THE PECULIARITIES OF DESIGNING WITH BRITTLE MATERIALS – WEAK POINTS AND DEFICIENCIES

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The principles and peculiarities of designing with brittle materials, particularly engineering ceramics, which follow from the probabilistic fracture mechanics approach required, are concisely reviewed. A critical examination of the underlying, commonly accepted assumptions reveals the shortcomings of standard design practice. The difficulties of considering real materials behaviour are briefly discussed. Attention is brought to the necessity for a further elaboration of design methodology and the provision of design relevant materials data.

INTRODUCTION

Despite the numerous advantageous properties of engineering ceramics, designers still hesitate to use them for load bearing applications. The main subjective reason seems to lie in the different methodology, based on probabilistic fracture mechanics, which is required for designing with brittle as opposed to conventional, ductile materials. In fact, there are also certain deficiencies in the design methodology and a lack of necessary material data. This will be reviewed in the present paper.

PROBABILISTIC FRACTURE MECHANICS

Ceramics are prone to brittle failure due to their intrinsically high yield strength and low fracture toughness. Their inability to relax stress concentrations at the tips of microscopic surface or volume flaws can result in any one of these flaws propagating catastrophically in a uniform tensile stress field, if the stress intensity factor $K_I = Y\sigma\sqrt{a}$ (for the applied stress σ normal to the flaw plane, flaw length a, and a geometrical factor Y) reaches a critical value, i.e. fracture toughness, K_{Ic} (Wachtman (1), Lawn (2)). Since the length of inherent flaws, a, is a stochastic variable, so is the tensile strength of a brittle material

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$$\sigma_{\mathrm{f}} = K_{\mathrm{Ic}} / Y\sqrt{a} . \tag{1}$$

Consequently, design with brittle materials requires a probabilistic approach.

Rearranging the failure criterion (1) gives the critical flaw length

$$a_{\mathrm{c}} = \left(K_{\mathrm{Ic}} / Y\sigma\right)^2 . \tag{2}$$

Any flaw longer than or equal to a_{c} would unstably propagate and be destructive under the applied tensile stress σ normal to the plane of the flaw.

Assuming that a component fails if any one flaw initiates fracture (the weakest link hypothesis), and that there is no interaction between flaws, the probability of failure, P_{f}, equals the probability of encountering at least one destructive flaw in the component, i.e.

$$P_{\mathrm{f}} = 1 - \exp(-N_{\mathrm{c}}) , \tag{3}$$

where N_{c} is the most probable number of destructive flaws, i.e. their mean number in a large set of identical components (Danzer (3), cf. Freudenthal (4)). This is obtained by integrating the local density of destructive flaws $n_{\mathrm{c}} = \int_{a_{\mathrm{c}}}^{\infty} g(a, \mathbf{r})\,\mathrm{d}a$ over the volume (where $g(a, \mathbf{r})$ is the frequency distribution density of flaw lengths at a point defined by the position vector $\mathbf{r}$) (3). Since a_{c} depends on the magnitude of stress at a certain point, so does the value of n_{c}. Assuming that relation (2) applies and that flaws of different lengths are uniformly distributed over the volume, i.e. $g(a, \mathbf{r}) \equiv g(a)$, it can be shown (4) that any distribution of flaw lengths which for $a \to \infty$ converges toward zero as fast as a^{-k}, where k is any constant, leads to the two-parameter Weibull distribution of strength

$$P_{\mathrm{f}} = 1 - \exp\left(-\frac{1}{V_0} \int_V \left(\frac{\langle\sigma\rangle}{\sigma_0} \right)^m \mathrm{d}V \right) , \tag{4}$$

with the Weibull modulus m and characteristic strength σ_0 directly related to the parameters of the distribution of flaw lengths $g(a)$, and V_0 being a certain reference (or unit) volume; the notation $\langle\sigma\rangle$ is used for the Heaviside function to explicitly indicate that the integration should be performed only over the region loaded in tension. As a matter of fact, the Weibull distribution of strength (4) of a ceramic material is commonly presumed, and the parameters m and σ_0 are directly determined as material constants by a statistical evaluation of measured values of strength.

