Method for Estimation of Crack Path Stability

D.A. Zacharopoulos

School of Engineering
Democritus University of Thrace
GR 671 00, Xanthi, Greece
dzachar@civil.duth.gr

ABSTRACT. *Prediction of crack growth path is a prerequisite for estimating the final shape of broken solids and structures. Crack path in broken specimens provides information for the loading conditions just before fracture. Experiments on brittle materials, pre-cracked specimens of the same geometry under similar loading conditions, however, may yield different crack trajectories at times. The existing theories for the prediction of the crack path are based on the perturbation method combining the analytical and finite elements methods. They require a knowledge of the toughness equations. Moreover, they can only be applied to specimens with simple geometry and loadings. A different approach is used in the present work. The finite element technique is used to calculate the strain energy density (SED) contours. The predicted trajectory of the crack during unstable propagation is assumed to coincide with the minimum of the strain energy density function according to the SED criterion. The degree of crack path stability depends on the sharpness of the SED oscillations. This simple method offers a reliable prediction of the crack path stability for two as well as three-dimensional problems with complex geometry structures and arbitrary loadings. To be specific, both the TPB and DCB specimens are analyzed. The findings are in good agreement with the theoretical and experimental results in the literature.*

INTRODUCTION

Instability related to a crack deviating from a straight path has been studied in [1,2]. A criterion for curved was also introduced. A curved crack path is said to be unstable if an infinitesimal deviation from the original path would lead to a different crack path. In what follows, the distinction of 'stable' and 'unstable' crack path will be made.

- ***A stable crack path prevails when the broken pieces of the fractured specimen by catastrophic destruction can be reproduced for similar tests under identical conditions.***

When the results are very sensitive to the prevailingly conditions, scattering of the results is likely. This could involve the highly constrained boundary conditions and/or the boundary conditions. Crack trajectory can depend on the material properties, the specimen geometry, the rate of loading and temperature. Stiffness of the machine also plays a role. These factors can be used to check the suitability of selecting criteria for determining crack trajectory. Dynamic crack propagation and bifurcation phenomena were investigated by utilizing the strain energy density theory together with the catastrophe theory in [3]. While crack path instability has been studied by others, they do not delve into the details connected with the shape of the crack path and instability. Most of the cases dealt with simple crack geometries and loading conditions. The practice is to consider a straight path crack being exclusively stable. Instability refers only to the situation when a crack starts to curve. There are situations where a propagating curve crack may be stable and a straight crack may be unstable. These situations will be elaborated further in this work.

Crack path instability will be studied in this work without confining to special cases such that the results are specimen and/or loading speific. To this end, the strain energy density theory [4-9] will be used. It applies to any material type, specimen configuration, loading types and dimensions. A computer finite element program is devised for plotting the strain energy density contours according to the crack geometry before the occurrence of unstable crack propagation.. The would-be crack trajectory would be mapped out by the minimum strain energy density contours as if the process of crack propagation took place instantly. This assumption has been found to yield excellent results [4] revealing minute details of crack path wiggling as it initiates from a smooth elliptical boundary. This is precisely what the experiments show. Furthermore, the theory yields information on crack path instability for very complex structures and loadings. Hence, the instability conditions of a mode I propagating crack may be stated as

- ***The path of crack propagation is considered stable and follows a straight or curved path. Bifurcation or crack forking (two branches) is also included as being an unstable behavior.***
- ***When the crack path is unstable, it can follow many curved path including the straight line.***

STABILITY OR INSTABILITY OF CRACK PATH

The SED theory predicts failure by fracture and/or yielding. It is based on the following hypotheses:

- ***Fracture initiates at the location where the $[(dW/dV)^{max}_{min}]_L$ appears when it reaches the critical value $(dW/dV)_c$.***
- ***Yielding initiates at the location where the $[(dW/dV)^{max}_{max}]_L$ appears when it reaches the critical value $(dW/dV)_d$.***

The dW/dV, represents the strain energy density. To facilitate the determination of the crack trajectory, a finite element computer routine is established to compute and plot the strain energy density contours. This provides a visual identification of the stationary values of the strain energy density function with a topographic map. With experience, the locations of the would-be crack path can be spotted very quickly. This will be demonstrated in the work to follow where stable or unstable crack path could be found.

