AN ALTERNATIVE CRACK PATH STABILITY PARAMETER

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It is well known that, under both static and fatigue loading, the directional stability of a Mode I crack in an isotropic material under essentially elastic conditions is governed by the *T*-stress, a stress parallel to the crack. If the *T*-stress is compressive, then a crack is directionally stable, and if the *T*-stress is tensile it is directionally unstable. In practice it is sometimes found that cracks are directionally stable even if the *T*-stress is tensile. Dimensional considerations mean that the *T*-stress does not provide a satisfactory measure of crack path stability. An alternative non-dimensional parameter, the *T*-stress ratio, T_R, is proposed. For a particular material there appears to be a critical value of T_R, T_{Rc}, below which a crack path is directionally stable.

INTRODUCTION

As is well known, the directional stability of a Mode I crack in an isotropic material under essentially elastic is governed by the *T*-stress. The *T*-stress (symbol *T*) is a stress parallel to the crack. It is also the coefficient of the second term in the series expansion for the crack tip stress field. Values of the *T*-stress are available for a range of configurations (Sherry et al (1)). The first term in the series expansion is a singularity, which dominates the crack tip stress field. Its coefficient in the usual form of the series expansion (Erdogan (2)) is the Mode I stress intensity factor, *K*. Stress field components are proportional to $K/r^{1/2}$, where *r* is the distance from the crack tip. The third and higher terms can usually be neglected. It is assumed here, as is usual in the literature, that the same criteria apply to both static and fatigue loading. In a two dimensional elastic analysis by Cotterell (3) it was shown that if the *T*-stress is compressive (negative) a crack is directionally stable. If the *T*-stress is tensile then a crack is directionally unstable. In practice, it is sometimes found that cracks are directionally stable even if the *T*-stress is tensile. A non dimensional function of the *T*-stress, *B*, which is a measure of the local biaxiality, is widely used (1). It is often called the biaxiality ratio. The term biaxiality ratio is also used for the applied biaxiality ratio (λ in Figure 1). An alternative parameter, the *T*-stress ratio T_R, is proposed. This can

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be used as a measure of crack path stability in a given material. The critical value of T_R, T_{Rc}, at which a crack path becomes unstable is calculated for some materials using experimental data taken from the literature.

A practical difficulty in the analysis of experimental crack path stability results is in deciding whether or not a particular crack path should be regarded as stable (Leevers et al (4)). The British Standard for fatigue crack rate testing (5) states that a crack path is only acceptable if it lies within a corridor defined by planes $0.05W$ (Figures 1-4) on either side of the plane of symmetry containing the crack starter notch root or roots. This critierion was adapted as a crack path stability definition by defining a stable crack as one which remained within the corridor defined in the British Standard. It is easy to apply, but has the disadvantage that it does not take into account changes in T_R as a crack grows.

The *T*-Stress Ratio

In the usual definition of local biaxiality(1) the crack length, a, is used as the characteristic dimension (for an internal crack a is the half crack length) and $B = T(\pi a)^{1/2}/K$. An alternative approach is to consider the direct stress, parallel to the crack, and near the crack tip. That due to the *T*-stress is simply T. The stress, σ_x, due to the stress intensity factor, on the crack line, and ahead of the crack, is (2) $K/(2\pi r)^{1/2}$. The *T*-stress ratio, T_R, may now be defined as the ratio of the *T*-stress to σ_x at some characteristic value of r, r_{ch}. Provided that r_{ch} is small T_R may be regarded as a geometry independent crack tip parameter. Taking $r_{ch} = 0.0159...$ mm leads, with stress in MPa and stress intensity factor in MPa$\sqrt{}$m, to the convenient expression:

$$T_R = 0.01T/K = 0.01B/(\pi a)^{1/2} \tag{1}$$

As an example consider an infinite panel containing a centre crack, length $2a$, with a uniaxial tensile stress, σ, perpendicular to the crack, as in Figure 1. From the solution by Westergaard (6), $K = \sigma(\pi a)^{1/2}$, and $T = -\sigma$. Hence $B = -1$, and:

$$T_R = -0.01/(\pi a)^{1/2} \tag{2}$$

which shows that there is a size effect, a point which is sometimes overlooked; for geometrically similar configurations T_R decreases arithmetically as the size increases. A similar point was made by Finnie and Saith (7) in a different context. Taking a as a typical value of 25 mm, then from Equation (2) $T_R = -0.0357$. For a particular material there should be a critical value of T_R, T_{Rc}, below which a crack path is directionally stable. A rather different approach by Finnie and Snaith (7) led to an equivalent criterion.

A biaxially loaded square plate containing a centre crack, length $2a$, (Figure 1) is sometimes used for crack path studies. Under uniaxial loading ($\lambda = 0$) the local biaxiality is given, by Leevers and Radon (8), as:

$$B = -[1 + 0.085(a/W)] \qquad 0 \leq a/W \leq 0.6 \tag{3}$$

TABLE 1 — Values of T_R for Square Centre Cracked Panel, W = 50 mm.

a/W	$\lambda = -1$	$\lambda = 0$	$\lambda = 1$	$\lambda = 2$	$\lambda = 3$
0	$-\infty$	$-\infty$	0	∞	∞
0.1	- 0.1616	- 0.0805	0.0006	0.0817	0.1628
0.2	- 0.1122	- 0.0574	- 0.0026	0.0522	0.1069
0.3	- 0.0892	- 0.0472	- 0.0053	0.0366	0.0786
0.4	- 0.0750	- 0.0413	- 0.0075	0.0262	0.0599
0.5	- 0.0649	- 0.0372	- 0.0095	0.0183	0.0460
0.6	- 0 0571	- 0.0342	- 0.0113	0.0116	0.0344

and under equal biaxial loading ($\lambda = 1$) by (1):

$$B = 0.029 + 0.03(a/W) - 2.87(a/W)^2 + 4.829(a/W)^3 - 3.125(a/W)^4 \quad 0.1 \le a/W \le 0.6 \qquad (4)$$

so combining Equations (3) and (4):

$$B = -[1 + 0.085(a/W)] + \lambda[1.029 + 0.115(a/W) - 2.87(a/W)^2 + 4.829(a/W)^3 - 3.125(a/W)^4] \quad 0.1 \le a/W \le 0.6 \qquad (5)$$

