Parametric Analysis of Oblique Edge Cracks under Cyclic Surface Loading

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ABSTRACT. *The problem of the contact between a cylinder rolling on a semi-plane with an inclined edge crack is solved by the Weight Function method. The Crack Opening Displacement components were obtained by using the analytical expression of the Green's Function. The Hertzian pressure distribution was assumed as load to determine the nominal stress distribution in the un-cracked body. The conditions of partial crack closure were analysed and the influence on the effective stress intensity factors* K_I *and* K_{II} *produced by the normal and tangential forces acting on the closed portions of the crack were included. By considering different friction conditions between the crack surfaces, the evolution of* K_I *and* K_{II} *during a typical loading cycle was analysed. The effects of the crack inclination and friction on the crack surfaces were investigated*

INTRODUCTION

Many machine components (e.g. gears, rolling bearings, rail wheels) suffer surface fatigue damage induced by contact loading. During the early stages of fatigue damage, inclined edge cracks [1-3] can be observed. The pressure induced by the elastic contact cyclically crossing the crack mouth is a typical loading condition in those cases. Moreover, it is common practice to improve surface fatigue resistance by surface treatments, many of which inducing local compressive residual stress [4]. As a consequence, both residual and externally induced stress fields can show complex through thickness variation with high gradients. The evaluation of the fracture mechanics parameters for such cracks is not a simple task, particularly when several crack lengths have to be considered in order to predict the fatigue life. The presence of lubricant, which can be pumped into the crack and trapped in it, makes the problem [5,6] even more challenging. The fracture analysis is complicated by the occurrence of mixed Mode [7] and partial crack closure [8]. Indeed, mutual normal and tangential forces in the closed portions of the crack are very effective and they have to be properly accounted for in the SIF (Stress Intensity Factor) evaluation [9].

The WF (Weight Function) approach was found to be particularly suitable for solving this kind of problem [10]. A general matrix like WF, having the form of a truncated power expansion, has been recently proposed by the authors for an inclined edge crack [11]. From that WF, analytical Green's functions were also obtained [12] by which the COD components can be directly obtained under a general load applied on the crack faces by direct integration. By introducing the Green's functions in an iterative procedure, the crack closure can be efficiently and accurately predicted as shown in [13]. After assuming a tentative partial closed region, the zones in contact are modified step by step until the compatibility conditions are met on the whole crack extension within a specified tolerance.

In the present paper this approach is extended to study the contact loading due to a cylindrical body rolling on the boundary of a semi-plane carrying an inclined edge crack. The effects of different crack inclination angles and friction conditions between the crack surfaces were analysed on the evolutions of K_I and K_{II} during the loading cycle. By assuming the Hertzian load induced by a cylinder rolling with no friction on the semi-plane, the nominal stress distribution in the un-cracked body was determined along the crack path. The nominal stress distributions are the only input required by the WF based procedure. The occurrence of the partial crack closure was demonstrated and the related influences on the SIFs exerted by the mutual normal and tangential forces acting on the closed portions of the crack evaluated.

PROBLEM DEFINITION

The problem is described in Fig. 1, showing a cylinder (having radius R) moving on a elastic semi-plane carrying an edge crack inclined by an angle q with respect to the inward normal at the surface.

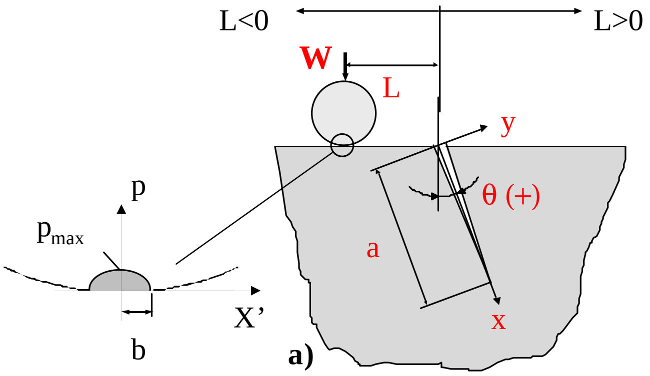

Fig. 1

After Hertz, the maximum pressure, p_{max}, and the half-width b of the contact region can be related to the applied load per unit depth W and the materials elastic constants. In order to evaluate the nominal stress distribution induced by the rolling cylinder in the un-cracked body, the Boussinesq solution and the superposition principle was employed [14].