Let O coincides with the point of failure initiation. It could be the crack tip. The hypotheses for the strain energy density theory will be used to determine the initial direction of crack growth as well as the crack path of propagation.

- ***Assume that fracture initiates from O along the direction OL by setting (OL)=r_c when $[(dW/dV)^{max}{}_{min}]_L$ reaches the critical value $(dW/dV)_c$. The predicted crack path during the unstable propagation corresponds to the loci of the points that start from L and connects all those having with the maximum gradient of (dW/dV). The end point is G, where the global minimum value of (dW/dV) prevails.***

On the map, the crack path is indicated by V shaped contours of the strain energy density. If the apices of the V_s points are joined by a line, then the resulting curve starts from the peak O and ends in vicinity of the point G. This curve is the trajectory of the propagated crack. Referring to the geographic map the curve, OG represents a gorge of a riverbed that starts from the hilltop O. The mapping of this gorge gives additional information for estimating the crack path stability according to the sharpness of the curve. It can be said that:

- ***The stability of the crack path can be deduced from the degree of the sharpness of the curve that describes the "gorge".***

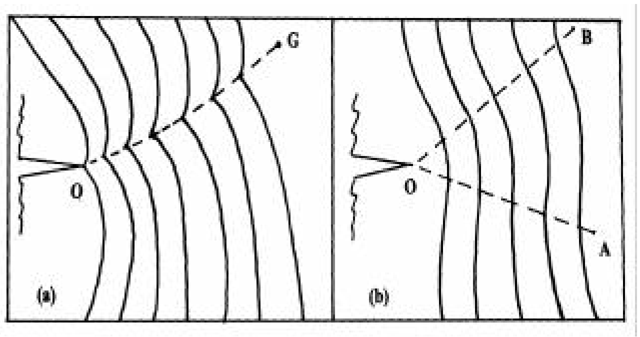

Figure 1. Maps of contours (dW/dV): a) Crack path stability. b) Crack path instability.

Sharpness may be defined by referring to Figure 1. It shows two possible patterns for the contours of the strain energy density around at the crack tip 0. Figure 1(a) shows a map of contours of strain energy density where the gorge can be distinguished very clearly. This would correspond to a stable crack path. In the Figure 1(b), the gorge cannot be distinguish clearly. As an analogy, the former case could be referenced as a small riverbed while the latter as a large riverbed. It is possible that for the case in Fig. 2(b) the crack path could follow a path enclosed by the angle AOB.

DISCUSSION OF RESULTS

To illustrate how the strain energy density theory can be applied to determine a stable or unstable crack path, two specific examples will be considered. They will be TPB and DCB test specimens. The results will be compared with the available experimental observations.

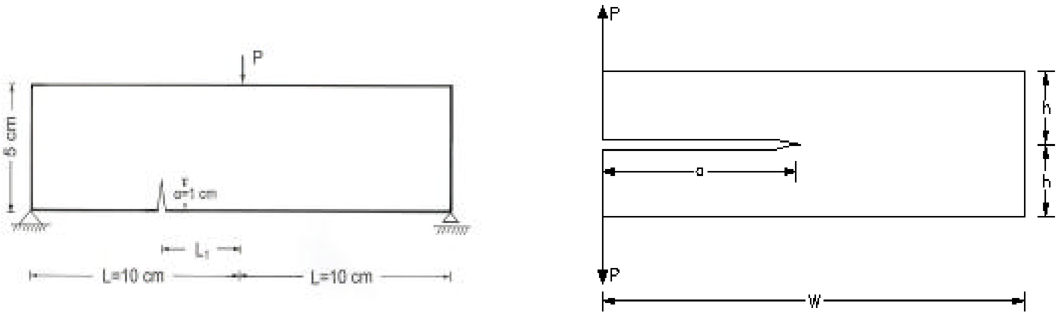

Figure 2. Geometry and loading configuration of the TPB-type and DCB-type specimens.

