

# Surface Crack Subject to Mixed Mode Loading: Numerical Simulations and Experimental Tests

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**ABSTRACT.** *Experimental observations of three-dimensional fatigue crack growth are compared to numerical predictions from the Boundary Element code BEASY. Fracture analysis results for a complex geometry specimens are presented with regard to 3D crack propagation and mode coupling effects. The specimens under consideration are cracked tension specimens, with a through crack or a quarter circular corner crack that proceed inclined to the remote loading direction. Mixed mode conditions along the crack edge are characterised: the stress intensity factors are determined using the crack opening displacement method and the crack growth direction is computed by the minimum strain energy density criterion. Effects of specimen complex geometry, caused by a part-through hole nearby the initial three-dimensional crack, on fracture crack path are analysed. A satisfactory agreement between numerical and experimental crack growth direction and crack growth rates are displayed, both for three-dimensional part-through corner cracks and through the thickness elliptical cracks.*

## INTRODUCTION

Damage Tolerance is used in the design of many types of structures, such as bridges, military ships, commercial aircraft, space vehicle and merchant ships. Damage tolerant design requires accurate prediction of fatigue crack growth under service conditions and typically this is accomplished with the aid of a numerical code. Many aspects of fracture mechanics are more complicated in practice than in two-dimensional laboratory tests, textbook examples, or overly simplified computer programs. Load spectrum, threshold effects, environmental conditions, microstructural effects, small crack effects, Multiple Site Damage (MSD) conditions, material parameters scatter, mixed loading conditions (material flaws or pre-cracks, which may have been introduced unintentionally during the manufacturing process, can have an arbitrary orientation with respect to the loading applied to the component) and complex three dimensional geometry, all complicate the process of predicting fatigue crack growth in real word applications. This paper focuses on one of these complications (see also [1-4]): complex three dimensional crack path assessment, under mixed mode loading.

In particular, a series of laboratory tests have been designed and implemented to evaluate three dimensional crack path prediction capabilities for a commercial code (BEASY), based on Dual Boundary Element Method (DBEM) [5-9]. With such code

the geometry of the test specimen and the shapes of evolving crack fronts, are not restricted to the simplified configurations found in the libraries of many commercial codes and the three dimensional crack propagation proceed automatically, with repeated remeshing at each crack growth step.

## NUMERICAL SIMULATION AND EXPERIMENTAL TEST

Experimental fatigue tests are performed on notched plates undergoing cyclic axial load. The crack initiation process and the crack propagation are monitored and a general traction load spectrum is applied to the specimen. Experimental crack paths are compared with those obtained with a numerical procedure based on DBEM (Dual Boundary Element Method) as implemented in the commercial code BEASY.

### *Three-Dimensional Corner Crack Propagation under Mixed Mode Loading*

A variable amplitude fatigue traction load (Table 1) is applied by a servo-hydraulic machine (Instron 8502), with a frequency  $f=10$  Hz, at ambient temperature, on an aluminium plate specimen (370x70x5, clamped, Fig. 1) with a part through the thickness hole (depth  $h=2.5$  mm, radius  $r=3$ mm, thickness  $t=5$  mm). An initial triangular notch is introduced on the specimen by a thin saw in order to localise the crack initiation, that proceed initially as a corner crack in a propagation plane perpendicular to the remote load direction. Von Mises stresses corresponding to the initial dimension of the corner crack monitored are illustrated in Figs 2a-b. The material fatigue parameters [10], necessary for the crack propagation numerical analysis, were previously obtained by experimental crack growth tests on holed cracked aluminium specimens coming from the same lot of the specimen under analysis. Such parameters are just slightly modified in order to improve the correlation. In particular, the only modification is related to the  $n$  coefficient of the NASGRO 2.0 formula (Eq. 1) which is decreased from  $n=2.61$  to  $n=2.58$ , while the overload shutoff ratio is kept at  $R_{s0}=1.31$  :

$$\frac{da}{dN} = \frac{C \cdot \Delta K^n \cdot \left(1 - \frac{\Delta K_{th}}{\Delta K}\right)^p}{\left(1 - \frac{\Delta K}{(1-R) \cdot K_c}\right)^q} \rightarrow C = 4.26E-11, n = 2.59 \quad (1)$$