In the present study the movements of the rolling cylinder either rightward or leftward on the semi-plane were considered. The friction forces between crack edges produce different loading histories at the crack tip, in dependence on the direction of the travelling load. It is worth noting that the load history experienced by a crack inclined with an angle $+q$ when the cylinder moves leftward is equivalent to that of a crack inclined by an angle $-q$ when the cylinder moves rightward. Therefore the inclination of the crack can be used to study the effect of the cylinder motion direction. The analysis was carried out for different values of the ratio *a/b* (Fig.1) and several crack inclinations were analysed to investigate the occurrence of an orientation of maximum loading. In the contact between crack edges two values of the Coulomb friction coefficient were assumed: moderate m=0.1 and high m=0.5. In the closed portions of the crack, both sliding and sticking portions were found depending on the friction coefficient. No difference was assumed between static and dynamic friction coefficients. The values of SIFs, contact stresses and COD components were normalized with the following characteristic values $K_o = W \cdot \sqrt{\frac{p}{a}}$, $S_0 = \frac{2W}{pa}$, $v_0 = \frac{a \cdot S_0}{E}$ respectively.

DISCUSSION OF SOME RESULTS

When neglecting the contact between the crack edges, negative values of K_I and v COD component (in the y direction of Fig. 1) are usually obtained, thus representing a physically unacceptable material overlapping. On the contrary, taking account of the normal pressure and of the friction force, the crack is partially closed for nearly all positions of the cylinder. Typical solutions for different load positions are reported in Fig. 2. Examples of SIFs evolutions vs the load position L/a (L representing the distance of the load see Fig. 1) for different *a/b* ratios are shown in Fig. 3 (θ= 40 °, cylinder moving rightward). The crack is subjected to mixed Mode (I+II) in a narrow range of load positions near the crack mouth (L>0) where K_I >0, and to pure Mode II in the remaining part of the *L/a* positions domain.

At large *a/b* ratio (i.e. for long cracks if b=const), a relative increment of the effect of Mode I is predicted (increases of the maximum K_I and extension of the *L/a* region where K_I is positive).

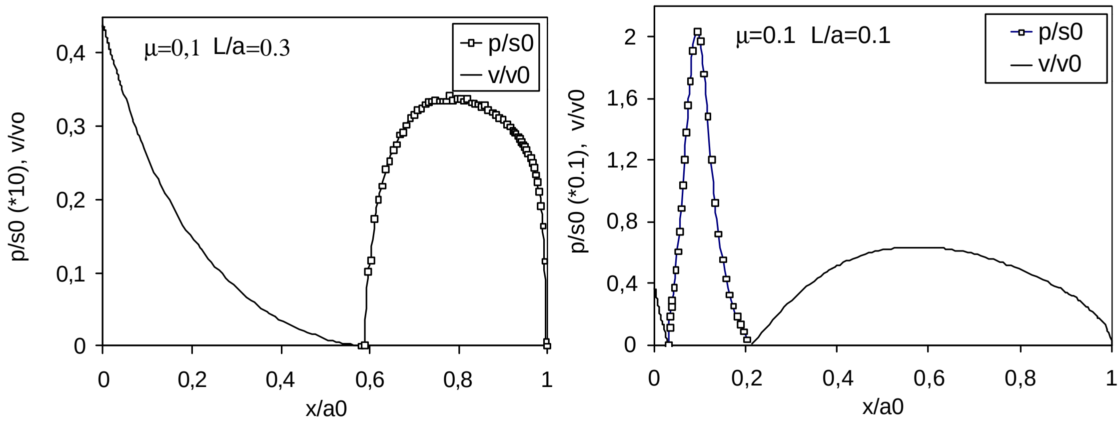

Figure 2. Typical COD (v) and contact pressure distribution a) crack mouth open (K_I=0) b) crack closed in an intermediate portion (K_I>0).