Three point bending specimen

Consider a cracked specimen shown in Fig. 2. It has a span 2L=20cm, a width 5cm and a thickness 1cm. The crack of length á=1cm is located at a distance L_1 from the midspan of the specimen. Three point bending will be considered with a concentrated load P. The mechanical constants are: E=3.4 GPa and ν=0.35 and $K_{Ic}=6.1\text{x}10^2$ $kNm^{-3/2}$

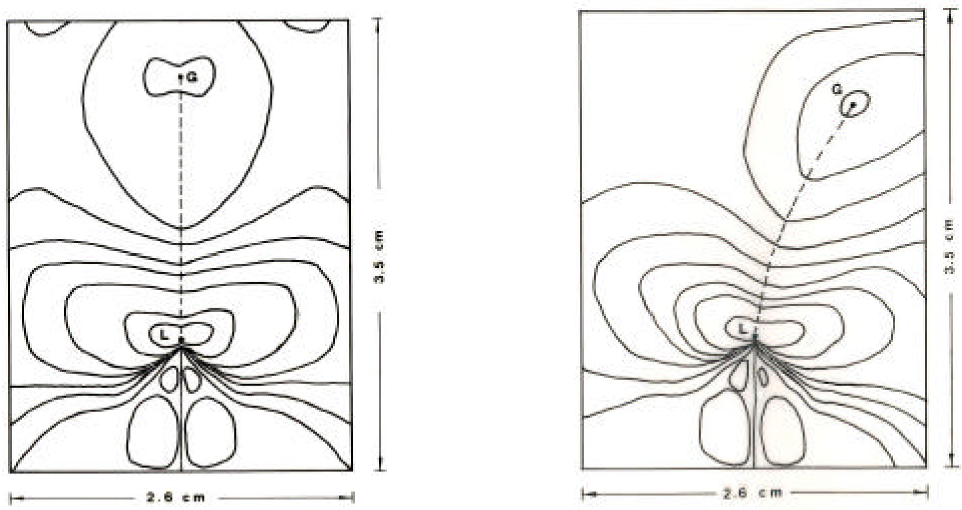

Figure 3. Contours maps of (dW/dV) and crack path in TPB-type specimen for L_1=0 and L_1=7cm.

Twelve–node isoparametric finite elements are used. The finite element program calculates the stress intensity factors with high precision. A graphical computer program is developed for finding the contours map of the strain energy density function.

For the TPB specimen, the crack is located at different distances from the midspan of the beam. Different mixed mode loading conditions can thus be simulated. An important characteristic of this specimen is that the crack tip stress field is tensile and it becomes compressive as the distance from the crack is increased. The results shown in Figure 3 inclusive for L_1=0 cm and 7 cm. The predicted crack path LG is identified on the SED contours. The predicted crack trajectory for all crack positions is stable.

Experiments have also been performed for in three-point bending test specimens made from Plexiglas with the dimensions mentioned above. The specimens were tested with a machine which controlled the increase of the applied load. Slow rate loading is also considered [8]. It was observed that the experimental results are very close to the theoretical predictions for critical loads and angles of initial crack extension. The experimental fracture trajectories were close to theoretical predictions in the neighbourhood of the crack tip. The deviations increased with the distance from the crack tip. The reinitiating of the crack became suddenly after the crack has been arrested near G which is the global minimum of the strain energy density in the contour map. Furthermore, the fractured pieces of the tested specimens for each case were identical and they are repeatable. The present study takes exceptions to the open literature result that the crack path tends to become unstable when the ratio ì =K_{II}/K_I increases. A possible explanation could be attributed to the fact that the strain energy theory accounts for both the influence of local and global effects on crack propagation while the conventional approaches only accounts for the local effect by using only the asymptotic stress expressions.

Double cantilever beam specimen

The DCB test specimen is shown in Fig. 2. The dimensions of the specimen which remain constant are the length W=15cm and the thickness B=1 cm while the crack length a and height h can vary. Note that the Young's modulus is E=3.0 GPa and the Poisson ratio is ν=0.33.

Approximately 80 elements are used. Highly refined of elements are used around the crack tip. A solution for the DCB specimen was given in [10-11]. Figure 4 plots ó/(6P/BW) against (a/W) by using ó/(6P/BW)=(a/W)/$(h/W)^2$. In Figure 4, the dotted curve corresponds to the work in [11] which refers it to unstable crack propagation because the trajectory is curved. The solid curve is obtained in the present work; it differs from that in [11] significantly. Two regions can be identified : one to the left and another to the right. If the combinations of ó/(6P/BW) and (a/w) are such that the data fall to the left region the crack trajectory would be regarded as unstable and the path would be curved. If the data fall to the right region, the crack trajectory would be stable and the path is straight. Regions that are further to the left of the solid curve will always yield curved crack path. This regarded as stable in the present work.

