LIFETIME PREDICTION UNDER CYCLIC LOADING FOR A CLASS OF ELASTO-VISCOPLASTIC DAMAGED MODEL

S. Bezzina

This paper deals with the implementation in finite element calculation of complex elasto-viscoplastic constitutive equations exhibiting non linear kinematic, isotropic hardening and fully coupled to ductile damage evolution model. The state variables are considered to evolve phenomenologically according to competing process between elastic properties, strain hardening (isotropic and kinematic) and continuous isotropic damage. First, constitutive laws are presented. Then, numerical example of failure under cyclic loading are presented and discussed.

INTRODUCTION

Continuum damage modeling has become one of active subject of research in recent years. Many researchers introduced damage variable as a scalar, vector, or tensor quantity for the effective representation of damage phenomena for several kinds of materials under creep, fatigue, or other loading conditions. (Leckie et al (1)) studied the creep rupture of structure with scalar damage variable. (Chaboche(2)) reviewed the general concept of damage mechanics relating to damage measure, description of the mechanical behavior of the damage material, and damage evolution equations. This review shows the guideline of research in continuum damage mechanics.
The modeling of the failure prediction of a structure loaded in isothermal fatigue is determined using the Manson-Coffin law (Manson (3)) identified for the material at the service temperature of the structure. The plastic strain amplitude chosen to calculate it is obtained by structural calculations at its most loaded point, when the stabilized cycle is reached. When the loading is very anisothermal or when it leads to complex stress and strain states, it is even more difficult to calculate the lifetime of the structure using the Manson-Coffin law.

* Université de Technologie de Compiègne, LG2MS URA 1505, Compiègne, BP 529.

One way of overcoming these problems is to use an approach derived from the thermodynamics of irreversible processes with internal variables like Continuum Damage Mechanics approach (CDM).

In this paper, After selecting internal state variables, we follow the thermodynamic framework of the CDM approach. The coupling between the elasto-viscoplastic behavior and the isotropic damage is made using the hypothesis of the total energy equivalence developed in the case of elasto-plastic behavior by (Saanouni et al (4)). First, constitutive equations are described, the damage evolution equation and the rate forms of the constitutive equations are presented. The application is limited to a simple case of traction-compression to illustrate the capability of the present approach to describe naturally the failure under cyclic loading.

ELASTO-VISCOPLASTIC DAMAGE MODEL

Typically, forming processes are accompanied by finite strain and notable damage accumulation. This is in contrast with the small strain regime where damage evolution is only essential for cyclic loading. The total strain is then assumed to be the sum of an elastic part and a viscoplastic part: $\varepsilon_e = \varepsilon - \varepsilon_{vp}$. The internal variables are the isotropic hardening (r, R), the kinematic hardening (α, X) and finally, the isotropic damage (D, -Y) . The extreme value of the damage variables D are 0 for a sound material and 1 for a volume element with null stress carrying capability.

THERMODYNAMIC POTENTIAL

The specific free energy Ψ, taken as the thermodynamic potential in which elasticity and plasticity are uncoupled, gives the law of elasticity coupled with damage and the definition of the associated variables related to internal variables as detailed in (bezzina (5)):

$$\rho\psi(\varepsilon_e, r, \alpha, D) = \rho\psi_e(\tilde{\varepsilon}_e) + \rho\psi_{vp}(\tilde{\varepsilon}_e, \tilde{r}, \tilde{\alpha}) \tag{1}$$

$$\rho\psi(\varepsilon_e, r, \alpha, D) = \frac{1}{2}\tilde{\varepsilon}_e : \Lambda : \tilde{\varepsilon}_e + \frac{1}{3}C\tilde{\alpha} : \tilde{\alpha} + \frac{1}{2}Q\tilde{r}^{(2)} \tag{2}$$

For all internal variables denoted v and their associated forces A we are $\tilde{v} = \sqrt{1-D}v$ and $\tilde{A} = \frac{A}{\sqrt{1-D}}$. Λ is the symmetric fourth order tensor properties and ρ is the density material. C and Q are respectively the kinematic and isotropic hardening moduli. The state laws are classically derived from the state potential as:

$$\sigma = \rho\frac{\partial\psi_e}{\partial\varepsilon_e} = \tilde{\Lambda} : \varepsilon_e \tag{3}$$

$$X = \rho \frac{\partial \psi_{vp}}{\partial \alpha} = \frac{2}{3} \tilde{C} \alpha \tag{4}$$

$$R = \rho \frac{\partial \psi_{vp}}{\partial r} = \tilde{Q} r \tag{5}$$

$$\tilde{\Lambda} = (1-D)\Lambda, \tilde{Q} = (1-D)Q, \tilde{C} = (1-D)C \tag{6}$$

$$-Y = \frac{J_2^2(\sigma)}{2E(1-D)^2} \left[\frac{2}{3}(1+\nu) - 3(1-2\nu) \left(\frac{\sigma_H}{J_2(\sigma)} \right)^2 \right] + \frac{1}{3} J_2^2(\alpha) + \frac{1}{2} Qr^2 \tag{7}$$