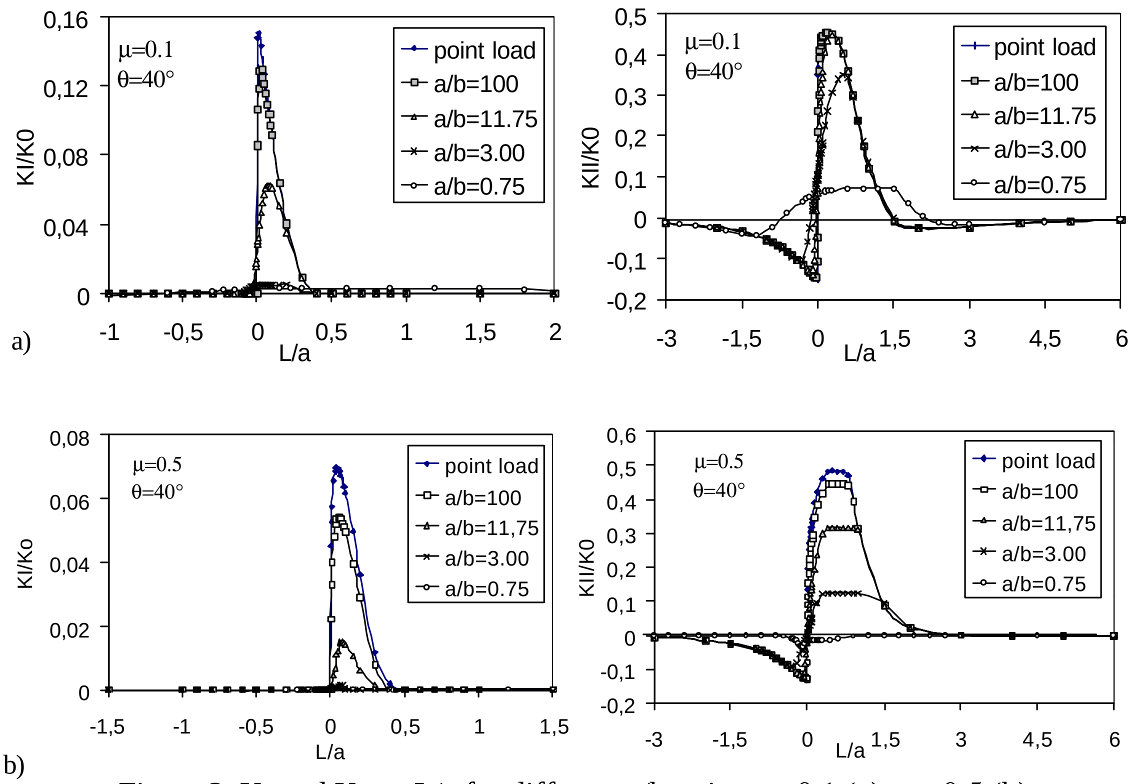

Figure 3. K_I and K_{II} vs L/a for different a/b ratios, μ=0.1 (a) , μ=0.5 (b).

For a Hertzian load, positive values of K_I (even though with low values) can be produced also for $L/a<0$ (only zero values are expected in these conditions under point like load [13]). The plateau of K_{II} vs L/a is a consequence of the sticking in the closed portions. This phenomenon can be observed at the lower a/b ratio for m=0.1, whereas it appears also at larger a/b ratios for m=0.5.

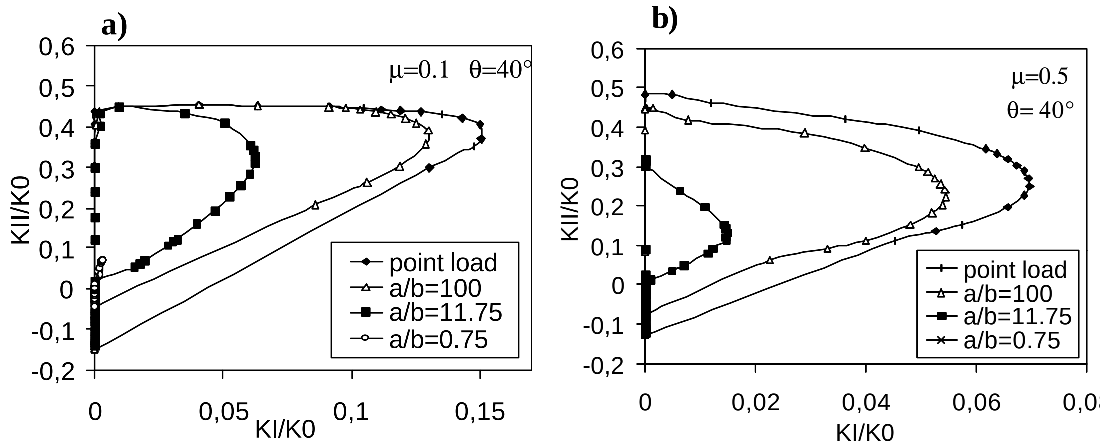

Figure 4. K_{II} vs K_I at different a/b for μ=0,1 (a) and μ=0,5 (b).