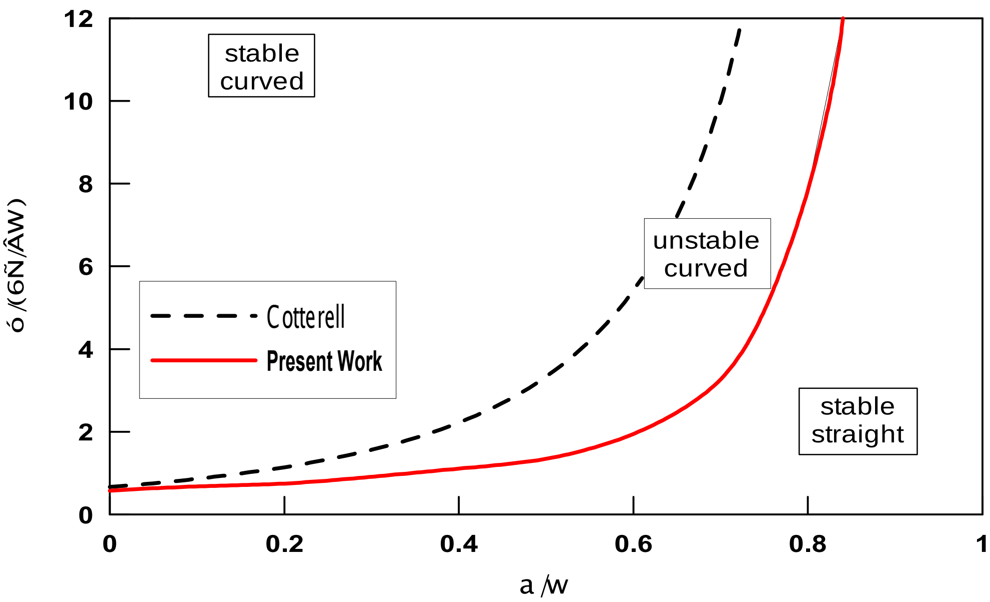

Figure 4. Classification map of crack path (in)stability for TPB-type specimen.

Figure 5 shows the SED contours for the DCB specimen having W=15cm, h=1.5cm and a=1.5cm. This gives a/W=0.1 and h/W=0.1. The path OG corresponds to the stable curved crack trajectory that falls to the region further to the left in Fig. 4. The case of straight and stable crack is predicted in Figure 6 where a/W= 0.80 and h/W= 0.53. This falls into the right region in Figure 4.

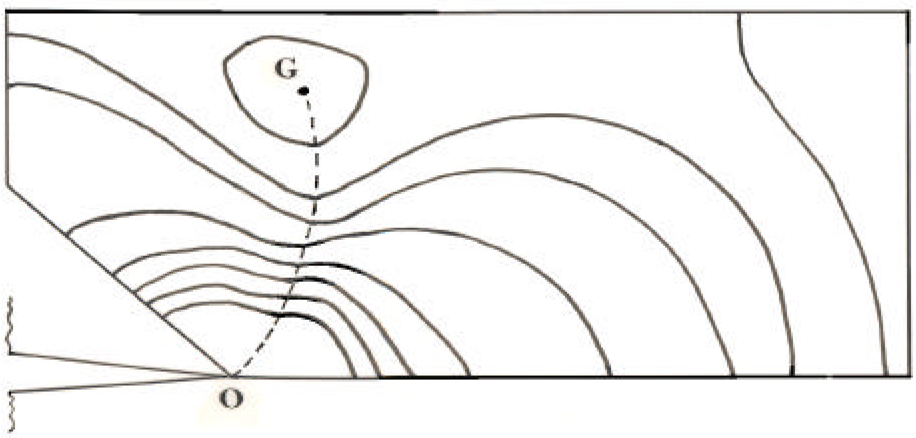

Figure 5. Contours map of (dW/dV) and crack path OG in DCB-type specimen with a/W=0.1 and h/W=0.1.

