Fatigue and Fracture Analysis of Superplastic Formed/Diffusion Bonded Structures: BEM Analysis

C.Di Pisa[1], M.H. Aliabadi[1] and A.Young[2]

[1] Queen Mary, University of London, Mile End Road E1 4NS, London, UK.
C.DiPisa@qmul.ac.uk, M.H.Aliabadi@qmul.ac.uk

[2] Structural Material Centre,Qinetiq, Farnborough, UK.

ABSTRACT. *In this paper a study of a Superplastic Formed/Diffusion Bonded (SPF/DB) aircraft wing section under working load is presented. Such structures operate under cycle load, therefore fatigue has to be considered. The presence of cracks has to be included since damages can occur either in production or during the operating life. The damaged structure is then considered and the stress intensity factors are evaluated for cracks of different sizes. Considering a load cycle which span from 0 to maximum load, the relationships between the stress intensity factor and the criteria for fatigue-life prediction and crack growth directions is soon derived. The aim of this work is to demonstrate the effectiveness of the boundary element method to investigate behaviour of cracks in complex large scale structures in presence of damages.*

INTRODUCTION

Superplastic Forming/Diffusion Bonding [4] is a process that provides the capability to manufacture complex structures as a single element, which otherwise would have been made out of a large number of elements. Typical application are to be found in the aircraft design where it is essential to provide structures with high strength and light weight. An example of SPF/DB structures can be found in figure 1; this component is a part of a larger X-core stiffened section. These structures operate under fatigue loading and cracks grow in fatigue. Key parameter for crack growth and fatigue-life is the Stress Intensity Factor (SIF), hence accurate methods to evaluate it are required.

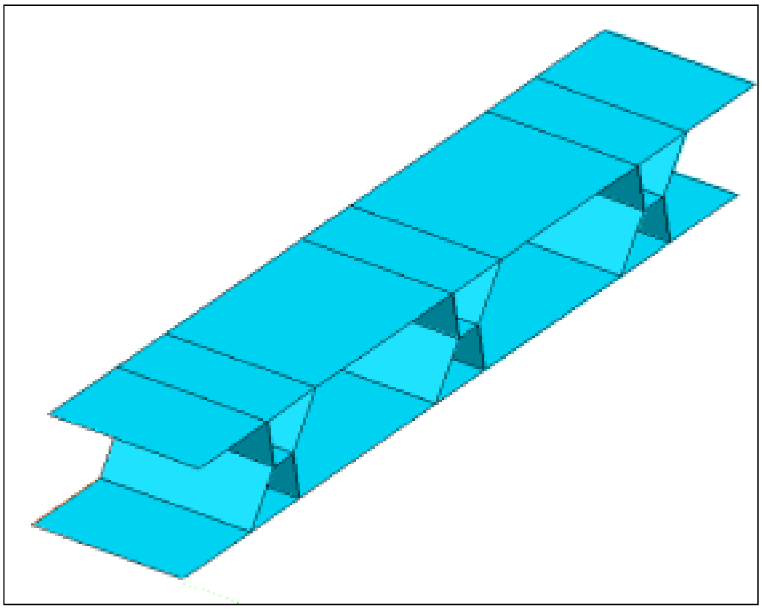

Figure 1: X-core structure

BOUNDARY ELEMENT METHOD FOR MODELLING SPF/DB STRUCTURES

Introduction

The growth of cracks under fatigue load (such as variable working load for aircraft structures) may also be described by the stress intensity factor, following Paris postulate [5] which relates the rate of growth per cycle of stress (da/dN) to the stress intensity range ΔK ($K_{\max} - K_{\min}$), when the stress is at its maximum level:

$$\frac{da}{dN} = C(\Delta K)^m$$

where C and m are to be found experimentally.

Typical rate diagram is shown in fig.2.

It is clear the central role that stress intensity factor plays at this stage therefore the importance of its correct evaluation. Although many numerical solution are available in literature, there is still a gap in the application to more realistic problem as the damaged X-core wing section presented.

The Boundary Element Method

The X-core stiffened structure examined here can be seen as an assembled plate structure; therefore the BEM for flat plates is used following Aliabadi [1] and Wen and Aliabadi [2].

