THE STRESS INTENSITY FACTOR FOR SMALL CRACKS AT MICRO-NOTCHES UNDER TORSION

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Existing approaches to the torsional fatigue strength of pieces containing defects were based on the stress concentration factor concept. However, the torsional fatigue limit of specimens containing small holes is controlled by the threshold condition of small cracks (6,7). Therefore, the value of τ_w/σ_w for specimens containing small defects must be studied from the point of fracture mechanics.

The scope of this paper is to address the calculation of SIF for a small crack emanating from a 3-D hole under biaxial state of stress by using the weight function method. The obtained results are in good accordance with experimental results on specimens with defects.

INTRODUCTION

Most of the theories about the fatigue strength of a material or a component under torsional loading deal with the definition of a limit stress of the material supposed to be defect-free. From this point of view the typical ratio of torsional -τ_{wo}- to bending - σ_{wo}- fatigue limits of smooth specimens is about 0.58 according to Sines theory (1).

If the cracks or defects inherently exist in the material, the ratio of the torsional fatigue strength to the bending fatigue strength should be different from that of plain (defect free) specimens. This is because the propagation direction of a crack emanating from a defect and its stress intensity factor depend on its size, orientation and shape.

The fatigue strength of a component containing defects or inhomogeneities subjected to axial or bending loads can be predicted based on the models of Murakami and Endo (2), who showed that at the fatigue limit small nonpropagating cracks are always present at the defect tip (2). Because of this fact the inhomogeneities can be treated like cracks and accordingly the fatigue limit – σ_w- has to be calculated as the cyclic threshold stress at which the crack emanating from the defects does not propagate. This type of analysis, which implies to calculate the SIF for a crack with an irregular shape and to model the relationship between ΔK_{th} and flaw size, can be easily done with the $\sqrt{area}$ model by Murakami and Endo (3).

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Existing approaches to the torsional fatigue strength of pieces containing defects were based on the stress concentration factor concept. Goodier (4) and Mitchell (5), respectively analysing spherical cavities and 2-D holes, calculated K_t in bending and in torsion and they predicted a ratio between the fatigue limits - τ_w/σ_w – of about 0.75.

However, Murakami and co-workers (6,7) showed that the torsional fatigue limit of specimens containing small holes is controlled by the threshold condition of small cracks in the direction (± 45° to specimens axis) perpendicular to the maximum principal stress (see Fig. 1). Therefore, the value of τ_w/σ_w for specimens containing small defects must be studied from the point of fracture mechanics. In (6) the experimental results were analysed by using the 2-D exact solutions of Murakami (8), who studied the SIF for a crack emanating from a hole subjected to a biaxial state of stress. Considering cracks with a/R in the range 0.1-0.3, the ratio between SIF under tension and torsion results to be about 0.8.

This approach to the problem of surface defects under torsional and multiaxial loading is extremely simple and it can be easily applied since it reduces the problem to a mode I fatigue strength estimation. However, in spite of the predicted value τ_w/σ_w= 0.8, the experimental results on 3-D artificial notches by Murakami et al. (6,7) and on natural defects by Mitchell (5) and Endo (9) give an average value of 0.85.

The scope of this paper is to address the calculation of SIF for a small crack emanating from a 3-D hole under biaxial state of stress by using the weight function method and the solutions for the stress distribution near holes and hemispherical pits. The final objective of this paper is not to calculate KI but rather to obtain a precise evaluation of the ratio τ_w/σ_w.

SIF FOR SMALL CRACKS AT HOLES AND SPHERICAL CAVITIES

Preliminary calculations

The stress distribution in the neighbourhood of a circular hole in a 2-D plate subjected to any stress pattern can be calculated as the superposition of two stress fields caused by the principal stresses acting on the plate. In particular the stress σ_y acting at point A is the sum of the two stress distributions from Kirsch (10) (Figure 2):

stress caused by s_1 $$\sigma_1 = S1 \cdot f_1(x,R) = S1 \cdot \left[1/2 \cdot (1 + \frac{R^2}{(R+x)^2}) + 1/2 \cdot (1 + 3 \cdot \frac{R^4}{(R+x)^4}) \right] \quad (1)$$

stress caused by s_2 $$\sigma_2 = S2 \cdot f_2(x,R) = S2 \cdot \left[1/2 \cdot (1 + \frac{R^2}{(R+x)^2}) - 1/2 \cdot (1 + 3 \cdot \frac{R^4}{(R+x)^4}) \right] \quad (2)$$

Some preliminary SIF calculations were carried out considering a crack emanating from a hole in a plate subjected to a tensile stress and to a shear stress. The SIF was calculated using the Bruekner WF (11) for an edge crack in a semi-infinite plate and the stress distribution $\sigma_Z = \sigma 1 + \sigma 2$ (tension: S2=0; shear S2=-S1). The outcomes of these calculations were then compared with the solutions of Murakami (8).

