PREDICTION OF FATIGUE CRACK BEHAVIOUR IN ELASTOPLASTIC MATERIALS

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A problem about the stress-strain state around the rectilinear fatigue crack, propagating in a plate has been solved by a method of singular integral equations within the model of thin plastic strips. The reversive plastic yielding of the material and the effects caused by plasticity induced crack closure have been taken into account. This solution is used for the analysis of the regularities both of the initial stage of fracture (short crack growth) and fracture at high-amplitude loading. The crack growth rate is calculated by the linear-elastic and elastoplastic criteria and proceeding from this, a correct use of the effective SIF range, as a universal parameter of fatigue crack growth, is evaluated.

INTRODUCTION

A reversive plastic yield of material in the process zone and the related effect of crack closure are the principal factors, that determine the behaviour of fatigue cracks under different loading conditions. These complex processes can be successfully modelled by the known δ_c-model, generalized for cyclic loading conditions (Panasyuk (1), Dugdale (2)). The investigation results of Wang and Blum (3), Newman (4) and others proved that the above approach is very effective for prediction of fatigue fracture kinetics and assessment of structural life time. However, the available methods of calculations are based on rather complicated numerical solutions. This fact significantly decreases the range of such methods application. A rather simple scheme for construction of the solution for the analysis of elastoplastic situation at the fatigue crack tip within the framework of δ_c-model is proposed in the paper of Panasyuk et al. (7). It incorporates a method of weight functions, reduction of the problem to Cauchy-type singular integral equation. Thus, the closed-type solution of a number of important modelled problems of practical value can be constructed. In this paper the mentioned method is used for modelling the effects of plasticity during fatigue crack growth.

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METHOD OF ANALYSIS

An internal rectilinear crack $2l$ in length in a plate, subjected to cyclic loading with amplitude $\Delta p\left(\Delta p = p_{\max} - p_{\min}\right)$ is considered (Figure 1). The material of a plate is assumed as perfect elastoplastic with the yield strength σ_s. In accordance with the δ_c-model, the plastic yield of the material is assumed to localize in thin strips on the crack extension. These strips are modelled as the cuts with stresses σ_s, applied to their edges. The principle of superposition allows to replace the action of the removed stresses p by the efforts, applied to the crack edges. A solution of the problem is found in terms of the Green's function $H(l,x,\xi)$, that determines the vertical displacement of the crack edges under the impact of a system of separate concentrated forces, applied symmetrically to the crack edges at points $x = \pm\xi$:

$$H(l,x,\xi) = \frac{2}{\pi E}\ln\frac{\sqrt{l^2 - x^2} + \sqrt{l^2 - \xi^2}}{\sqrt{l^2 - x^2} - \sqrt{l^2 - \xi^2}} \tag{1}$$

In particular, the above leads to the following dependence for crack edges displacement under maximum loading of a cycle ($p = p_{\max}$):

$$u_{\max}(x) = p\int_0^{l+l_p} H\left(l + l_p, x, \xi\right)d\xi - \sigma_s\int_l^{l+l_p} H\left(l + l_p, x, \xi\right)d\xi \tag{2}$$

where l_p is the length of the monotone cyclic region.

The situation at the minimum loading of a cycle ($p = p_{\min}$) can be modelled similarly. It is assumed that the stresses within the cyclic plastic region ($l < |x| < l + l_{pf}$) are equal to the yield strength under compression. Besides, the crack closure effect caused by the plastic stretches of height $u_{res}(x)$, on the crack edges, is also taken into account (Figure 1). These stretches are formed as the crack passes through the plastic strain region in front of its tip. As a result the unloaded crack edges link up completely or partially in a certain section ($l_c < |x| < l$), interacting by stresses $\sigma_{con}(x)$. Thus,

$$\begin{aligned} u_{\min}(x) = u_{\max}(x) - \left(p_{\max} - p_{\min}\right)\int_0^{l+l_{pf}} H\left(l + l_{pf}, x, \xi\right)d\xi + \\ + \int_l^{l+l_p} \sigma_{con}(\xi) H\left(l + l_{pf}, x, \xi\right)d\xi - 2\sigma_s\int_l^{l+l_p} H\left(l + l_{pf}, x, \xi\right)d\xi \end{aligned} \tag{3}$$