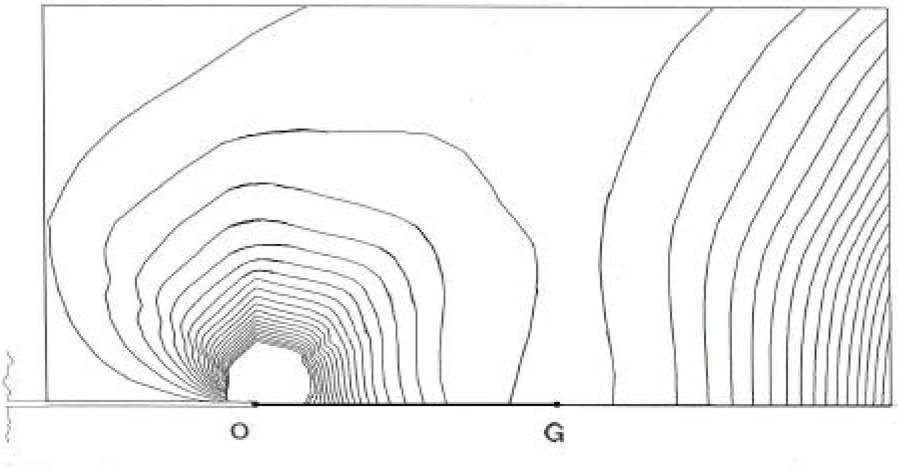

Figure 6. Contours map of (dW/dV) and crack path OG in DCB-type specimen with a/W=0.80 and h/W=0.53.

Finally, the SED contours for the DCB specimen with a/W= 0.4 and h/W=0.3 is presented in Fig. 7. Note that the contours do not yield distinct stationary values for the SED function. This means that the crack path would be unstable. The trajectory of fracture cannot be predicted with certainty

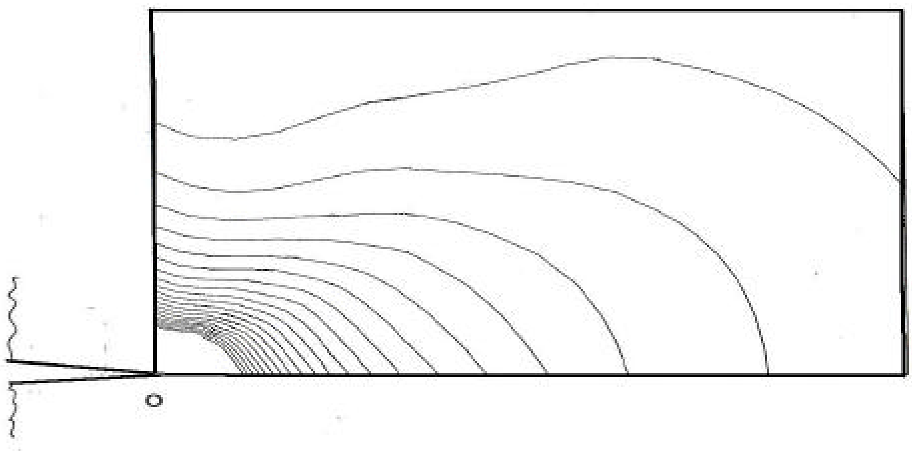

Figure 7. Contours map of (dW/dV) and fuzzy predicted crack path in DCB-type specimen with a/W=0.4 and h/W=0.3.

CONCLUSIONS AND PERSPECTIVES

Method that is based on the strain energy density theory has been used to predict the conditions under which the propagating would follow a straight or curved path even though the load is applied symmetrically with respect to the original crack plane. The stability aspects of the predicted crack path are also discussed with reference to the reproducibility of the results. It is not identified strictly with the obtainment of a straight or curved crack path. A more general view is adopted. It is concerned with the sharpness of the SED contours where the stationary values of the SED function are clearly observable while in other cases the stationary value of SED may be obscured owing to the combinations of the load and geometric factors that fall into the border line region where the slightest deviation of the parameters would yield a different crack path. It might be that in collaboration with other methods it could play an important role in the solution to the problems of crack path stability. Further investigation is needed to examine the region where the crack path stability governing parameters are sensitive. This may suggest a shift in scaling of the initial defect size. This is well known for instability problems of thin shells and fatigue crack propagation. The application of the method in interface problems, between different materials with respect to their fracture toughness, is already on its way. In the future we will try with further inquiring work to locate the factors that can be controlled by the proposed method and we will exhaust its limits.

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