Stresses and Strains in Correspondence of a Transverse Crack in a Shaft: Effect of Crack Closure

N. Bachschmid¹ and E. Tanzi²

¹ Politecnico di Milano, Via La Masa, 34 I 20158 Milano, nicolo.bachschmid@polimi.it ² ezio.tanzi@mecc.polimi.it

ABSTRACT. The results of an extensive experimental investigation on a cracked shaft specimen have shown two different aspects which are generally disregarded in cracked shaft analyses: a consistent crack closure effect, which affects the well known crack breathing behaviour, and, in conditions where the crack should be completely closed by compressive stresses on the crack faces, high stress intensity factors in positions close to the crack lips, showing that the contact occurs only in small areas close to the lips. Both effects can be simulated in simple ways by linear models.

INTRODUCTION

In the frame of a research project cofinanced by EDF (Electricite de France, R. & D., Department Analyses Mecaniques et Acoustique) several transverse cracks have been produced in shaft specimens, and their effect on the dynamical behaviour of rotating shaft lines, in which the cracked specimen had been introduced, has been analysed experimentally and has been simulated with suitable models.

A transverse crack in a rotating shaft loaded with stationary bending loads, as it occurs in horizontal axis heavy rotors like steam turbines and generators, opens and closes periodically (1per rev.), in other words breaths. The stiffness of the shaft changes periodically according to the breathing mechanism; the rotation dependent stiffness change generates vibrations, with harmonic components which depend on the shape and depth of the crack and of its breathing behaviour.

Therefore the interest in measuring experimentally the breathing behaviour in one of the aforesaid cracked specimens. To this scope a series of strain gauges have been applied close to the crack and also directly across the crack lips, and the horizontal cracked specimen has been loaded with different stationary loads, and has been rotated in different angular positions in order to excite the breathing of the crack.

Smaller loads were not able to open the crack as it resulted in all the different measuring points: the crack closure effect generates an internal bending moment which holds the crack closed. Only when the external bending moment overcomes the internal bending moment, then the lips of the crack start to open.

When the crack is closed, with an external bending moment which sums up with the internal bending moment, then the measured compressive strain is much higher than the theoretical strain calculated assuming a linear compressive stress distribution over the cracked section. This can be explained by assuming that when the crack is closed, the

contact occurs only on a smaller portion of the cracked surface, or on the crack lips only, determining higher strains associated also to stress intensity factors. This aspect is also related to the crack closure effect. Despite the fact that crack closure effects have been studied by several researchers (see [2]), their influence on rotating shafts has never been modelled suitably, as far as the authors know.

In the following pages the experimental results are presented and are compared to calculated results.

Despite the highly non linear stress and strain distribution in the cracked area, and the non linear breathing behaviour, the overall load-strain behaviour results to be quite linear: the overall load –deflection can therefore easily be represented by a linear model.

DESCRIPTION OF THE EXPERIMENTAL APPARATUS

The dimensions of the cracked shaft are shown in Fig. 1. The crack has been initiated by means of a small slot generated by electro-erosion, and has been propagated roughly half way the shaft cross section by applying a constant bending load to the rotating beam. The part in which the slot had been machined was removed by turning.



Figure 1. Schematic drawing of the test rig (dimension are in mm).

The final cracked section has the shape shown in Fig. 2, as it resulted from ultrasonic test measurements. The cracked shaft has been clamped at one end but can be rotated around his axis by steps of 15° each. At the other end a vertical load has been applied and has been increased by steps of 152 N. The corresponding bending moments in correspondence of the cracked section range from a minimum of 37.4 Nm to a maximum of 1351 Nm and are given in Table 1, along with the theoretical strains, calculated assuming linear stress distribution.

Strain gauges from A1 to A11 are applied each 15° as close as possible to the crack, in section C (cracked) in Figs. 1 and 2, strain gauges from A12 to A15 are applied to section I (integer) which is sufficiently far away from the crack to not be influenced by its breathing behaviour, and are used as reference signals. Strain gauges from A16 to A28 are applied each 15° in correspondence of the crack but on the integer part opposite to the crack.



Figure 2. Strain gauge positions in section C (the shaded area is uncracked).

Strain gauges from B31 to B39 have been applied across the lips, as it is shown in Fig. 2 right. These strain gauges of type B indicate compressive strains when the bending moment closes the crack, but indicate the relative displacement of the crack lips (and not the strain) when the crack is opening. They generally have failed when stretched beyond their linear elastic range, in the higher load range.