Dependence (3) with combination with the condition of crack edges junction in the contact region

$$u_{\min}(x) = u_{res}(x) \quad , \quad l_c \le |x| \le l \tag{4}$$

form the integral equation for evaluation of the contact stresses $\sigma_{con}(x)$. By differentiating both equations (3) and (4) with respect to x, the above equation can be reduced to the following conventional singular equation

$$\int_{l_c}^{l} \frac{\sigma_{con}(\xi)\sqrt{\left(l+l_{pf}\right)^2-\xi^2}}{x^2-\xi^2} d\xi = \pi f(x) , \quad l_c \le |x| \le l \tag{5}$$

where f(x) is the given function. The solution of this problem, bounded at point $x = \pm l$ according to Muskhelishvili (6), gives the following expression for $\sigma_{con}(x)$:

$$\sigma_{con}(x) = \frac{1}{\pi} \frac{4x\sqrt{l^2-x^2}}{\sqrt{x^2-l_c^2}\sqrt{\left(l+l_{pf}\right)^2-x^2}} \int_{l_c}^{l} \frac{\sqrt{\xi^2-l_c^2}}{\sqrt{l^2-\xi^2}} \frac{f(\xi)d\xi}{\xi^2-x^2} \tag{6}$$

Proceeding from the condition of stresses boundedness at the end of the monotone and cyclic plastic regions, the length of these regions l_p and l_{pf}, respectively, is determined. Parameter l_c is evaluated from the additional condition, that presupposes that contact stresses are positive and tend to zero at the ends of the contact region $x = \pm l_c$. Function $u_{res}(x)$ is also unknown and is determined by a step-by-step analysis. It is assumed that at each step of the crack increment, a height of a stretch, formed on the new surface, is equal to the value of plastic crack edges displacement $u_{\min}(x)$ immediately in front of the crack tip.

The proposed system of calculational dependences is used for evaluation of the stress-strain state parameters in a plate, containing a crack. The main parameters are the following: crack tip opening displacement (CTOD) at minimum $\delta_{\min} = 2u_{\min}(l)$ and maximum $\delta_{\max} = 2u_{\max}(l)$ loading of a cycle; crack opening stresses, that correspond to such values of external loads $p = p_{op}$, at which the contact stresses on the surface are completely removed and the process of active deformation of the material, immediately in front of the crack tip, begins.

RESULTS AND DISCUSSIONS

Initial stage of fatigue crack propagation. The proposed solution is used for the internal crack in a plate, initiating from the original defect of length $2l_0$ (Figure 1). This situation is a model for short fatigue crack growth, formed in the material due to fracture of hostile inclusions, microvoids extension, cracking along the phase boundaries etc. These initial defects do not contain plastic stretches on their edges, which are the principal cause of crack closure. Therefore the cyclic compound of stresses and deformations at the their tip is higher. When mode I fatigue crack begins to initiate from such a defect, the influence of closure sharply increases. Figures 2 and 3 show the dependence of $p_{op}/p_{\max}$ and $\Delta\delta/\delta_0$ (δ_0 is maximum CTOD value for a cycle for the initial crack) on the crack length. In particular, as the crack grows, the CTOD value at first decreases, falls down to the minimum value, and then begins to grow. This tendency agrees with the regularities of variation of the short fatigue crack growth rate, observed in experiments (Ritchie and Lankford (7)). It is the main cause of the short crack effect, according to which the short cracks grow quicker and at lower values of SIF than the long ones.

A stage of steady crack growth. A process of crack closure stabilizes as the crack length increases to the value, approximately equal to the size of the plastic zone near the initial defect. After that the values of $p_{op}/p_{\max}$ and $\Delta\delta/\delta_{\max}$ do not longer change with the crack length. These steady values, calculated as a function of the maximum stresses and stress ratio of the loading cycle are given in Figures 4 and 5. The results are in good agreement with the published data (4). The proposed method allows to analyze the high-amplitude loading, including the conditions close to complete yield. This fact is important when modelling the low-cycle fatigue processes.

Crack growth rate criteria. Figure 6 illustrates the comparison of the fatigue crack growth rates v_1 and v_2, calculated by the linear-elastic and elastoplastic criteria, using dependences

$$v_1 = C\left(\Delta K_{ef}\right)^n \quad , \quad v_2 = C_1\left(\Delta\delta\right)^{n/2} \tag{7}$$

It was considered that both dependences were established according to the results of one and the same basic experiment, conducted under low-scale yield condition, i.e. they coincide at $p_{\max}/\sigma_s \to 0$. As a result the relationship between parameters C and C_1 is established. The curves show that the ΔK_{ef}-based calculation gives a somewhat underestimated value of the crack growth rate. At $p_{\max}/\sigma_s \leq 0.6$ an error is insignificant, and the estimate is suitable for practical usage. At higher loading amplitudes, especially at negative value of the stress ratio, a more accurate estimate is necessary.