The strength of many ceramic materials deteriorates with time due to subcritical crack growth (SCCG) which causes the mean number of destructive flaws N_{c} to continuously increase, which can lead to delayed failure. This should also be considered in the assessment of failure probability. SCCG in ceramics is most frequently explained as being a consequence of thermo-mechanically activated environmental attack on atomic bonds at a crack tip (Ritter (5), (2)). The observed rates of SCCG may be approximated by a power low $\mathrm{d}a/\mathrm{d}t = C\,K_{\mathrm{I}}^{\,n}$, where C and n are material parameters for specific environmental conditions. Thus, it is possible to calculate the time a flaw of initial length a_{i} under the action of a constant stress σ would need to reach the critical length a_{c}. Taking this into account formula (4) can be generalized for the time-dependent cumulative probability of failure under an invariable load (Danzer (6)):

$$P_{\mathrm{f}} = 1 - \exp\left(-\frac{1}{V_0} \int_V \left(\frac{\langle\sigma\rangle}{\sigma_0}\right)^m \left(1 + \frac{\langle\sigma\rangle^2}{C_1} t\right)^{m/(n-2)} \mathrm{d}V \right), \tag{5}$$

with $C_1 = 2 / [C\,(n-2)\,Y^2 K_{\mathrm{Ic}}^{\,n-2}]$.

If necessary, the reliability of a ceramic component can be assured by proof testing. By subjecting a component, for a short time, to a pre-determined load level appropriately higher than that which it would experience in service, components that would fail in less than the required lifetime can easily be detected and rejected (Davidge (7)). It is, however, not always possible to appropriately 'scale' all relevant conditions, such as thermal stresses or environmental conditions.

PECULIARITIES OF DESIGNING WITH BRITTLE MATERIALS

The peculiarities of designing with brittle, as opposed to ductile, materials become evident from the character and structure of the basic formula for design calculations (5):

- The calculation of strength is probabilistic by its nature.
- The volume integration brings about the size effect, i.e. the increase in the probability of failure with increasing size of component, under an equal stress level.
- Only tensile stresses are decisive for the risk of brittle failure. Therefore, brittle materials are more appropriate for components loaded mainly in compression and tensile stresses should be minimized by proper design.
- Since the Weibull modulus m enters (5) as a power, the probability of failure is very sensitive to small variations in the highest tensile stresses that arise. Therefore avoiding design-conditioned stress concentrations becomes even more important than when designing for ductile materials. This is particularly important as any excessive stress can not be relaxed by local plastic deformation.
- Since stresses at every point in a component enter into the calculation of strength, a precise knowledge of the stress distribution over the volume is required and not just the maximum values. This can generally only be achieved by numerical methods.
- Unlike ductile materials ceramics subjected even to an invariable loading suffer strength degradation (by SCCG) and the probability of failure increases with time.

Although instantaneous brittle fracture and SCCG are responsible for the failure of the majority of ceramic components, other damaging mechanisms may also be important and have to be considered by a designer. Ceramics, for instance, are prone to edge chipping under a contact load (Almond and McCormick (8)), but other kinds of contact damage may also appear, e.g. indentation fracture (2) and a variety of wear mechanisms (Jahanmir (9)). Impact damage can be especially critical. Another problem arising from the brittleness and other thermal-elastic properties of engineering ceramics is their comparatively low resistance to thermal shock. The limiting factor in high temperature structural applications of ceramics may be creep (1). Cyclic fatigue is far less pronounced than in ductile materials and is caused by different mechanisms (1).

The design of a ceramic component should also be appropriate for the specific way of fabrication (Creyke et al. (10)).

WEAK POINTS AND DEFICIENCIES

Even if design calculations are performed only to avoid instantaneous or delayed brittle failure, the application of formula (5) still involves some uncertainties, and can even be inappropriate if some of the commonly accepted assumptions concerning a) the underlying brittle failure criterion (1), b) the assumed Weibull distribution of strength, and c) the kinetic law of SCCG, are violated.

The failure criterion (1) and consequently formulae (4) and (5) hold for mode I fracture under a uniform uniaxial stress state of a material with unchanging fracture toughness K_{Ic}, in the absence of any internal stresses. The problem of defining a proper criterion for brittle fracture under mixed mode loading, and thus specifying an equivalent stress for a multiaxial stress state, is still subject to debate (Thiemeier and Bruckner-Foit (11)). As a first approximation, the equivalent stress can be taken to be the first principal stress, which is plausible only if other principal stresses are not of comparable magnitude. Nevertheless, the brittle failure criterion (1) is applicable only to tensile stresses, and for exceptionally high compressive stresses design calculations have to be based on completely different principles. *R*-curve behaviour, i.e. an increase in fracture toughness with crack advancement, characteristic for many engineering ceramics, enables propagating cracks of a certain length to stop growing (1). Similarly, in a non-uniform stress field, a propagating crack can also be arrested by entering a less stressed or compressive region. Taking all this into account considerably complicates the assessment of reliability. Internal stresses could be more easily introduced into the calculation, but are generally unknown.