E and ν are Young's modulus and Poisson's ratio of the virgin material. $J_2(\sigma)$ is the Cauchy stress equivalent $J_2(\sigma) = \sqrt{\frac{3}{2}} S:S$, S is the stress deviator $S = \sigma - \sigma_H \delta_{ij}$, and σ_H is the hydrostatic stress.

It appears that when D = 1 the associated forces σ, X and R vanish illustrating that the medium cannot support any effort. By referring to equation (7), the damage energy release rate which is equivalent to the energy release rate G in fracture mechanics contains three terms: the classical contribution of elastic energy, and two news terms representing the release of the stored (kinematic and isotropic) energy.

FLOW RULES AND DAMAGE EVOLUTION

The complementary laws in terms of the selected state variables are applied to derive the viscoplastic flow rule in damaged material. The equipotential surface function is decomposed into viscoplastic (φ^*_{vp}) and damage-related (φ^*_D) component as:

$$\varphi^*_{vp} = \frac{K}{(n+1)} \left\langle \frac{1}{K} \left[f + \frac{3}{4} \frac{a}{C} \tilde{X} : \tilde{X} - \frac{1}{3} aC\tilde{\alpha} : \tilde{\alpha} + \frac{1}{2} \frac{b}{Q} \tilde{R}^2 - \frac{1}{2} bQ\tilde{r}^2 \right] \right\rangle^{(n+1)} \tag{8}$$

and

$$\varphi^*_D = \frac{S}{(s+1)} \frac{1}{(1-D)^{\beta}} \left(-\frac{Y}{S} \right)^{(s+1)} \tag{9}$$

$$f(\tilde{\sigma}, \tilde{X}, \tilde{R}) = J_2(\tilde{\sigma} - \tilde{X}) - \tilde{R} - \sigma_y \tag{10}$$

K, n, S, s, β are material parameters. By applying the generalized normality rule and after some developments detailed in (Bezzina (5)), we can get the evolution of stress and the internal variables as:

$$\dot{\sigma} = \tilde{\Lambda} : \dot{\varepsilon} - \dot{\tilde{\lambda}}_{vp}(1-D)\left[\frac{3}{2}\Lambda : \frac{S-X}{J_2(\sigma - X)} + \frac{Y^*}{(1-D)}\tilde{\sigma}\right] \tag{11}$$

$$\dot{X} = \dot{\tilde{\lambda}}_{vp}(1-D)\left[C\frac{S-X}{J_2(\sigma - X)} - (a + \frac{Y^*}{(1-D)})\tilde{X}\right] \tag{12}$$

$$\dot{R} = \dot{\tilde{\lambda}}_{vp}(1-D)\left[Q - (b + \frac{Y^*}{(1-D)})\tilde{R}\right] \tag{13}$$

$$\dot{D} = \dot{\tilde{\lambda}}_{vp}\frac{1}{(1-D)^{\beta-\frac{1}{2}}}\left(\frac{Y}{S}\right)^s \tag{14}$$

where $\dot{\lambda}$, $\dot{\tilde{\lambda}}_{vp}$ are defined as:

$$\dot{\tilde{\lambda}}_{vp} = \frac{\dot{\lambda}}{\sqrt{1-D}}, \dot{\lambda} = \left\langle\frac{f}{K}\right\rangle^n \quad \text{if } f>0 \tag{15}$$

It is worth nothing that the damage evolution equation (14) is valid for many forms of damage evolution like fatigue or creep. In a synthetic paper, (Lemaitre (6)) show that many other laws of damage can be obtained by this form, especially laws of fatigue damage for great or weak number of cycles and the law of creep damage.