Load	Bending Moment in section C (N/m)	Max stress in section C (MPa)	Max strain in section C (με)
Load S	37	1.11	5.29
Load A	105	3.11	14.80
Load B	182	5.40	25.72
Load C	271	8.04	38.28
Load D	426	12.64	60.18
Load E	558	16.56	78.88
Load F	690	20.49	97.58
Load G	822	24.42	116.28
Load H	954	28.35	134.98
Load I	1087	32.27	153.68
Load J	1219	36.20	172.38
Load K	1351	40.13	191.08

Table	1.
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EXPERIMENTAL RESULTS

Typical results are shown in Fig. 3 for measuring point A6 in which the maximum effect of the crack is expected: 0° means crack axis (which passes close to measuring point A6) directed vertically downwards (crack closed), 180° means crack axis directed vertically upwards (crack open).



Figure 3. Strain gauge A6: strain versus rotation.

Starting from load D a flat zone in strain level (around 140 microstrains) is clearly recognisable in between $0^{\circ} - 60^{\circ}$ and in between $300^{\circ} - 360^{\circ}$, for a total range of angular positions of 120° . For higher loads the flat zone extends to higher values of angular position range (up to 165°). In this range of angular positions where the strain does not change, the crack is supposed to be definitely open. The strain for the open crack is not zero but a tensile strain of 140 microstrain is measured on the open crack lips. The reason is that when the strain gauges have been applied to the shaft with no external loads, an internal pre-stress was present on the crack lips due to the crack closure effect, generating a compressive strain of 140 microstrains. Only the loads which generate a similar value of tensile stresses are able to open the crack. Similar but generally higher values (up to 240 microstrains) of compressive strains due to the crack closure effect have been found in the other measuring points A3, A4, A5 and A7. The behaviour differs when we are in positions which are closer to the crack ends, which means also closer to the crack tip, as will be shown in measuring point A10.

When the crack is closed (at the angular position of 180°) the compressive stress with the maximum load of 350 microstrains is much higher with respect to the theoretical value of 190 microstrain. This can be attributed to the fact that only a smaller part of the cracked surface is in contact, generating locally higher stresses and also stress intensity factors which would be absent if the crack faces were completely in

contact each other over the complete crack area. This strain magnification is present for all the different loads proportionally with the same value, but its value is different for different measuring points and tends to unity (no strain magnification) close to the crack ends (in measuring points A1 and A10), which could be expected. The value of the stress intensity factors is represented in Fig. 4, along with the depth of the crack below the measuring points.



Figure 4. Stress intensity factor versus number of straingauge (A1 to A10).

It is interesting to notice that these stress intensity factors are roughly independent from load. In the measuring points close to the crack ends, when the loads generate tensile stresses, during the gradually opening of the crack, rather high stress intensity factors have been measured which are due to the closeness of the crack tip to the measuring points. This can be seen in measuring point A10, represented in Fig.5.



Figure 5. Strain gauge A10: strain versus rotation.

Here the reference angular position has been shifted in order to have at 180° the crack closed (with the load directed towards the measuring point A10); the region where the crack is open is not any more symmetrical with respect to the position 180°, because the crack extension is not symmetrical with respect to the reference axis. The maximum compressive strain of 200 microstrains is close to the theoretical value, but the maximum tensile strain of 380 microstrains is much higher than the theoretical value. The crack closure effect hardly can produce higher stresses in this measuring point with respect to point A6, therefore we have again a strong stress concentration factor, due to the closeness of the measuring point to the crack tip. Similar values are found in point A1, something smaller values are found in A2 and A9.

The strains in measuring point A16 which is diametrically opposite to point A6, are represented in Fig. 6. Considering again the maximum load we have with closed crack (angular position 0°) a maximum tensile stress of 240 microstrains, which is something higher than the theoretical value of 190 microstrains: this is caused by the fact that not all the cracked area is effective as has been pointed out before in closed crack configuration.



Figure 6. Strain gauge 16: strain versus rotation.

With crack completely open (angular position 180°) the compressive stress reaches 310 microstrains: this value cannot be compared with the theoretical value calculated assuming linear stress distribution over the reduced section area, because the presence of the crack introduces a highly non-linear stress distribution, as can be seen also in Fig. 7 where the stress distribution due to a bending moment applied to a shaft with a 50% deep crack is shown in the case of open crack compared to the case of closed crack. These results have been calculated with a 3D non-linear model of the cracked shaft.