The result, in accordance to Schjive analyses (12), was that the approximate solutions had a precision of 5% for $a/R<0.2$. However, the ratio KI_{tens}/KI_{shear} is almost exact up to a/R= 0.2 and the maximum error is less than 5% in the range 0.2<a/R< 1 (see Figure 3). It has to be remarked that KI_{tens}/KI_{shear} tends to 1 because the effect of σ_2 decreases as the crack grows.

3-D Corner cracks

Because of the good outcomes of 2-D analysis, the straight edge approximation was also used for calculating the SIF for small cracks at cylindrical holes and hemispherical surface pits under tension and torsion. Figure 4 shows the shape of the corner crack.

We approximate the stress distribution in the neighbourhood of 3-D defects by the following equation:

$$\sigma(x,R) = \frac{C_1}{3} \cdot S1 \cdot f_1(x,R) - C_2 \cdot S2 \cdot f_2(x,R) \tag{3}$$

whose coefficients C_1 and C_2 are the stress concentration factors on the edge of the 3-D cavity caused by the remote stress fields S1 and S2. This equation is compared in Figure 5 with the solutions of the stress distributions around cylindrical holes and hemispherical surface pits by Murakami et al. (13) and by Noguchi et al. (14, 15). These three curves are in good agreement.

The calculation of KI was then carried by using the Weight Function for a corner crack by Shiratori et al. (13) who gave a SIF solution:

$$K_I = \sum_{n=0}^{3} A_n \cdot K_n \tag{4}$$

the stress distribution being described by a polynomial of the type (η=1-x/a):

$$\sigma = \sum_{n=0}^{3} A_n \cdot \eta^n \cdot \sigma_o \tag{5}$$

The application of Shiratori's WF was firstly checked on the calculation of SIF for a small corner crack at a hole in a finite plate. The comparison of the results with Newman's solutions (16) gave a precision similar to the previous 2-D analyses.

The whole procedure then allowed us to calculate KI at the tips of small cracks at cylindrical holes and hemispherical pit under tension and shear. The resulting KI_{tens}/KI_{shear} data for semicircular cracks (a/b=1) are shown in the Figures 6 and 7.

CONCLUDING REMARKS

The obtained results clearly show that the typical ratios of KI_{tens}/KI_{shear} fall in the range 0.83-0.87, with an average value of 0.85.

The experimental results obtained on defects are summarised in Figure 8, where the ratio between fatigue limit under torsion and bending - τ_w/σ_w - is plotted against S_u. It is worth remarking that the data of Murakami et al. were obtained with drilled holes (d/h=1), those of Mitchell with natural defects and those of Endo pertain to nodular cast iron.

The data in general fall within the predicted range, except the one of (7) which is a medium carbon steel having a strong texture in the rolled direction, i.e. in the direction of specimen axis.

The good agreement between the present theoretical KI_{tens}/KI_{shear} ratio and the experimental results confirms the fact that the fatigue limit of components containing defects can be accurately predicted by treating the defects as cracks. The method of analysis here proposed can be easily applied to any combination of biaxial stresses in order to precisely estimate the fatigue limit of surface and sub-surface defects.

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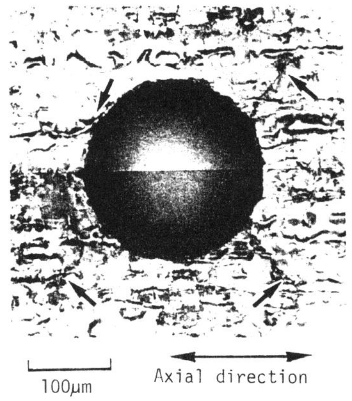

Figure 1 – Nonpropagating cracks from a micro-notch at torsional fatigue limit (7)

