A REVIEW OF HIGH-CYCLE FATIGUE MODELS UNDER NON-PROPORTIONAL LOADINGS

T. Lagoda and E. Macha *

The paper contains a review of multiaxial fatigue criteria under non-proportional loadings. The criteria have been divided into three groups: stress, strain and energy based criteria. It has been stated that fatigue life is influenced by not only individual statistical characteristics of stress/strain state components but by their joint characteristics as well. The latest proposals connected with fatigue life calculations in multiaxial loading tend to including changes of both stresses and strains in the critical plane. They are especially energy criteria.

INTRODUCTION

Non-proportional random or cyclic loadings cause fatigue damages in machine elements. They are usually different than those caused by proportional loadings. This problem is important for design calculations and is subjected to detailed experiments, mainly under cyclic loadings with phase displacement. Łagoda and Macha (1) analysed more than 120 publications concerning non-proportional high- and low-cycle loadings. Here the authors presented only the most important comments and remarks resulting from the review for high-cycle fatigue.

STRESS MODELS

In 1921 Mason and Delnay were the first who tested influence of phase displacements on fatigue life of materials

In 1945 Nishihara and Kawamoto tested influence of phase displacements on fatigue strength of some materials under cyclic bending with torsion. They found that for hard and mild steels and cast iron the fatigue limit increased as the phase displacement increased: in the case of duralumin it did not change. They proposed two models for determination of the equivalent normal stress amplitude, depending on the ratio of fatigue limits under bending

*Technical University of Opole, POLAND

and torsion and no a certain number of coefficients.

Later influence of the phase displacement between bending and torsion on the fatigue strength was analysed by, among others, Little (1969). He proposed a criterion based on the actual amplitude of shear stress, including the phase displacement angle and amplitudes of normal (from bending) and shear (from torsion) stresses.

Numerous papers by Troost and El-Magd (1974÷1986) concern an idea of the critical plane. Their criterion has a form of the square function of quotients of stress and the fatigue limit in normal and shear stresses respectively.

Grubisic and Simburger (1975) tested CK45 steel under biaxial tension-compression, tension-compression with torsion and bending with torsion. For elasto-plastic materials they proposed to determine the equivalent stress (with participation of shear and normal stresses) as a linear function of amplitude of normal and shear stress in the critical plane determined by the shear stress. The efficient stress fatigue coefficient is calculated as a mean-square value in all the possible planes. In this criterion the mean values of shear and normal stresses are taken into account by modifications of the limit contour.

Zenner et al. (1977÷1985) proposed the criterion of shear stress intensity, similar to that proposed by Grubisic and Simburgera, but they did not modify the limit contour. They included the mean value of the normal stress modifying the amplitude of the shear stress. This criterion is well supported by the results obtained by Zenner et al. for 34Cr4 steel and by the tests done by Lempp, Mielke and Issler for different combinations of stresses, frequencies and course shapes (sinus and trapezoid).

McDiarmid (1981÷1991) proposed a criterion being a linear combination of the shear stress amplitude and the maximum amplitude of the normal stress (with weight participation) in the plane of the maximum shear stress.

Other stress criteria proposed by many authors are less supported by the results of experiments. Timszin and Hazanow (1973) tested 30HGSA steel, Sonsino and Grubisic (1985÷1987) - notched specimens made of Fe-1,5%Cu steel, next Sonsino and Pfohl (1990 ÷1991) did the tests under different frequencies of loadings. Lee (1985) tested SM45C, 42CrMoN, 34Cr4V steels. Non-proportional bending with torsion was tested also by Ohloson (1985) - for 4338-02 aluminium, Dang Van (1989) - for materials tested by Nishihara and Kawamoto, Froustey and Lassere (1989÷1992) for 30NCD16 steel applied in aircraft industry and XC18 and 25CD4 materials. The latest papers by Papadopulos and Morel (1996) contain an extended proposal by Dang Van.

Sanetra and Zenner (1991) tested specimens made of 30CrNiMo8 steel subjected to synchronous and asynchronous cyclic loading with constant and variable amplitude under combined bending with torsion. Their results are very intereating and were reanalyzing by Łagoda and Macha.