The NASGRO 2.0 formula is combined with the Generalised Willenborg model in order to allow for retardation effects. The numerical and experimental crack growth rates exhibits a satisfactory correlation (with the exception of the final part) as evident from Fig. 3: in such a figure experimental and numerical curves are plotted, showing the elliptical crack front semi-axis length  $a$  (on the plane surface) and  $c$  (along the plate thickness) against number of cycles, correspondingly to the given load spectrum. Von Mises stresses, at the moment in which the corner crack becomes through the thickness,

are shown in Fig. 4 on a magnified deformed plot and a photo of the specimen is illustrated in Fig. 5.

When the crack becomes through the thickness but still with an elliptical front the two break-out point positions, on the two specimen faces, are indicated as  $a$  and  $b$ . It is important to clarify that the transition between part through and through crack, in which there is the plastic rupture of the residual ligament (when  $c$  is almost equal to the thickness), is not correctly simulated by LEFM (linear elastic fracture mechanics) due to the large plastic deformations involved. Such plastic effects explain the slant shape of the final part of the crack front which deviate from the plane of crack propagation (Fig. 6) in the direction of maximum shear (ductile materials).

Table 1. First part of the load spectrum on specimen N°1 (still corner crack).

	Block 1	Block 2	Block 3
Number of cycles	15000	25	9000
$P_{\max}$ [kN]	22.29	28.98	22.29
$P_{\min}$ [kN]	11.15	11.15	11.15



Figure 1. Specimen anterior part.

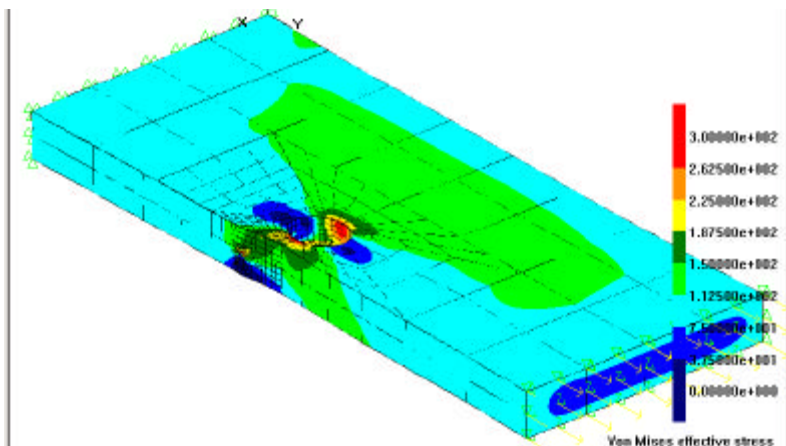


Figure 2a. Von Mises stress under  $P_{\max}=17$  kN

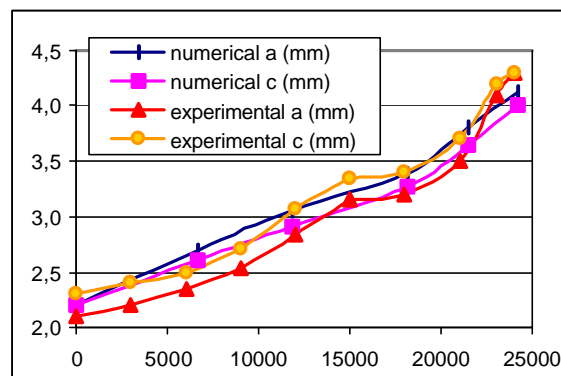
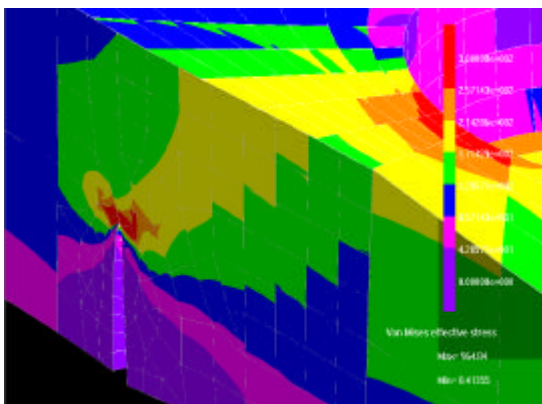


Figure 2b. Close up of the initial crack monitored. Figure 3. Crack length versus cycles.