By comparing the K_I experienced by the cracks in the two friction conditions, the typical coupling effect between normal displacement and tangential stresses due to the non-symmetrical problem can be observed. A significant difference with respect to the ‘point like load’ solution appears already at relatively high *a/b* values for *m*=0.5, whereas for *m*=0.1 the K_{II} evolutions are similar to that of the point like load solution, up to relatively low *a/b* values. The sticking phenomenon is promoted under a Hertzian load because, when *a* and *b* are comparable, the normal contact stress distributions, arising between crack edges to avoid overlapping, are smoother functions of the posistion [15] as compared to the similar distributions produced under point loading. In Fig. 4 K_{II} vs K_I loci experienced by the crack are reported. The closure effect can be appreciated by comparing the maximum values of K_I and K_{II}.

Effects of Cylinder Movement Direction

As the dynamic friction force contrasts the relative sliding of the crack edges, different SIF histories are induced by the rightward (+) and leftward (-) movement of the cylinder. In Fig. 5 an example of SIFs obtained for *q*= +70; the corresponding loci K_I vs K_{II} are shown in Fig. 6. The effect of the direction is strong, showing a higher crack loading in the case of a cylinder moving rightward. As expected, the difference increases when increasing the friction coefficient.

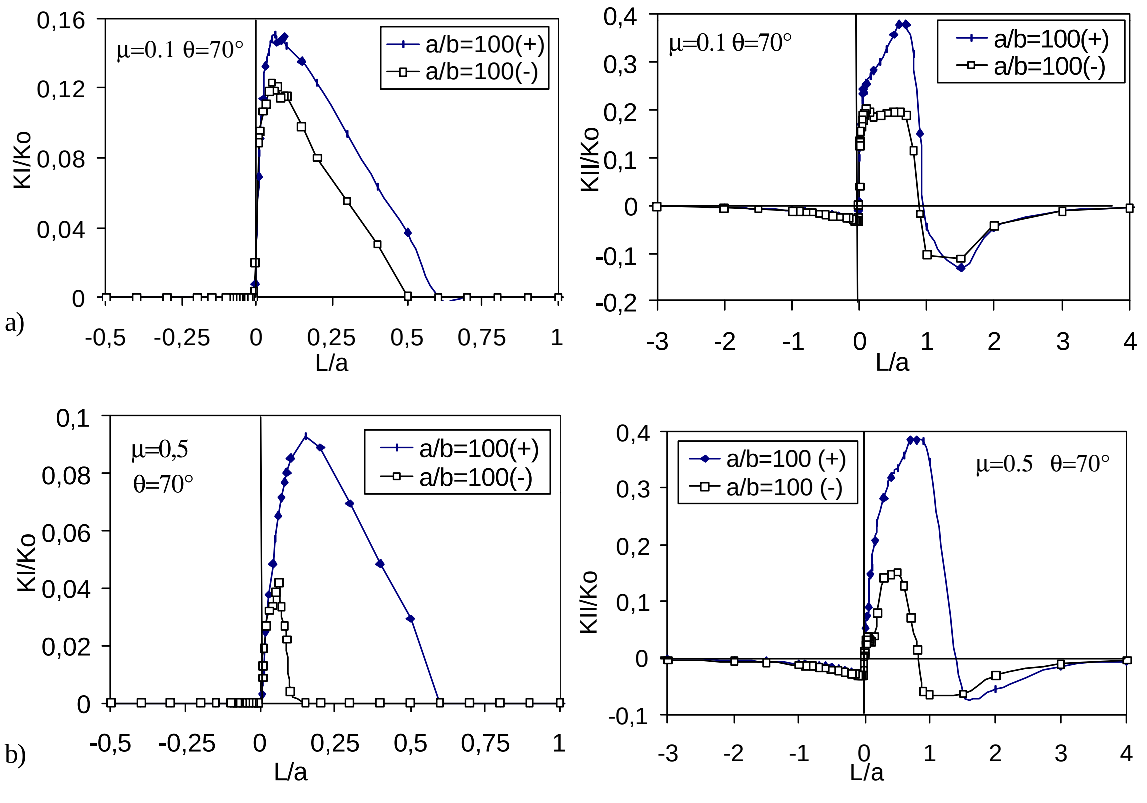

Figure 5. K_I and K_{II} evolutions vs L/a with a friction coefficient μ=0.1 (a) and μ=0.5 (b) for different directions of cylinder movement (+ rightward, - leftward) .