Thum and Kirmser (1943) done the first tests under tension-compression with torsion. They tested steel notched specimens. Later Löwisch (1993) tested 16MnCr4 steel.

Not many research workers tested biaxial tension-compression. For example, Dietman et al. (1991) tested St35 steel and Sempruch and Szala (1992) analysed the data for 25CrMo4 steel.

Hoffman and Seeger (1991) tested biaxial tension-compression and tension-compression with torsion in St37 steel under stress concentrators. They proposed to calculate the fatigue life including changes of the local strains and stresses.

Macha proposed a general stress criterion including a combination of shear and normal stresses in the critical plane. He assumes that:

1. The fatigue fracture is influenced by a combination of normal, $\sigma_\eta(t)$, and shear, $\tau_{\eta s}(t)$ stresses in direction $\bar{s}$ in the critical plane with normal $\bar{\eta}$.
2. The direction $\bar{s}$ coincides with the mean direction of the maximum shear stress in the critical plane.
3. The maximum value of a linear combination of stresses $\tau_{\eta s}(t)$ and $\sigma_\eta(t)$ under multiaxial random loadings satisfies the following equation

$$\max_t \{ B\, \tau_{\eta s}(t) + K\, \sigma_\eta(t) \} = F \tag{1}$$

The author analysed his criterion in some papers.

STRAIN MODELS

Sanetra and Zenner tried to describe the experimental data for the high-cycle fatigue with use of such parameters as the maximum principal strain, the maximum shear strain and applying Kandil-Brown-Miller (KBM), Lohr-Ellison (LE) parameters and Huber-Mises-Hencky criterion. Also Tipton tried to apply the strain parameters used by Sanetra and ASME and Mowbray parameters. He was not, however, successful.

Brown and Wang (1993) propose a general criterion. For high-cycle fatigue it can be written as

$$\frac{0.5(\Delta\gamma_{max}) + S(\delta\varepsilon_n)}{1 + \upsilon' + (1 - \upsilon')S} = \frac{\sigma_f - 2\sigma_{n,mean}}{E}(2N_f)^b \tag{2}$$

where $\Delta\gamma_{max}$ and $\delta\varepsilon_n$ are the shear strain range and the normal strain excursion between the two turning points of the maximum shear strain, respectively, on the plane of maximum shear strain. The parameter S is a material constant and $\sigma_{n,mean}$ is the mean value of the stress normal to the maximum shear plane.

Macha proposed a general strain criterion including combined shear and normal strains in the critical plane, like in the stress criterion (1)

$$\max_t \{ b\, \gamma_{\eta s}(t) + k\, \varepsilon_\eta(t) \} = f \tag{3}$$

ENERGY MODELS

Andrews and Brown (1989) tested AISI 316 steel using the following criterion proposed by American code (ASME)

$$\overline{E} = \left[\left(\Delta\sigma_{max} \Delta\varepsilon_{max} \right)^2 + \left(\Delta\tau_{max} \Delta\gamma_{max} \right)^2 \right]^{1/2} \tag{4}$$

For A516Gr70 carbon steel, Ellyin et al. (1993) propose an energy criterion based on plastic and elastic energies; for high-cycle fatigue and neglecting plastic strain energy the criterion takes a form

$$\Delta W^{e+} = \chi(\rho) N_f^{\alpha} + c(\rho) \tag{5}$$

where $\chi(\rho)$ i $c(\rho)$ are linear functions of the strain ratio.

Socie, Fatemi and Bennantine (1988-1991) propose to count the cycles and cumulate damage for high-cycle fatigue according to the following rule including changes of both strain and normal stress

$$\frac{\Delta\gamma}{2}\left(1 + \frac{\sigma_{n,max}}{R_m}\right) = \frac{(\tau_f)^2}{G}(2N_f)^b \tag{6}$$

where $\sigma_{n,max}$ is the maximum normal stress on the maximum shear plane.

Lately (1996) Łagoda and Macha (2) have been proposed a new energy approach based on the power density of stress. Power density of stress delivered to the material by rate of specific work of stresses on strain, being a fatigue damage parameter changing at time in a continuous way, makes it possible to reduce multiaxial random stress state to the equivalent uniaxial one.

