-(5) are equivalent to construction of its envelope, i.e., using the function of one variable $\sigma_y(x)=-AH(2a)^{1/2}e^{-1/2}x^{-1}$ instead of $\sigma_y(x,t)$ in the calculations of the stress intensity factor one can obtain the same result (formula (5)).

Hence, an asymptotic quasistatic analysis of multiple fracture under the action of a thermal shock can be performed within the framework of a model of fracture in which the time is excluded and all cracks are considered in the limit equilibrium state under the action of the stresses being the envelope of the real time dependent stresses. Such an approach can be used for modeling thermal fracture in finite bodies under different schemes of loading.

SOME ESTIMATES FOR FINITE BODIES

As an example let us consider the disk cracking under the action of the heat flux towards its external cylindrical surface. The disk end planes are heat-insulated. This scheme is associated with the experimental studies of thermal fracture of high strength ceramics (4), (5). We will use the quasisteady envelop of the nonsteady temperature distribution along the disk radius in the parabolic, $T_p(r)=T_{in}+(\Delta T)(r/R)^n$, $n\sim2$, and logarithmic, $T_l(r)=T_{out}+(\Delta T)\ln(r/R)$, forms (5) where R is the disk radius, T_{in} and T_{out} are the temperature at the inner and outer parts of the disk, respectively.

The disk can be represented as a combination of the wedge-shaped thin beams separated by the thermal cracks. One can show that the stresses σ_θ for the wedge-shaped thin beam are equal to

$$\sigma_\theta = A(\pi a)^{1/2}\alpha n(r/R)^n(H_o/R)), \quad \sigma_\theta = A(\pi a)^{1/2}\alpha(H_o/R) \tag{8}$$

for the distributions $T_p(r)$, $T_l(r)$, respectively, where H_o is the beam width at the disk surface. The current width of the beam equals $H(r)=H_o(r/R)$.

The condition of the limit equilibrium of a central (bisector) crack in a wedge-shaped beam can be written as follows $\sigma_\theta(H)^{1/2}=K_{Ic}$. Then relations between H_o and the length of the crack in the state of the limit equilibrium have the following form

$$H_o = [K_{Ic}R/nB(1-(\ell/R)^{n+1/2}]^{2/3}, \quad H_o = (K_{Ic}R/B)^{2/3}(1-(\ell/R)^{-1/3} \tag{9}$$

for the parabolic and logarithmic temperature envelops, and $B=A(\pi a)^{1/2}$.

Finaly, the relative crack density is equal to

$$n_r \approx (1-(\ell/R))^{8/3}, n=2; \quad n_r \approx (1-(\ell/R))^2, n=1 \tag{10}$$

for the parabolic envelop and

$$n_r \approx (1-(\ell/R))^{4/3} \tag{11}$$

for the logarithmic one.

These dependences are given in Fig. 3 along with the experimental data obtained by Lanin (9) on thin disks of heatproof ceramic materials.

CONCLUSION

The suggested asymptotic approach was used to analyze the integral characteristics of the multiple cracking under thermo-shock. On the other hand, it enables to explain the observed difference of the fracture process at shock cooling when the crack grow from the disk surface to its center and shock heating when cracking starts at the disk center. As a rule, in the second case the specimen is separated on parts while in the first one full fracture rarely occurs (4), (5). Within the considered beam model this effect can be associated with the conditions of the growth of the cracks of maximum size. One can show (Goldstein; Osipenko (10)) that the cracks growing from the disk central zone grow in an unstable manner almost up to the disk surface while the stress intensity factors for the edge cracks monotone decrease with their length increase providing the crack arrest.

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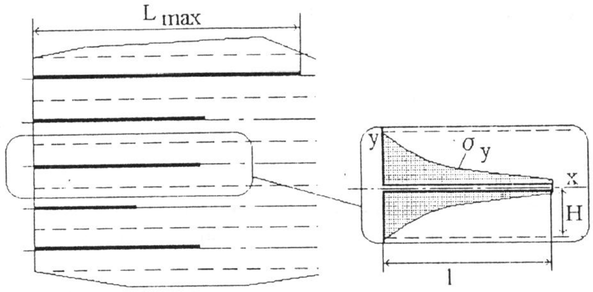
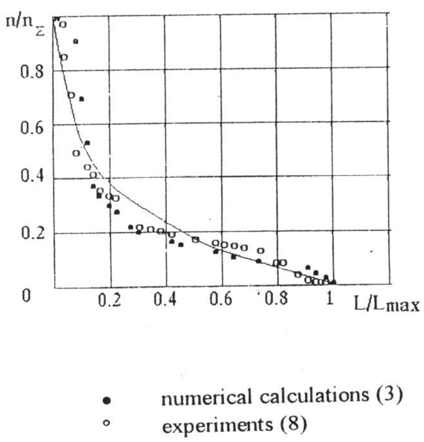


Figure 1. Scheme of multiple cracking



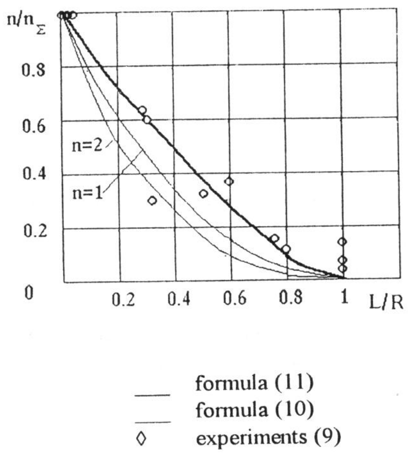


Figure 2. Dependence of the crack density in the halfplane

Figure 3. Dependence of the crack density in the disk