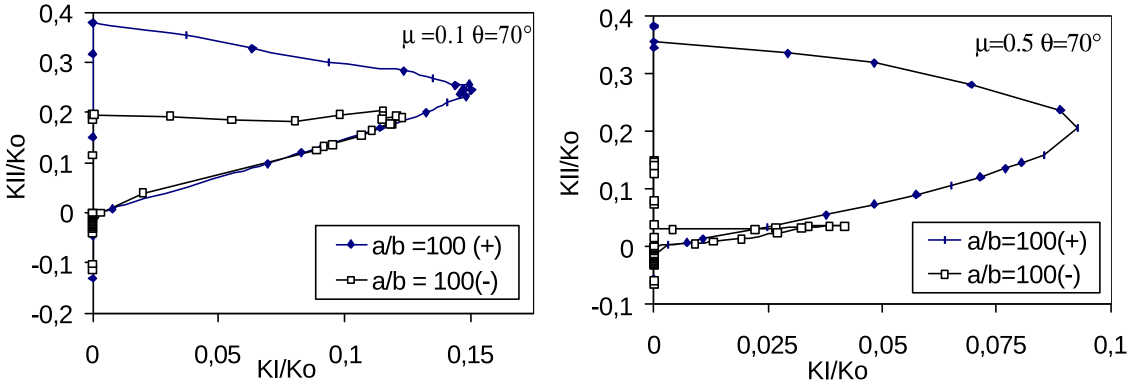

Figure 6. K_I vs K_{II} for the two friction conditions μ=0.1 and μ=0.5.

Effects of the Crack Inclination Angle

The effect of crack inclination was analysed in the whole range θ =[–70° , + 70°] in which the WF was determined with the cylinder moving rightward. The calculated DK_I and DK_{II} ranges are reported in Fig. 7 as a function of the angle θ. For both DK_I and DK_{II}: the positive crack inclination is the most critical, for any *a/b* ratio (including the point like load), particularly with high friction. The most critical condition for DK_I is reached at an angle higher than 60° in dependence of the friction. For crack inclination ranging between 0° and 20°, the maximum values for DK_{II} are observed.

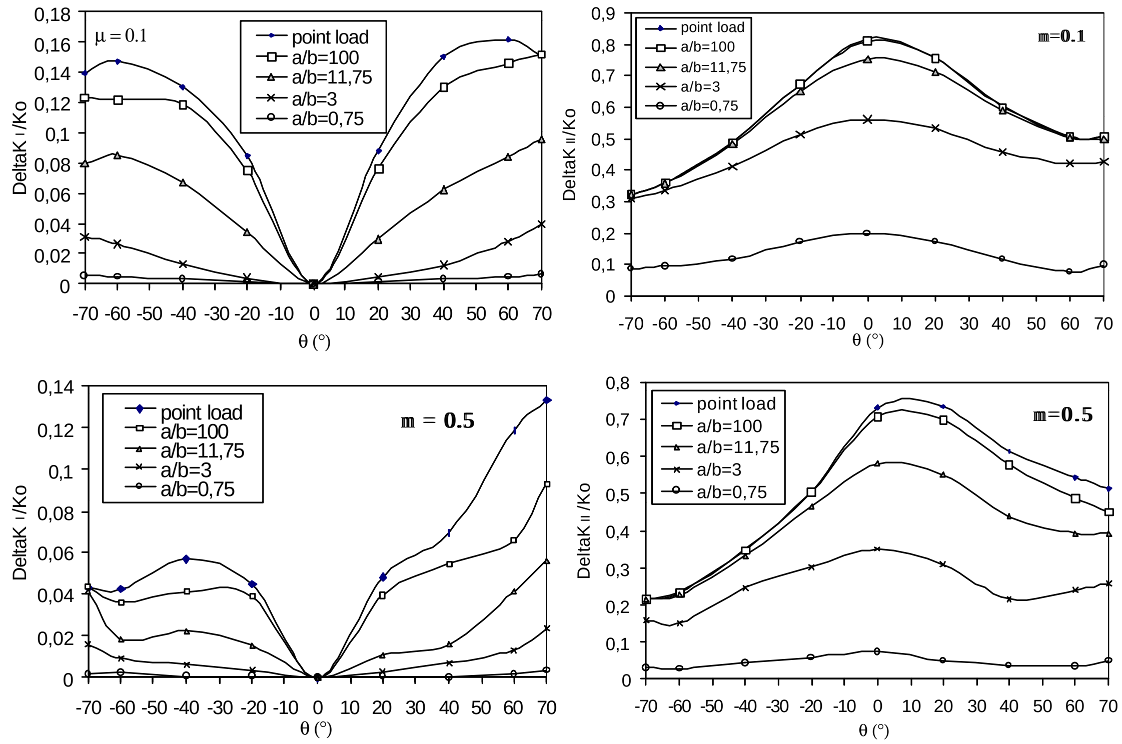

Figure 7. $DeltaK_I$ and $DeltaK_{II}$ vs the crack inclination angle θ for μ=0.1 and μ=0.5.

