EXPERIMENTAL STATISTICAL ANALYSIS OF SCATTER IN FATIGUE CRACK GROWTH LIFE UNDER STATIONARY RANDOM LOADING

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Three series of fatigue crack growth tests under random loading were carried out. In each series, different loading histories were obtained from the same random process. The bandwidth of the three random processes studied was different. The fatigue lives obtained in each series were analysed statistically and compared with those for the other series and with the simulated results provided by a strip yield fatigue crack growth model. The influence of the bandwidth of the random process on the statistical distribution of crack growth life and the accuracy of the predictions of the strip yield fatigue crack growth simulation model are analysed.

INTRODUCTION

Reliable predictions of the duration of fatigue crack growth processes under random loading is hindered by a number of variables many of which are also random in nature. As a result, crack growth lives can only be statistically approximated. Prominent among the random variables that affect fatigue life statistics are the initial crack length (Palmberg et al. (1), Jouris and Shaffer (2)), the parameters that define crack growth law (Virkler et al. (3)), the growth threshold (K_{th}) and load variation (Ten Have (4), Schütz (5)).

The loads borne by a variety of real-life systems such as off-shore structures, aircraft, wind turbines and motor cars can only be determined statistically. Characterising the loads that act on a given system requires the knowledge of the different loading regimes the system will sustain, and their frequency and order of appearance. In addition, because loads vary randomly in each regime, one must also know the statistical features of load variations. These data are the base to define a load history that is representative of the loads the system will support during its life, which will usually be one of finite length.

Simulated and experimental fatigue life analyses of elements involve the iterative application of the representative load history until failure occurs. However representative,

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the load history will never be more than a sample of the whole population of elements with identical statistical properties. Using a different load history will lead to an also different fatigue life, the magnitude of the difference depending on the characteristics of the material concerned, the statistical properties of the load variation and the history length, which is defined by the number of cycles, N_h. Some fatigue life analyses are addressed by using standardised load histories (Potter and Watanabe (6)). The number of cycles used varies appreciably among histories depending on the expected service life of the element in question, the type of load it is to bear and even the year the load history was defined.

Iterative application to failure of a finite-length history can lead to error for two main reasons. First, it introduces a sequence effect that may depend on the length of the load history; as a result, a different representative load history will give an also different result. Thus, the fatigue lives obtained with one load history on constancy of all other parameters potentially influencing the result will be nothing but one of the many possible life values. In order to ensure reliable fatigue life estimations under real loading from simulations or tests involving definite random histories one must know the potential dependence of the results on the particular representative history used. To this end, one must determine where the actual results can be rather different from those predicted by a specific load history or whether the actual scatter will be insignificant.

In previous work, Domínguez and Zapatero (7) used numerical simulations to analyse the influence of bandwidth and length of the load history on fatigue crack growth life under random loading. This paper reports the results of an experimental analysis of these effects that was aimed at determining the accuracy of simulation methods for predicting the statistics of fatigue crack growth life under random loading. A description of the tests performed and the results obtained (as well as some statistics) is followed by a discussion of the results and some interesting conclusions drawn from them.

TESTS AND RESULTS

Tests were conducted in four series (S_1 to S_4) using load histories obtained from three Gaussian stationary random processes of zero mean and with a different bandwidth each. The random loading processes studied were defined in terms of their power spectral density function (*psd*). Bandwidths were characterised by using the irregularity factor, ε, defined as the ratio of the frequency of mean up-crossing to the peak frequency. The factor varied from 0 to 1: the closer to unity, the more narrow the band and *vice versa*. The ε values used in the four series were 0.64 (S_1), 0.77 (S_2 and S_3) and 0.85 (S_4).

An overall 20 different load histories consisting of 25000 cycles each (N_h=25000) that were obtained from the three random processes studied were numerically simulated for series S_1, S_2 and S_4. For series S_3, 30 histories with N_h=5000 cycles were simulated from the same *psd* than series S_2. The forms and values of the *psd* used were identical with those corresponding to the same ε value employed in previous work (7). After load histories were numerically obtained, they were increased by a positive mean load (μ = 4850 N) in order to avoid compression loads. The standard deviation for the set of histories was σ = 1080 N. Each of the 90 load histories obtained was applied to a different specimen and crack length was monitored throughout the crack growth. The tested material was 2024-T351 aluminium alloy. Specimens were of the Compact Tension (CT) type, 50 mm wide and 12 mm thick. All specimens were pre-cracked under cyclic loads of the same amplitude to a length of 15 mm. Then, each specimen was subjected to a different load history to a length of 25.3 mm was analysed. The load history used in each test was repeated an indefinite number of times until the final crack length was reached.