Values of the local biaxiality were calculated using Equation (5) for λ values of - 1, 0, 1, 2 and 3. For a/W = 0 values were calculated using the Westergaard solution for an infinite plate (6). Values of T_R were then calculated using Equations (1) and (5), taking W as 50 mm, and these values are shown in Table 1. For all values of λ T_R decreases arithmetically as a/W increases.

T_R values were also calculated for the compact tension specimen (Figure 2), the single

TABLE 2 — Values of T_R for Three Specimens.

a/W	Compact Tension Specimen (W = 50 mm, H = 30 mm)	Single Edge Notch bend Specimen (W = 50 mm)	Double Cantilever Beam (a/H = 20, H = 30 mm)
0.1		- 0.0289	
0.2	0.0053	- 0.0130	0.0650
0.3	0.0171	- 0.0033	0.0632
0.4	0.0211	0.0038	0.0657
0.5	0.0204	0.0093	0.0656
0.6	0.0191	0.0140	0.0676
0,7	0.0181	0.0185	0.0666
0.8		0.0237	

edge notch bend specimen (Figure 3), and a double cantilever beam specimen (Figure 4). Values of B used in calculations were taken from References 1 and 8. The results obtained are shown in Table 2. In the compact tension specimen a crack is normally directionally stable. Taking W as a typical value of 50 mm, T_R does not exceed about 0.022. This implies that for many materials T_{Rc} is at least 0.021. In the single edge notch bend specimen a crack is nearly always directionally stable. This is not surprising since T_R values are generally lower than for the compact tension specimen. It is well known (8) that a crack in a double cantilever beam specimen is directionally unstable, and this is confirmed by the high T_R values. As would be expected from St Venant's principle values of T_R are largely independent of a/W.

Some early fatigue crack growth tests carried out by Pook and Holmes (9) on Waspaloy, a nickel based gas turbine material, were re-analysed. The specimens used were biaxially loaded square plates. The results of the re-analysis showed that a crack path is stable for $T_R \leq 0.0122$, and unstable for $T_R \geq 0.0144$, that is T_{Rc} is about 0.013. The results of some similar static tests on PMMA (4) were also re-analysed. These results showed that a crack path is stable for $T_R \leq 0.0122$, and unstable for $T_R \geq 0.0144$, so T_{Rc} is about 0.013. Re-analysis of some fatigue tests carried out by Ramulu and Kobayashi (10) on 7075-T6 aluminium alloy double cantilever beam specimens (a/H = 3.57) showed that T_{Rc} appears to be about 0.041.

Conclusions

Crack path stabilty may be conveniently characterised by the critical value of the T-stress ratio, T_{Rc},

There is a size effect for geometrically similar configurations in that crack path stability increases with specimen size.

Re-analysis of experimental data taken from the literature, for both static and fatigue loadings, gave T_{Rc} values for various materials ranging from 0.013 to 0.041. A general consideration showed that for many materials T_{Rc} is to be at least 0.021.

SYMBOLS USED

a Crack length

B Local biaxiality

K Mode I stress intensity factor

r Distance from crack tip

r_{ch} Characteristic value of r

T T-stress

T_R T-stress ratio

T_{Rc} Critical value of T_R

W Specimen width

λ Applied biaxiality ratio

σ Uniaxial tensile stress

σ_x Stress due to stress intensity factor

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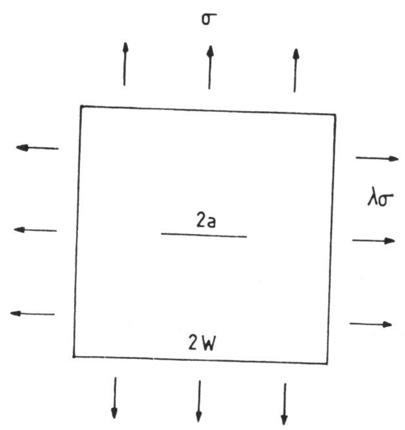

Figure 1 Biaxially loaded square plate

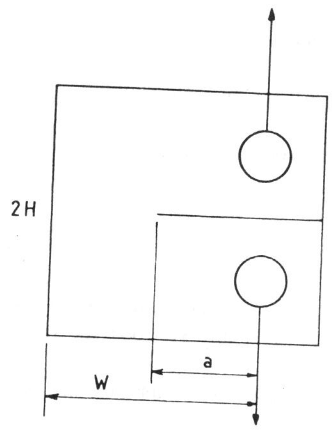

Figure 2 Compact tension specimen

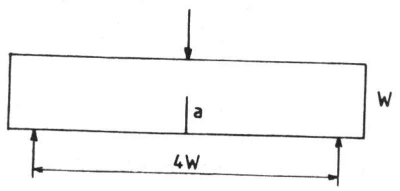

Figure 3 Single edge notch bend specimen

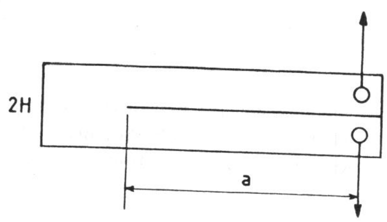

Figure 4 Double cantilever beam specimen