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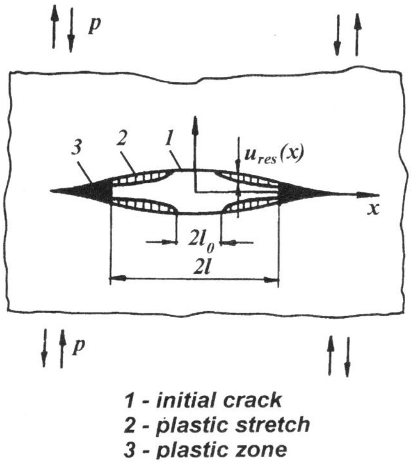

Figure 1 Fatigue crack in a plate

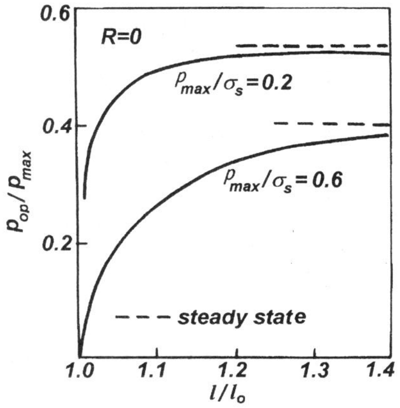

Figure 2 $p_{op}/p_{\max}$ vs. crack length

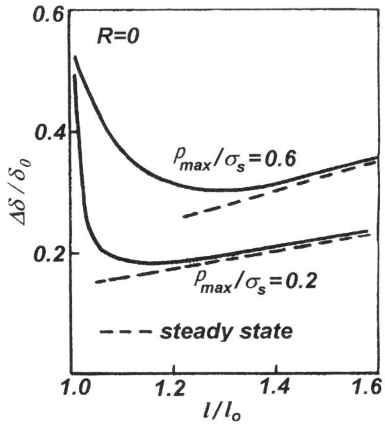

Figure 3 $\Delta\delta/\delta_0$ vs. crack length (δ_0 is CTOD for initial crack)

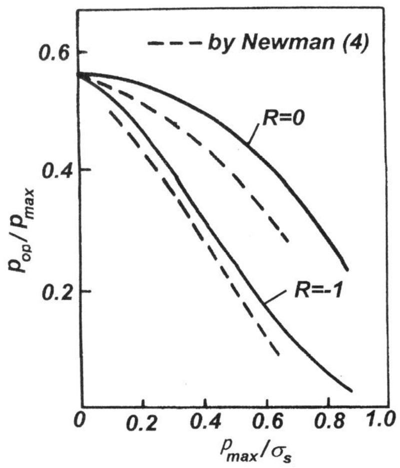

Figure 4 $p_{op}/p_{\max}$ for a steady growing crack

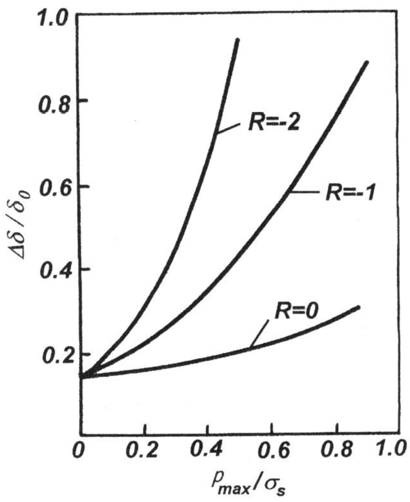

Figure 5 $\Delta\delta/\delta_{max}$ for a steady growing crack

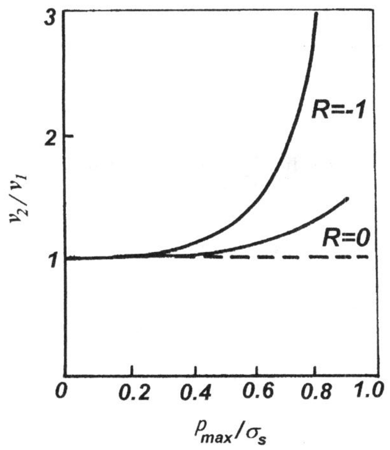

Figure 6 Comparison of the crack growth rates by linear (v_1) and nonlinear (v_2) criteria