Figure 1 shows the crack growth life obtained in each test. The values corresponding to each of the four series are joined by a line. The figure also shows the mean number of cycles required for failure to be reached, μ_{life}, in each series (horizontal straight lines). Figure 2 shows the coefficient of variation for fatigue life in each series ($COV_{life} = \sigma_{life}/\mu_{life}$). For comparison, it also shows the numerical values obtained by using the strip yield fatigue crack growth model (Newman (8)). The *x*-axis represents the irregularity factor values for the load histories in each series. Table I gives the statistical parameters for the four test series, as well as the mean values for the loading peaks (μ_P) and the ranks established by using a rain-flow counting procedure (μ_R).

DISCUSSION

Figure 1 shows the variation of fatigue life in each test of the four series. As can be seen, the mean fatigue life and its scatter in each test group varied with the bandwidth of the random process from which histories were produced. A comparison of the results for series S_2 and S_3 reveals that the mean fatigue life for the two series was virtually identical. Consequently, history length had no appreciable influence on the resulting fatigue life at least in this case. In previous work (7), using other bandwidths and N_h to obtain simulated fatigue lives, the mean fatigue life was also scarcely affected by history length.

On the other hand, bandwidth is markedly influential on fatigue life: crack growth duration increases strongly with increasing bandwidth (see Fig. 1). One plausible explanation for this finding is that, among load histories with the same standard deviation, those with the greatest bandwidths have the smallest mean value of peaks and loading rages (see Table I).

Scatter in the results for each test series was determined via COV_{life}. As can be inferred by comparing the COV_{life} values for series S_2 and S_3 in Fig. 2, N_h has a marked influence on scatter. The authors obtained similar results with other values of N_h (7). The effect of bandwidth on COV_{life} is very weak. Although COV_{life} decreases when the irregularity factor increases from $\varepsilon = 0.64$ to $\varepsilon = 0.77$, the change is only moderate. It increases from $\varepsilon = 0.77$ to $\varepsilon = 0.85$, but even less markedly. A similar result was previously obtained by simulation, using the strip yield fatigue crack growth model (8) with a constraint factor $\alpha = 1.5$. Simulated series analysed by the authors elsewhere (7) showed that changes in COV_{life} with decreasing bandwidth are appreciable for short load histories but not for long ones.

TABLE I - Statistical parameters for the data sets studied.

Series	ε	N_h (cycles)	μ_P (N)	μ_R (N)	μ_{life}	σ_{life}	COV_{life}
S1	0.64	25000	5718.5	1725.2	277166	13726.8	0.0495
S2	0.77	25000	5894.5	2077.4	168299	5280.1	0.0314
S3	0.77	5000	5895.8	2075.7	169978	15533.4	0.0914
S4	0.85	25000	6006.6	2300.4	146972	5502.4	0.0374

Indefinitely repeating finite-length histories until failure occurs introduces a sequence effect that is absent from the actual stationary process. The highest peaks in each history are sequentially repeated, so they may influence crack growth. The effect obviously increases with decreasing number of cycles in the history. Consequently, the influence of the extreme value in each load history on the resulting fatigue life was also analysed. In order to examine the influence of the extreme value in each history on fatigue life, the correlation between the two parameters was determined. The correlation coefficient was found to be $\rho = 0.96$ for series S_3 and much smaller ($\rho = 0.52$) for S_2, where histories were 25000 cycles long (see Fig. 3). The effect of bandwidth on the correlation between the extreme value of each history and the fatigue life it yields is unclear. A comparison of the ρ values for series S_1, S_2 and S_4 reveals a high correlation ($\rho = 0.71$) for the process with the smallest bandwidth (S_4) that decreases to $\rho = 0.52$ for S_2 which used a somewhat greater bandwidth and increases to $\rho = 0.65$ for the process with the greatest bandwidth (S_1). Because S_1 contains a large number of cycles of small amplitude and an also large number of peaks with values below the closure stress, the number of peaks that influence the closure stress is relatively small. As a result, the extreme values in the load histories will be more influential which will be reflected in increased correlation coefficients.

Other issues examined in this work were the type of fatigue life statistical distribution prevailing in each test series and the influence of bandwidth and load history length on it. To this end, the growth lives obtained in the tests were fitted to various distribution functions, *viz.* log—normal distribution, and Gumbel, Frechet, and Weibull distributions for minima. The goodness of fit was determined by Kolmogorov's test, the results of which are given in Table II. As can be seen, the fitting differences among the Frechet, Gumbel and log—normal distributions were very small, with no clear trend allowing any such function to be adopted as representative of the fatigue life statistical distribution. On the other hand, the Weibull distribution provided the poorest fit to the data, those for series S_2 excepted. In any case, 20 data is too small a sample size to allow precise fitting of a probability density function. The same study was repeated at different crack lengths throughout the crack growth process and similar results were obtained. Figure 4 shows the distribution for the series involving the greatest bandwidth (S_1) on log—normal probabilistic paper. As can be seen, the fit was very good.