The displacement boundary integral equations for 2D plane stress can be written as follow:

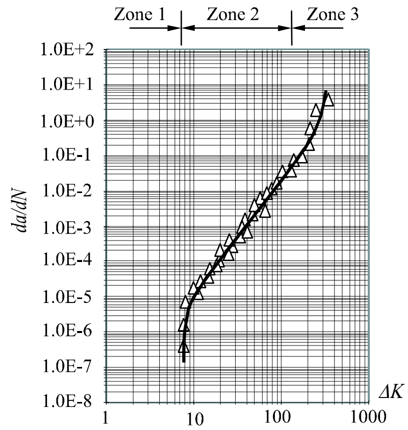

Figure 2: Typical fatigue crack growth rate diagram

$$
\begin{aligned}
& c_{\theta\alpha}(x')u_\alpha(x') + \int\!\!\!\!-_{\Gamma} T_{\theta\alpha}(x',x)u_\alpha(x)d\Gamma(x) \\
= & \int_\Gamma U_{\theta\alpha}(x',x)t_\alpha(x)d\Gamma(x) + \frac{1}{h}\int_\Omega U_{\theta\alpha}(x',X)f_\beta(X)d\Omega(X)
\end{aligned} \quad (1)
$$

and for plate bending

$$
\begin{aligned}
& c_{ij}(x')w_j(x') + \int\!\!\!\!-_{\Gamma} P_{ij}(x',x)w_j(x)d\Gamma(x) \\
= & \int_\Gamma W_{ij}(x',x)p_j(x)d\Gamma(x) + \int_\Omega W_{i3}(x',X)q_3(X)d\Omega(X)
\end{aligned} \quad (2)
$$

where u_α are in-plane displacements, w_j are the rotations in x and y, t_α in-plane tractions, p_j are the bending moments and the shear tractions, q_3 is the internal pressure. $T_{\theta\alpha}(x',x)$ and $U_{\theta\alpha}(x',x)$ represent the Kelvin fundamental solutions for plane stress elasticity, while $P_{ij}(x',x)$, $W_{ij}(x',x)$ are the Reissner plate fundamental solutions [1]. Greek indices vary from 1 to 2, Roman indices from 1 to 3.

The Dual Boundary Method

The Dual Boundary Method (DBM) for modelling cracks in plates was formulated by Dirgantara and Aliabadi[3]. The main idea of DBM is to model the crack as two separate surfaces facing each other, with coincident discretisation points. Then the displacement boundary integral equations are applied onto the upper surface Γ^+ meanwhile the traction boundary integral equations onto the lower surface Γ^-. The latter have been found via the application of the Hooke laws of elasticity to the derivative of the displacement boundary integral equations. The resulting equations for uniform pressure and free-traction crack are the following:

$$
p_\alpha(x') + n_\beta(x'^-) \int\!\!\!\!=_{\Gamma} P_{\alpha\beta\gamma}(x'^-,x)w_\gamma(x)d\Gamma(x)
$$

$$
\begin{aligned}
&+n_\beta(x'^-) \fint_\Gamma P_{\alpha\beta 3}(x'^-,x) w_3(x) d\Gamma(x) \\
&= n_\beta(x'^-) \fint_\Gamma W_{\alpha\beta\gamma}(x'^-,x) p_\gamma(x) d\Gamma(x) \\
&+n_\beta(x'^-) \int_\Gamma W_{\alpha\beta 3}(x'^-,x) p_3(x) d\Gamma(x) \\
&+n_\beta(x'^-) \int_\Omega W_{\alpha\beta 3}(x'^-,x) q_3(x) d\Omega(x)
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
&p_3(x') + n_\beta(x'^-) \fint_\Gamma P_{3\beta\gamma}(x'^-,x) w_\gamma(x) d\Gamma(x) \\
&+n_\beta(x'^-) \bbint_\Gamma P_{3\beta 3}(x'^-,x) w_3(x) d\Gamma(x) \\
&= n_\beta(x'^-) \int_\Gamma W_{3\beta\gamma}(x'^-,x) p_\gamma(x) d\Gamma(x) + \\
&n_\beta(x'^-) \fint_\Gamma W_{3\beta 3}(x'^-,x) p_3(x) d\Gamma(x) + \\
&n_\beta(x'^-) \int_\Omega W_{3\beta 3}(x'^-,x) q_3(x) d\Omega(x)
\end{aligned} \tag{4}
$$