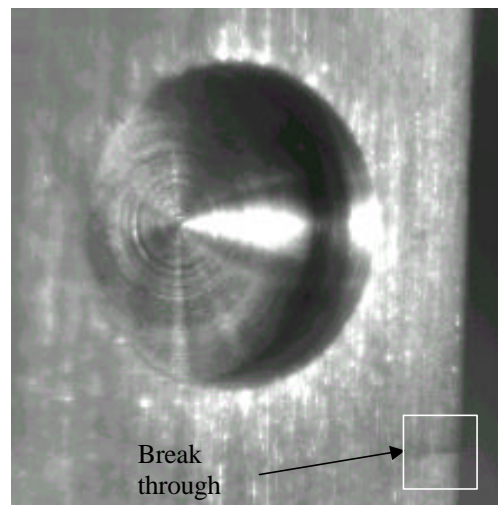
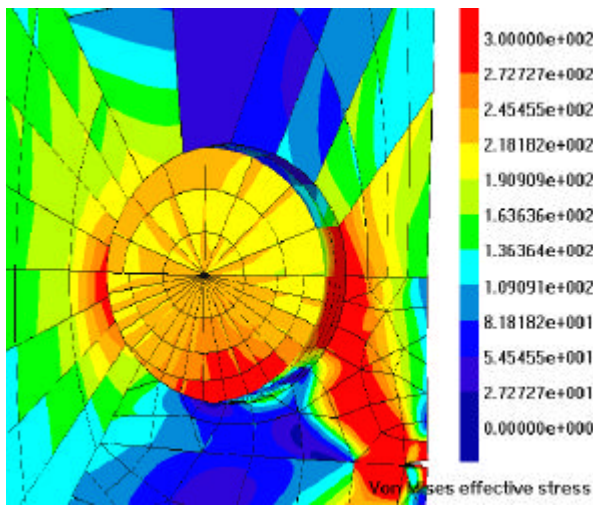


Figure 4. Von Mises stress ( $P_{\max}=17$  kN).

Figure 5. Initial monitored through crack.

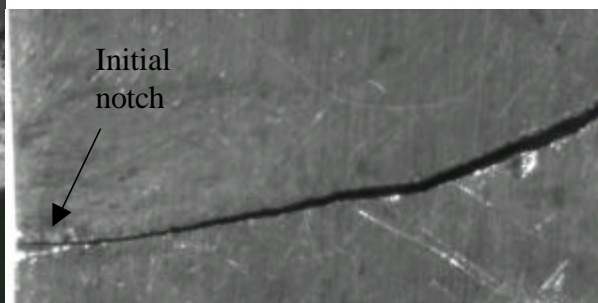
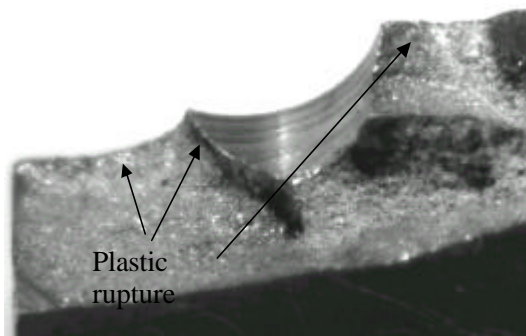


Figure 6. Plastic break through.

Figure 7. Crack path on specimen posterior part.

### ***Through Crack Propagation under Mixed Mode Loading***

The initial through elliptical crack considered for the simulation is inclined of  $7^\circ$  towards the hole, as coming from the numerical simulation and correspondingly to the experimental path (Figs 4, 7). Three thousands constant amplitude cycles ( $P_{\max}=17$  kN and  $P_{\min}=1.7$  kN) are applied and the comparison between numerical and experimental crack length versus number of cycles is illustrated in Fig. 8: again NASGRO 2.0 formula (Eq. 1) has been adopted with the aforementioned fatigue parameters (retardation model is not needed). It is possible to notice a strong correlation with reference to the posterior emerging front (crack size  $a$ ) whilst some difference exists on the anterior front, but this is due to the fact that what is visible on the surface (where the crack is measured) does not correspond exactly to the crack propagating front, because (as previously mentioned) where the crack intersect the anterior surface, the elliptical crack front undergoes a plastic rupture that create an inclined final crack surface (Fig. 6). For these reason the real propagating crack front, which is not visible on the surface, is only apparently closer to the hole than in the numerical simulation