Let us assume that the fatigue fracture is influenced by that part of power density of stress delivered to the material, $p_\eta(t)$, which corresponds to the rate of specific work of the normal stress $\sigma_\eta(t)$ on the normal strain $\varepsilon_\eta(t)$ and power density $p_{\eta s}(t)$ corresponding to the rate of specific work of the shear stress $\tau_{\eta s}(t)$ on the shear strain $\varepsilon_{\eta s}(t)$ in direction $\overline{s}$ on the plane with normal $\overline{\eta}$.

Then, depending on the assumed position of the fatigue fracture plane and participation of normal and shear power densities, we obtain different forms of the criterion. In its general form it is similar to equations (1) i (2)

$$\max_t \{ \chi\, p_{\eta s}(t) + \lambda\, p_\eta(t) \} = \psi \tag{7}$$

In a particular case

a) when the fatigue fracture is caused by the part of power density of stress delivered to the material, which corresponds to the rate of specific work of the normal stress $\sigma_\eta(t)$ on the normal strain $\varepsilon_\eta(t)$ in the expected fracture plane with normal $\overline{\eta}$, and the expected direction of the fracture plane coincides with the mean direction of the maximum principal strain $\varepsilon_1(t)$ from criterion (7), we obtain

$$p_{eq}(t) = p_\eta(t) = \sigma_\eta(t)\overset{\circ}{\varepsilon}_\eta(t) \tag{8}$$

b) if the fatigue fracture is influenced by that part of power density of stress delivered to the material, which corresponds to the rate of specific work of shear stress $\tau_{\eta s}(t)$ on the shear strain $\varepsilon_{\eta s}(t)$ in direction $\overline{s}$ on the plane with normal $\overline{\eta}$, and direction $\overline{s}$ on the expected fracture plane coincides with the mean direction of maximum shear strain $\varepsilon_{\eta s\,max}(t)$ (the expected fracture plane with normal $\overline{\eta}$ is determined by the mean position of one of two planes of maximum shear strain $\gamma_1(t)$) from criterion (7) we have

$$p_{eq}(t) = 4p_{\eta s}(t) = 4\tau_{\eta s}(t)\overset{\circ}{\varepsilon}_{\eta s}(t) \tag{9}$$

c) when we have a linear combination of normal and shear power densities of stresses in the critical plane, from criterion (7) we obtain a general expression for the equivalent power density course

$$p_{eq}(t) = Ap_\eta(t) + Bp_{\eta s}(t) = A\sigma_\eta(t)\overset{\circ}{\varepsilon}_\eta(t) + B\tau_{\eta s}(t)\overset{\circ}{\varepsilon}_{\eta s}(t) \tag{10}$$

where coefficients A, B have different values, depending on the critical plane selection.
We determine the equivalent power density histories according to equations (8) - (10) and next these histories are integrated in time. Then the energy cycles are counted and fatigue damages are cumulated according to the energy curve described with the equation

$$W_a = \frac{(\sigma_f)^2}{2E}(2N_f)^{2b} \tag{11}$$

CONCLUSIONS

There are three groups of multiaxial fatigue criteria: strain, stress and energy. For high-cycle fatigue the stress criteria are usually applied. The criteria proposed by Grubisic, Zenner, McDiarmid, Troost-El-Magd and Macha are based on the critical plane and they include a change in orientation of principal axis directions at time. Hoffman and Seeger try to join the changes of strain and stress in a given direction in order to determine the fatigue life. A new, interesting energy approach is also proposed by Socie and his co-workers.

From the review of the proposed stress criteria of multiaxial fatigue it appears that the problem of influence of phase displacements between the stress state components on the fatigue life has not been solved yet.

The strain criteria are seldom applied for high-cycle fatigue description and the results obtained with their use are rarly satisfactory.

Andrews et al. and Ellyin et al. propose to apply the energy criteria. None of the known energy criteria of multiaxial fatigue includes random loadings.

The proposed new energy approach based on the power density of stresses introduced into the material by the rate of specific work of stresses on strains in the critical plane seems to be the most proper for the problem solving.

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