and

$$
\begin{aligned}
&t_\alpha(x') + n_\beta(x'^-) \bbint_\Gamma T_{\alpha\beta\gamma}(x'^-,x) u_\gamma(x) d\Gamma(x) = \\
&n_\beta(x'^-) \fint_\Gamma U_{\alpha\beta\gamma}(x'^-,x) t_\gamma(x) d\Gamma(x)
\end{aligned} \tag{5}
$$

where $p_\alpha = M_{\alpha\beta} n_\beta$, $p_3 = T_{3\beta} n_\beta$ and $t_\alpha = N_{\alpha\beta} n_\beta$. The kernels $P_{\alpha\beta\gamma}(x',x)$, $W_{\alpha\beta\gamma}(x',x)$ are obtained from the linear combination of the derivatives of $P_{ij}(x',x)$ and $W_{ij}(x',x)$ respectively; $U_{\alpha\beta\gamma}(x',x)$, $T_{\alpha\beta\gamma}(x',x)$ are linear combination of the derivatives of $U_{\theta\alpha}(x',x)$ and $T_{\theta\alpha}(x',x)$.

To satisfy the additional continuity requirements for the traction boundary integral equations onto Γ^- discontinuous elements have been used [3].

Stress Intensity Factor

In plate theory five stress intensity factors have to be computed accordingly to the five crack modes (see Fig.3). The extrapolation of crack surface displacements technique (see [3]) is used to evaluate the SIF. The relationship between the crack opening displacements and the SIF can be expressed as:

$$
\left\{ \begin{array}{c} \Delta u_1 \\ \Delta u_2 \\ \Delta \varphi_1 \\ \Delta \varphi_2 \\ \Delta w_3 \end{array} \right\} = \left[\begin{array}{ccccc} \frac{8\sqrt{r}}{E\sqrt{2\pi}} & 0 & 0 & 0 & 0 \\ 0 & \frac{8\sqrt{r}}{E\sqrt{2\pi}} & 0 & 0 & 0 \\ 0 & 0 & \frac{48\sqrt{2r}}{Eh^3} & 0 & 0 \\ 0 & 0 & 0 & \frac{48\sqrt{2r}}{Eh^3} & 0 \\ 0 & 0 & 0 & 0 & \frac{24(1+\upsilon)\sqrt{2r}}{5Eh} \end{array} \right] \left\{ \begin{array}{c} K_1^m \\ K_2^m \\ K_1^b \\ K_2^b \\ K_3^b \end{array} \right\} \tag{6}
$$

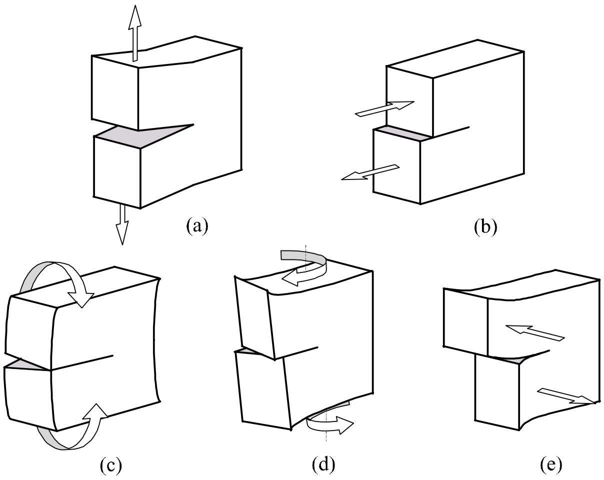

Figure 3: Five crack modes: a) opening and b) sliding due to membrane loads; c) opening and d) sliding due to bending and torsion loads; e) tearing due to shear load [6]

where K_{1m} and K_{2m} are membrane stress resultant intensity factors and K_{1b}, K_{2b} and K_{3b} are bending stress resultant intensity factors. Stress intensity factors are then evaluated following Dirgantara and Aliabadi [3].