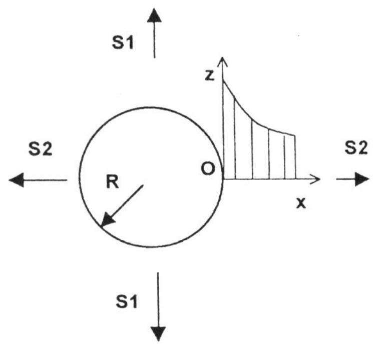

Figure 2 – Circular hole in a 2-D plate subjected to biaxial loading

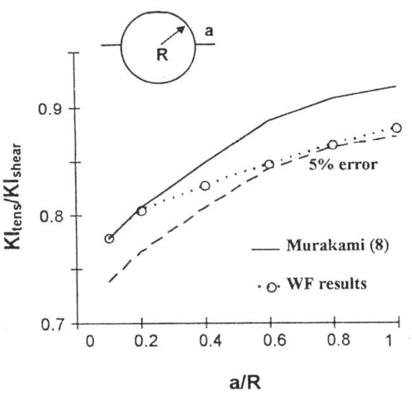

Figure 3 – KI_{tens}/KI_{shear} for a small crack at a hole in a 2-D plate

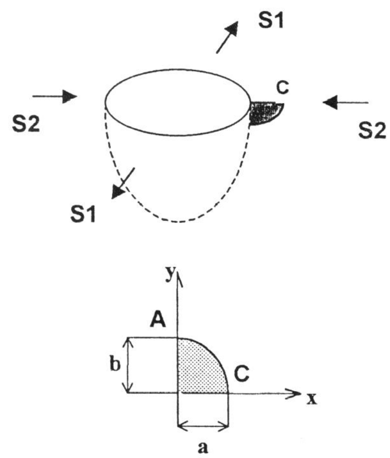

Figure 4 – Geometry of the small cracks examined

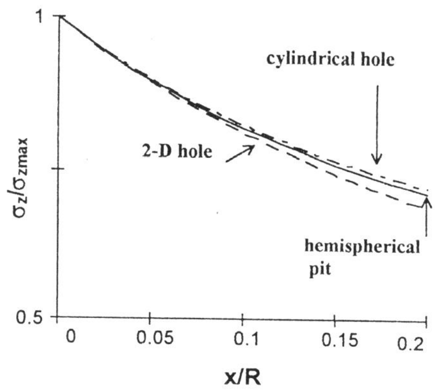

Figure 5 – Stress distribution around 3-D notches under tensile stress

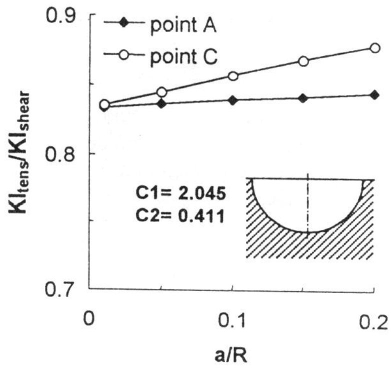

Figure 6 - KI_{tens}/KI_{shear} for a small crack (b/a=1) at a hemispherical pit.

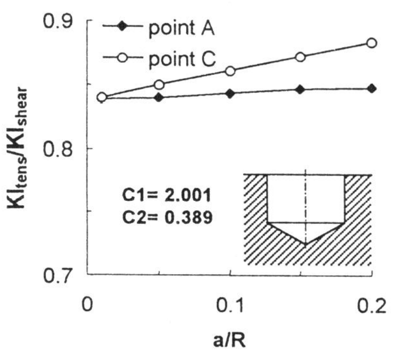

Figure 7 - KI_{tens}/KI_{shear} for a small crack (b/a=1) at a cylindrical hole.

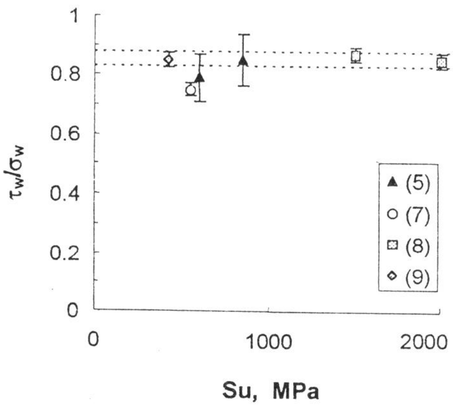

Figure 8 – Predicted τ_w/σ_w range against experimental results.