As has been stated, the commonly used Weibull distribution of strength of brittle materials (4) follows from the more general distribution function (3) provided certain assumptions are valid. The conditions even for the latter to be physically justified, however, (i.e. the weakest link hypothesis and no interaction between flaws) are not always fulfilled. Evidence of this can be the non-appearance of a size effect. This occurs whenever flaws are so densely distributed that they do interact. The distribution of flaws may also not comply with the additional conditions for strength to be Weibull-distributed. This can be due to a gradient distribution of flaws, which arises especially in large ceramic components, but also due to the presence of two or more different flaw populations. The consequences of both of these effects on the distribution of strength are discussed in Danzer et al. (12) and Danzer and Lube (13), respectively. Finally, the previously discussed objections about the applicability of the brittle failure criterion (1) (or (2)) may also implicate a non-fulfilment of this essential pre-condition for the Weibull distribution of strength.

The problem of the applicability of the Weibull distribution to the strength of brittle materials is decisive for the correct assessment of reliability. However, it is almost impossible to establish whether the strength is Weibull-distributed or not on the basis of a practical number of equal specimens (13). Results from the typically fewer than 100 nominally equal specimens usually tested all fall within such a narrow range of distribution function of strength that they can equally well be described by the Weibull as well as any other flexible, e.g. gaussian, distribution. These results most likely also fall into a narrow range of relatively high failure probabilities. Thus any extrapolation of probability of failure to practically relevant lower probabilities of failure and/or to components of size different from that of the specimens, is doubtful. However, testing of at least two sets of specimens of different size would adequately reveal whether there is a size effect, which is necessary for the Weibull distribution of strength to be justified. Furthermore, two or more ranges of strength, corresponding to different ranges of failure originating flaw lengths, are investigated in this way, which then enables the applicability of the Weibull or any other distribution to be evaluated by statistical methods (13).

Describing the multitude of possible mechanisms and parameters affecting SCCG by a simple empirical power law is certainly questionable. Even phenomenologically, three regions of dependence of the rate of crack advancement on stress intensity factor can generally be distinguished, and there is often an indication of a certain threshold value. Furthermore, environmentally accessible surface cracks grow much faster then those in the volume interior. This leads to the development of a distinct flaw population with further implications on the distribution of strength. Anyway, the greatest uncertainty in the accounting for SCCG arises from its strong dependence on environmental conditions. Thus, the parameters of SCCG determined under laboratory conditions may be totally inappropriate in service.

Apart from the above criticism directed at the validity of commonly accepted assumptions upon which standard calculations for design with ceramics are based, there is an acute problem in acquiring the necessary material data. Not only the parameters of the subcritical crack growth and other application relevant material data such as thermal properties or creep data, but even the Weibull parameters of the strength distribution (if it applies at all) have to be determined almost in every single case. This is because data bases are generally not available, data sheets are often incomplete and unreliable, the quality of ceramic materials is not standardized and, as a matter of fact, is often batch-dependent. Thus a reliable design with ceramics usually has to start with a time-consuming and costly material testing programme during which unsolved problems concerning the methods of testing of brittle materials may be encountered. For instance, fracture toughness, as a basic mechanical property of a ceramic material, can be determined by various methods, often delivering different results. There is, therefore, an urgent need for the standardization of engineering ceramics and methods of testing, leading ultimately to the establishment of data bases.

CONCLUSIONS

There are some difficulties in the probabilistic fracture mechanics modelling of failure behaviour of brittle materials, which is the basis for the calculation of strength, i.e. the reliability of load bearing ceramic components. These difficulties mainly arise from uncertainties about the applicability of commonly accepted assumptions, that sometimes are not appropriate at all. Difficult to account for are e.g. brittle failure under mixed mode loading, R-curve behaviour in a ceramic, crack arrest in a non-uniform stress field, the effect of environmental conditions on subcritical crack growth, and particularly the consequences of flaw distribution features on the distribution of strength. A further serious problem is a lack of necessary material data (and reliable testing methods). An additional effort and further elaboration of basic design concepts and methodology is required to bring it near to an engineering discipline.

Note.–Realizing the above discussed problems ESIS TC6 has recently agreed to start a round robin with the aim of initiating a data base containing design relevant ceramic materials data.

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