The maximum values of the stress intensity factors through the plate thickness are to be found on the plate surfaces for $modeI$ and $modeII$, while on the middle plate for $modeIII$. They can be related to the stress resultant intensity factors as follow:

$$\begin{aligned} K_I^{Max} &= \left|\frac{K_{1m}}{h}\right| + \frac{6}{h^2}\left|K_{1b}\right| \\ K_{II}^{Max} &= \left|\frac{K_{2m}}{h}\right| + \frac{6}{h^2}\left|K_{2b}\right| \\ K_{III}^{Max} &= \frac{3}{2h}\left|K_{3b}\right| \end{aligned} \tag{7}$$

Considering the minimum load as zero the ΔK is then evaluated as:

$$\Delta K = K^{\max} \tag{8}$$

CASE STUDIES

The X-core structure proposed is composed of flat plates of same material (Titanium alloy,Young modulus $E = 110,000 Mpa$; Poisson ratio $v = 0.3$) and different thickness, so to build a structure which is $w = 171mm$ by $d = 50mm$ deep and $h = 17.9mm$ high. It is subject to a cycle bending moment which range is from o to $477kN \times mm$. The bending load is generated by the application of linear distributed in-plane forces ($q = 156N/mm$) of compression and tension at the tips of the lower and upper skin, respectively (see Fig.4). For the numerical simulation symmetry in the xy plane has been considered.

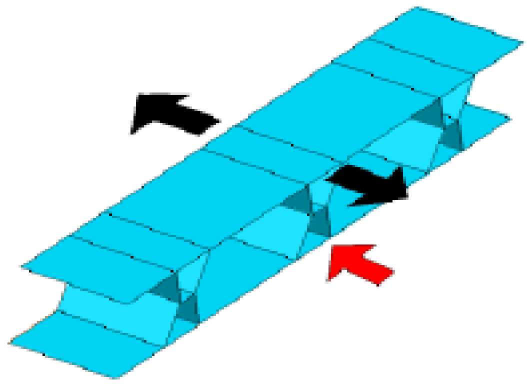

Figure 4: X-core: bending load

A central crack of various size has been placed in the middle plate of the top skin which is found to sustain the most severe normal stress.
Although the model is fully mixed-mode, the relevance on mod$eIII$ has not been proven yet to be significant, so it is ignored. The σ_{yy} contour for a crack of size $a = 5mm$ is shown, the stress concentration near crack tips is clear (Fig.5). The normalized $\Delta K^{Max}/K_o$, where $K_o = q\sqrt{\pi a}/t$ are plotted for the individual deformation modes against crack size in Fig. 6 (note that $\Delta K_{II}^{Max}/K_o$ has been magnified of a factor of 25).

Although the skins are the elements under the most severe conditions, the concentration of shear stress in the webs makes worthwhile a closer investigation. Therefore a crack ($a = 2mm$) as been placed into the upper central web, inside the stress concentration area. The resulting K^{Max} are normalized by $K_o = Mh_e\sqrt{\pi a}/I_e$, where h_e and I_e are the thickness and the inertia momentum of an equivalent structure. These results are then found, again for the individual deformation modes: $\Delta K_I^{Max}/K_o = 0.35$ and $\Delta K_{II}^{Max}/K_o = 0.237$.

CONCLUSION

The numerical prediction of the SIF is essential to provide an efficient analysis of crack behaviour in modern structures such as aircraft components. The boundary element method has already given good results when applied to simpler geometries. The new challenge is to show its flexibility to adapt to more complex geometries. As