APPLICATIONS

The theoretical model presented above was implemented in the general purpose Finite Elements code SIC (Système Interactif de Conception) developed at the University of Compiegnge. In this paper, we limit ourselves to simple illustration of the developed model. Under the assumption of plane strain, the simple case of axial symmetric traction compression loading with the imposed displacement equal to 0.001, a constant strain rate of $\dot{u} = 0.001s^{-1}$ and a time period of 40 seconds is considered. Material parameters correspond to a mild steel material and are provided by (Benallal (7)), those corresponding to a law of damage are chosen as suggested by (Lamaitre (6))

E = 144000 Mpa, ν = 0.3, $\sigma_y = 50$Mpa, $K = 50$Mpa, $n = 8$, $Q = 20000$Mpa,
b = 100, C = 21000 Mpa, a = 300, S = 10, s =1, $\beta = 1$ $D_c = 0.999$

Full implicit time integration scheme is used for the integration of the constitutive equations (generalized midpoint rule). Calculations without coupling to damage show the classical stabilized response after 5 cycles. In the coupling case, figure 1 shows the variation of the Cauchy stress σ_{22} versus the total strain ε_{22}. We note that the presence of damage generate continuous degradation of the mechanical properties until the total failure of the volume element. Figure 2 shows the variation of the Cauchy stress equivalent at the summit of cycles. The rapid variation of the rupture is caused by the jump of damage variable value at the end of lifetime as indicated in figure 3 where we are reported the damage variation as a function of number of cycles. It can be seen that calculation have been cried out for 96 cycles until the total rupture (i.e. when damage attains his critical value $D=D_c$).

Figure 3 shows the predicted variation of peak back-stress X_{22} versus the inelastic strain ε_{22}^{p}. The response indicate that the value of the internal variable starting from zero, increases with increasing hardening, reaches a maximum value and goes to zero when damage approaches $D = D_c$. The same result is obtained for the variation of the isotropic hardening R. Note that this is not the case in the theory used by (Chaboche (2)) and by other authors where the internal stresses X and R remain unaffected by the damage.

CONCLUSION

Using the framework of thermodynamic with internal variables, a new energy-based between the continuum damage mechanics and the classical elasto-viscoplastic behavior is proposed. These approach is based on coupled strains and damage constitutive equations and takes into account the redistribution of the stresses due to damage evolution. Its application to failure of structures under cyclic loading is illustrated and discussed. This paper helps to overcome with the limitations of classical analyses of cyclic loading such as the Manson-Coffin law.

REFERENCES

(1) Leckie, F. A. and Hayhurst, D. R., Proc. Royal Soc., Vol. 340., 1974, pp. 323-347.

(2) Chaboche, J. L., ASME Journal of Applied Mechanics, Vol. 55., 1988, pp. 59-64.

(3) Manson S. S., McGraw Hill Book, 1966.

(4) Saanouni, K., Forster, C., Ben hatira, F., Int. J. of Damage. Mech., Vol. 3., 1994, pp. 140-169.

(5) Bezzina, S., Modélisation théorique et numérique du procédé de découpage, Ph.D. Thesis, Université de Technologie de Compiègne, France, 1996.

(6) Lemaitre, J., C. R. Acad. Sci. Paris, t. 305., Serie II, 1987, pp. 1125-1130.

(7) Benallal, A., Eng. Comp., Vol. 3., 1986, pp. 323-330.

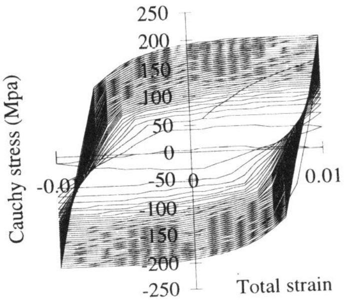

Figure 1 Cauchy stress-strain loops.

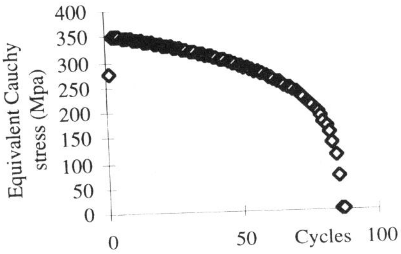

Figure 2 Cauchy stress equivalent as a function of number of cycles.

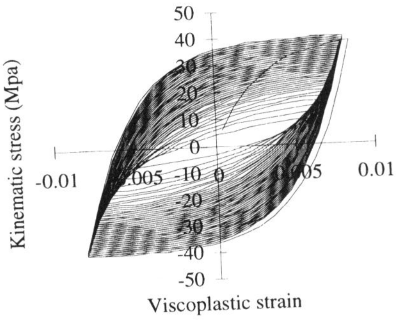

Figure 3 Kinematic stress-viscoplastic strain loops.

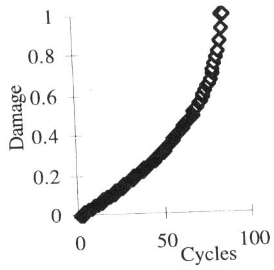

Figure 4 Evolution of the damage D as a function of number of cycles.