where plastic effects are not considered (Figs 9-11). After this block cycle the crack break through into the hole (Fig. 9) and this is also provided by the numerical simulation: as a matter of fact we can compare the rupture line inside the hole, clearly visible in Fig. 10 with the red strip, which highlight the highest Von Mises stress area, that goes from the crack tip to the hole border and along the hole lateral surface to the bottom edge (Fig. 11). The aforementioned contour plot is related to a number of cycles less than 3000 (approximately 2000) because whilst on the surface there is the plastic crack rupture the crack keeps on propagating inside the specimen (Fig. 12) and the final shape after 3000 cycles, obtained by simulation (Fig. 13) is well in accordance with the experimental one (Fig. 12).

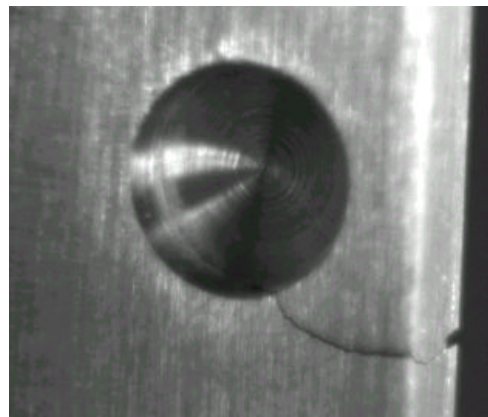
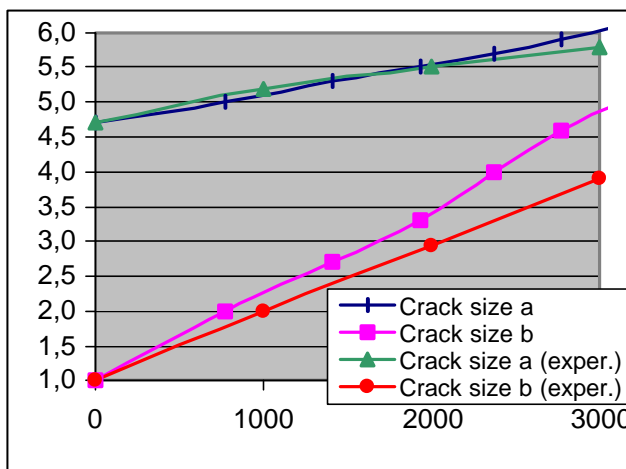


Figure 8. Crack length (mm) versus cycles.

Figure 9. The crack penetrate the hole

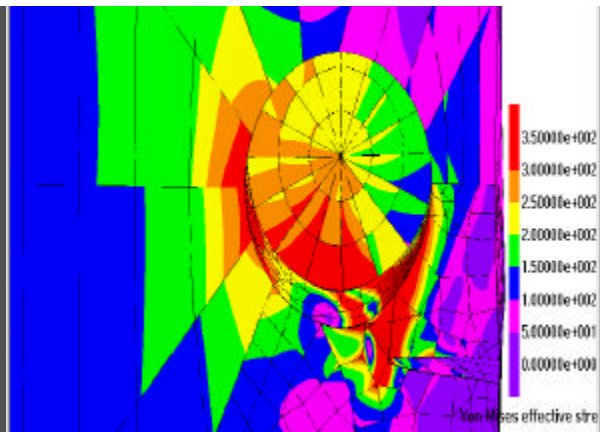
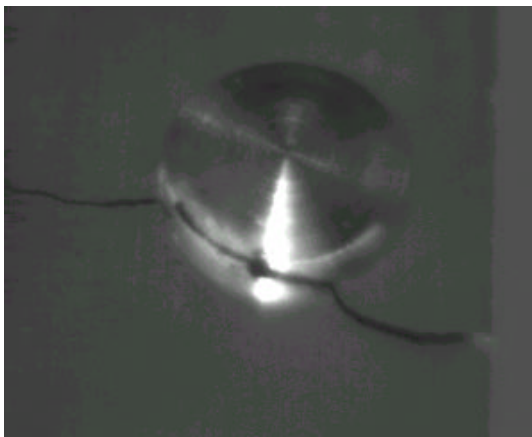


Figure 10. Final scenario before rupture.