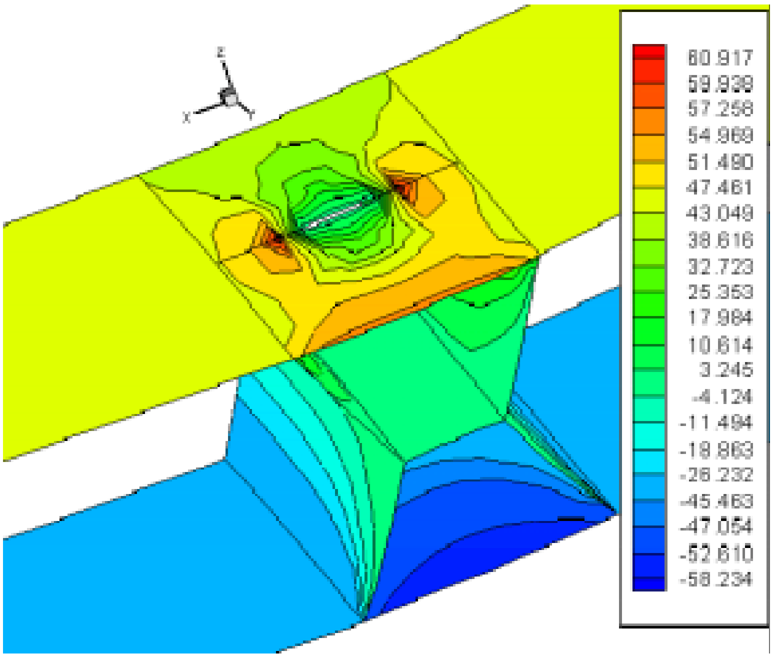

Figure 5: X-core: σ_{yy} contour near crack

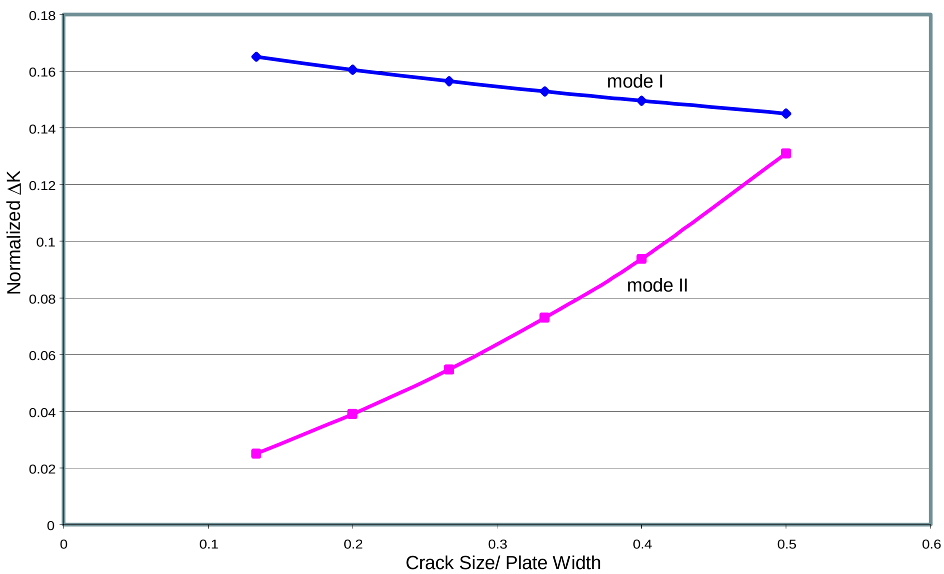

Figure 6: Normalized ΔK for individual deformation mode

present the efforts are focused on the SIF evaluation from which crack growth direction and fatigue life expectation will be derived from. The BEM proposed here has shown a good capability to analyze real aircraft components. The results obtained are encouraging although no comparison has yet been made with experimental results.

References

[1] Aliabadi, M.H.(2001), *The Boundary Element Method, vol II: application to solids and structures.* Wiley, Chichester.

[2] Wen, P.H., Aliabadi, M.H.,Young, A., *Damage Tolerance Assessment of Multilayered Advanced Airframe Structures: Boundary Element Analysis.*(to appear in Engineering Fracture Mechanics in 2003)

[3] Dirgantara,T. and Aliabadi,M.H. (2002), *Stress intensity factors for cracks in thin plates.* In: *Engineering Fracture Mechanics* **69**, pp1465-1486,

[4] Bottomley, I. E.(1995),*Superplastic forming and diffusion bonding of aircraft structures.* In: *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering*, **209**, 3, pp 227-231.

[5] Paris, P.C.(1964), *The fracture mechanics approach to fatigue*, In: *Fatigue, an interdisciplinary approach*, pp 107-132.

[6] Dirgantara, T. and Aliabadi, M.H. (2000), *Crack Growth Analysis of Plates Loaded by Bending and Tension using Dual Boundary Elementh Method.* In: *International Journal of Fracture*, **105 (1)**, pp 27-47.