Other interesting results are provided by the strain gauges (series B) which were applied across the crack. One example is given in Fig. 8 left, where the measured strains in point B35, which is very close to point A6 in the middle of the crack, are represented, and in Fig. 8 right where the compressive strains in the same point are represented in an enlarged scale.



Figure 7. Calculated axial stress distribution for the 50% deep crack in case of open and closed crack.



Figure 8. Strain gauge B35: strain versus rotation. Left overall, right enlarged compressive strains

Load B is not able to open the crack, and indications of A6 and B35 are similar. With load C the crack starts to open a little: 2000 micro-strain are measured in point B35, which compared with 140 micro-strains of point A6 indicates clearly the relative displacement of the crack lips. During the compression (Fig. 8 right) with all loads the measured strains are much higher than those measured in point A6: this could be due to the fact that despite the crack closure effect the crack lips on the external surface are still a little open, so that the indication of the gauge is firstly due to the relative displacement of the lips and later only to the material strain.

With higher loads the indication of the gauges starts to become affected by errors: during the compression with load C the indication seems unreliable, but with load D it could be again reliable, if compared with other measuring points. Load D and still more load E are stretching the gauge beyond its elastic limit. Higher loads produced the failure of the gauge. A non-linear dependency from loads of relative displacement of the crack lips during opening can be noticed; but this will not affect the overall deflection behaviour which always resulted quite linear.

The behaviour is obviously different when approaching the end of the crack (in points B 31 and B39) where the relative movement of the crack lips is strongly reduced, so that the gauges do not fail anymore or fail at very high loads.

An overview of lips opening in all measuring points is given in Fig. 9: the measured strains displacements have been distributed symmetrically with respect to the crack.



Figure 9. The opening of crack lips as measured by the series of B gauges.

THE SIMPLIFIED MODEL

The model used for calculating the breathing behaviour and the reduced stiffness of the cracked shaft is described in detail in [1]. Here only some results are recalled. The model of the breathing mechanism allows to evaluate with high accuracy, if compared with 3D non linear calculations, the open and closed parts of the crack. It is then assumed that only the closed parts together with the integer parts of the cracked cross section contribute to the resisting area and to its second moments of area. The values of the second moments of area are then used for evaluating the stresses due to the bending moment, in the same manner as it would have been in a constant cross section beam, which is obviously totally unrealistic. The highly non-linear stress distribution is not considered.

The crack closure effect has been simulated with an external bending moment which tends to hold the crack closed, generating a maximum of 140 micro-strains of compressive strain.

These strain values have been corrected with the stress intensity factors of Fig. 4. Now a comparison is possible between calculated results and experimental results, taking account of crack closure effects and stress intensity factors.

COMPARISON OF CALCULATED RESULTS WITH EXPERIMENTAL RESULTS

Figure 10 left shows the comparison of calculated results obtained with the above described simplified model, with theoretical results in measuring point A6 for the maximum load. It is surprising how good the simplified model is able to reproduce the experimental behaviour. This happens in almost all measuring points. The quasi-linear breathing behaviour model can be considered completely validated with these experimental results. In the diametrically opposite measuring point the results of the comparison are shown in Fig. 10 right.

The calculated maximum compressive strain is much higher than the measured one, because the model assumes unrealistic linear stress/strain distribution, as pointed out before. A suitable reduction coefficient could be used for getting a better fitting.



Figure 10. Comparison of calculated with experimental result: left in point A6, right in point A16.

CONCLUSIONS

Some crack closure effects on the breathing behaviour of a crack in a rotating shaft have been shown. The overall behaviours of the cracked shaft can be easily modelled with suitable external bending moments and the local behaviour with additional stress intensity factors.

REFERENCES

- 1. Bachschmid, N. and Tanzi, E. (2003) Calculating the dynamical behaviour of rotating beams affected by transversal cracks. Presented at Int. Conf. "FCP 2003".
- 2. Borri, B., Carpinteri, A. and Chiaia, B. (1998) Contact, Closure and Friction Behaviour of Rough Crack Concrete Surface. *Fracture mechanics of concrete structures*, Vol. 3, pages 1635-1644, Freiburg.