Figure 11. Von Mises stresses.



Figure 12. Final crack scenario.

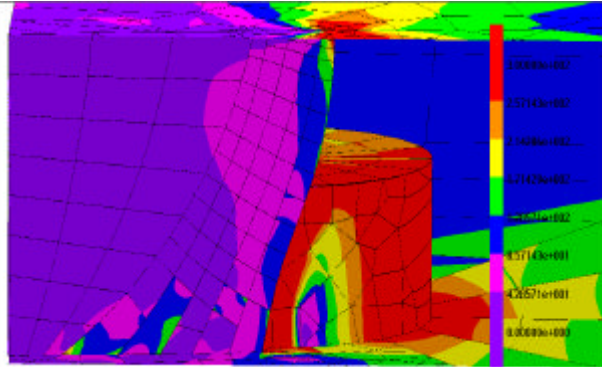


Figure 13. Simulated final crack front.

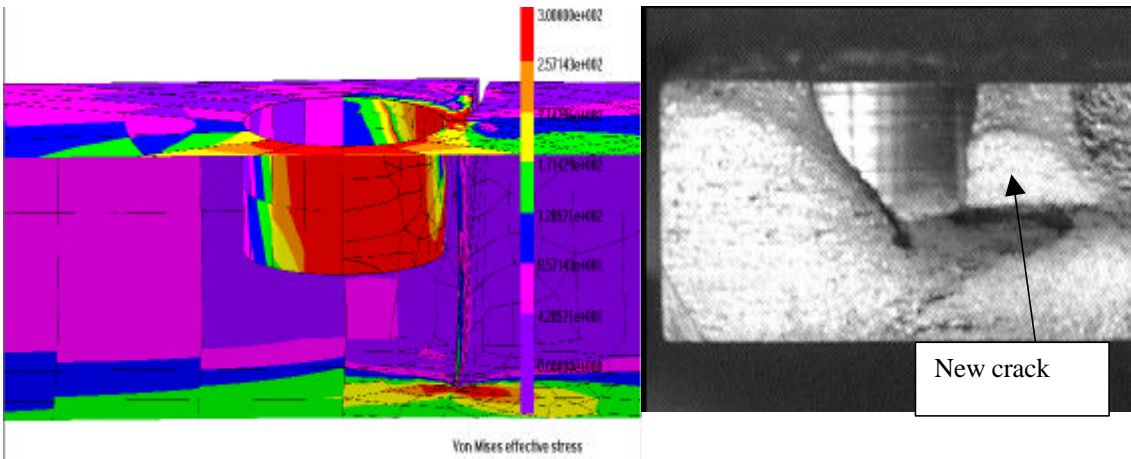


Figure 14. Simulated final crack front.

Figure 15. Plastic break through in the hole.

From Fig. 14 it is possible to notice, as previously said, that the plane of crack propagation is not intersecting the hole, whilst the break through into the hole happens along a plane perpendicular to the hole surface where the material between the crack front and the hole surface fails by plastic rupture (Fig. 6). After this phase another crack is initiated at the bottom of the hole (Fig. 15) and marker loads are necessary in order to assess the crack propagation rates; again it is possible to notice the plastic break through with a slant shape (Fig. 6). The overall scenario at the end of this phase is characterised by extensive plastic deformation (Fig. 16) in the area surrounding the hole (lateral and bottom part) and this make impossible to go on with the numerical crack growth simulation which is based on LEFM (Linear Elastic Fracture Mechanics).

In Fig. 16 the overall contour plot of Von Mises stresses is presented, when the crack break through into the hole.

## CONCLUSIONS

Interesting correlation between experimental complex crack path and numerical ones are obtained, also with reference to the plastic collapse phenomena if restricted to the initial phase. Moreover a satisfactory correlation is also obtained between experimental and numerical crack growth rates even in case of a general load spectrum applied to the specimen.