CONCLUSIONS

This paper reports new data on fatigue crack growth under stationary random loading in both narrow- and broad-band processes. It contributes new knowledge for determining the effect of using different representative load histories for a given process on fatigue life scatter. In the light of the above-discussed results, the following conclusions can be drawn:

(a) The length of the load history used is highly influential on the results of crack growth tests under random loading. For a given process, using a shorter load history substantially increases scatter in crack growth life. This can lead to predicted life values differing markedly from actual mean lives if a single, random history is used in the fatigue test.

(b) On the other hand, the mean fatigue life is scarcely affected by the length of the load history used.

TABLE II - Results of Kolmogorov's fitting test for the fatigue life distributions of the four test series.

Series	Log-normal	Frechet	Gumbell	Weibull
S1	0.0880	0.0840	0.0806	0.1321
S2	0.1085	0.1582	0.1525	0.0730
S3	0.1192	0.0916	0.1099	0.1747
S4	0.0963	0.1049	0.0990	0.1466

(c) The influence of the load history bandwidth on fatigue life scatter is only significant in broad-band processes.

(d) The effect of the extreme value in each history depends markedly on N_h. Such an effect decreases strongly with increasing N_h, at least in the cases studied here.

(e) The statistical distribution of the fatigue lives obtained by using different histories representing the same process can be fitted to various distribution functions of which the log—normal one provides slightly better results than the rest.

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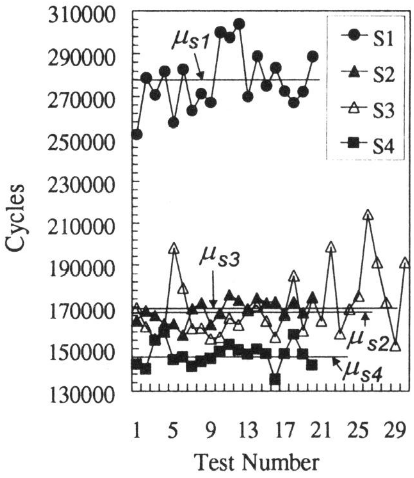

Figure 1 Crack growth lives in the four series of tests.

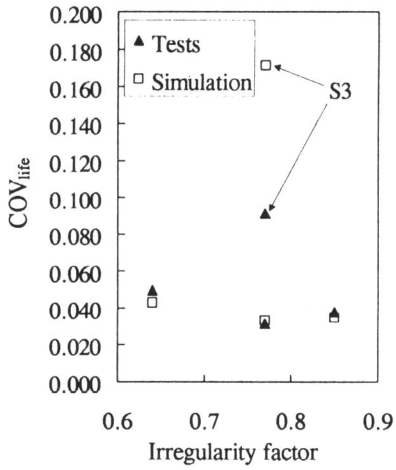

Figure 2 COV_{life} in each series (Tests and Simulation).

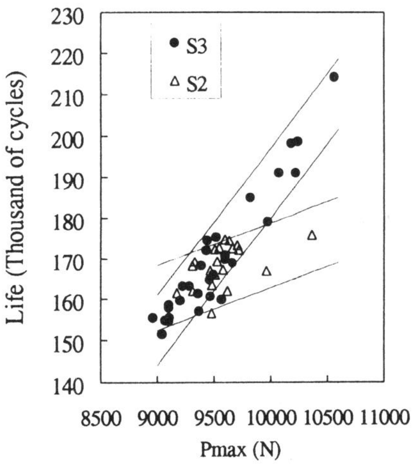

Figure 3 Correlation coefficient (ρ) between extreme value and fatigue life.

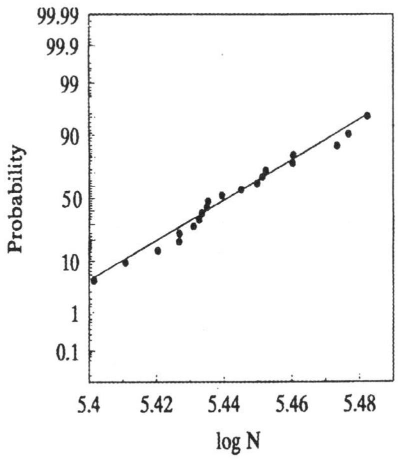

Figure 4 Distribution of life for series S1 on log-normal probabilistic paper.