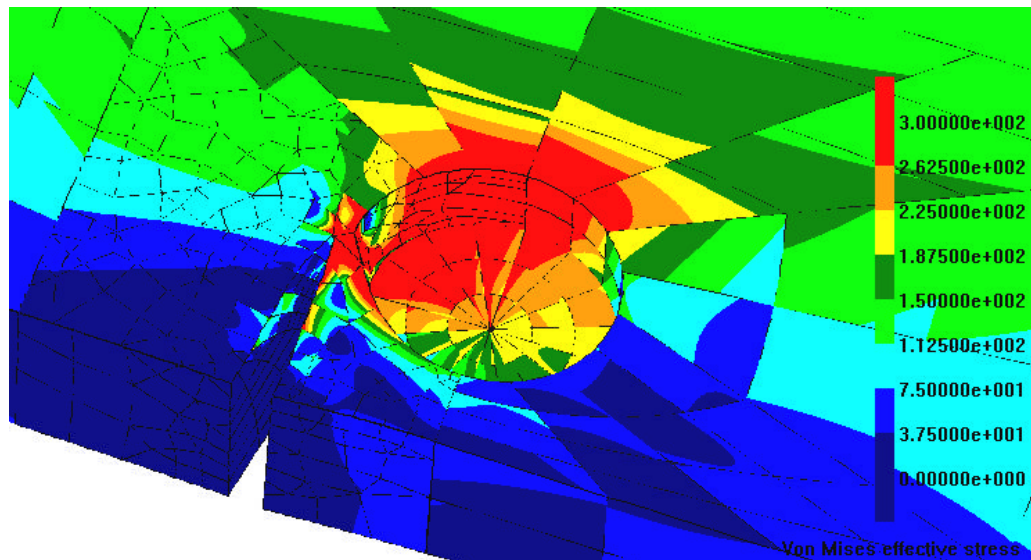


Figure 16. Von Mises stresses after 3000 cycle of through crack propagation.

## REFERENCES

1. Riddell, W.T., Ingraffea, A.R. and Wawrzynek, P.A. (1997) Experimental observations and numerical predictions of three-dimensional fatigue crack propagation. *Engineering Fracture Mechanics* **58**, 293-310.
2. Dhondt, G., Chergui, A. and Buchholz, F.-G. (2001) Computational fracture analysis of different specimens regarding 3D and mode coupling effects. *Engineering Fracture Mechanics* **68**, 383-401.
3. Hsien Yang Yeh, Chang H. Kim (1995) Fracture Mechanics of the Angled Elliptic Crack under Uniaxial Tension. *Engineering Fracture Mechanics* **50**, 103-110.
4. Cali, C., Citarella R. and Perrella, M. (2003) Three-dimensional crack growth: numerical evaluations and experimental tests. ESIS STP book *Biaxial/Multiaxial Fatigue and Fracture* **31**, pp. 341-360, Eds Andrea Carpinteri, Manuel de Freitas and Andrea Spagnoli, Elsevier.
5. Apicella, A., Citarella, R., Esposito, R. and Soprano, A. (1998) Some SIF's evaluations by Dual BEM for 3D cracked plates. *Proceedings of the Int. Conference AMME*, Poland.

6. Apicella, A., Citarella, R., Soprano, A. (1999), 3D Stress intensity factor evaluation by Dual BEM, *Proceedings of the International Conference Fracture and Damage Mechanics*, UK.
7. Mi, Y., Aliabadi, M.H. (1992), Dual Boundary Element Method for Three Dimensional Fracture Mechanics Analysis, *Engineering Analysis with Boundary Elements*, 10, 161-171.
8. Mi, Y., Aliabadi, M.H. (1994), Three-dimensional crack growth simulation using BEM, *Computers & Structures*, *Computers & Structures*, 52, 871-878.
9. Mi, Y. (1996), Three-dimensional analysis of crack growth, *Topics in Engineering*, 28, Computational Mechanics Publ., Southampton, U.K.
10. C. Cali, R. Citarella, M. Perrella (2003), Crack growth under general load spectrum: numerical simulation and experimental tests, *Proceedings of the XI International Conference CMEM 2003*, 12-14 may, Halkidiki, Greece.