The friction forces contribute to reduce the difference of DK_{II} as a function of the inclination angle.

In order to give a measure of the fatigue effect experienced by the crack tip when the cylinder moves on the surface, a range of equivalent extremes SIF values, K_{max} and K_{min}, was defined as $\Delta K_{eq} = K_{max} - K_{min}$. By observing that both K_I and K_{II} are positive in the region of Mixed mode, where K_{II} reaches its maximum, the following expression was used $K_{max} = \max\sqrt{K_I{}^2 + K_{II}{}^2}$ where K_I>0; while K_{min} was assumed equal to the minimum value of K_{II}, where K_I=0. The results of the analysed friction conditions are reported in Fig. 8. For any analysed condition, an asymptotic trend toward the 'point like load' solution can be verified for a/b>10. As expected, K_{II} components produce the predominant effect on ΔK_{eq} and the fatigue growth appears to be dominated by the Mode II loading.

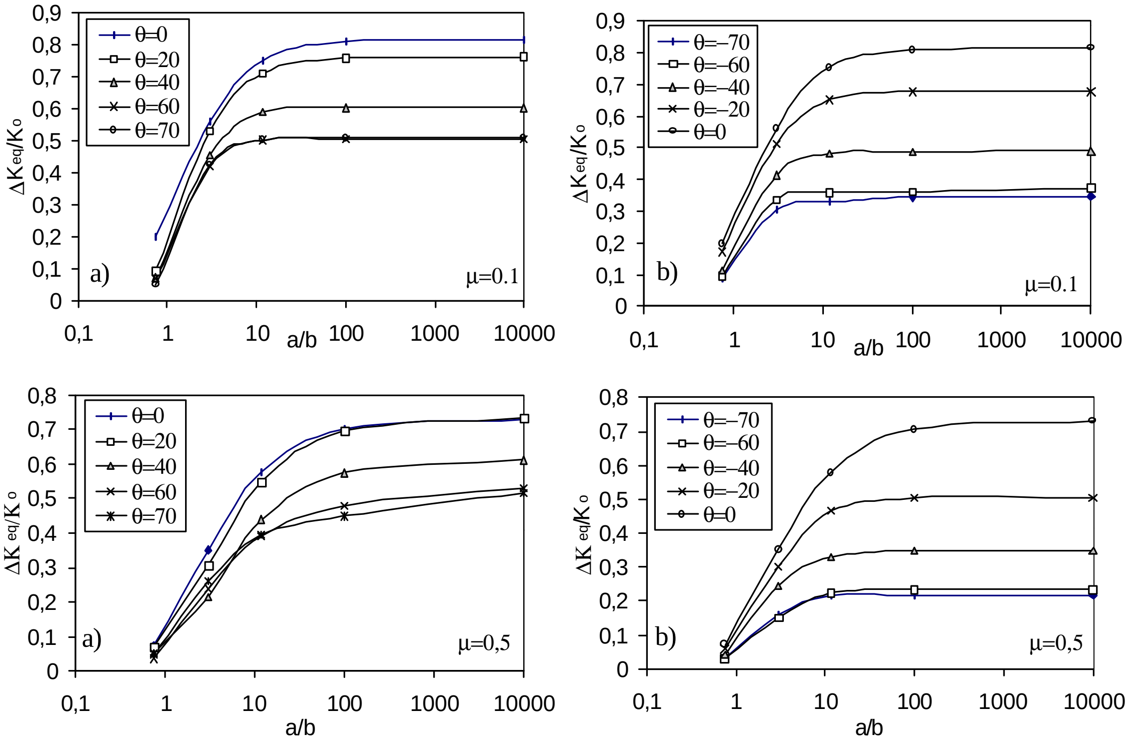

Figure 8. Equivalent SIFs ranges vs a/b for positive (a) and negative (b) inclinations.

CONCLUSIONS

The WF method was demonstrated to be an effective tool for solving the problem of an inclined edge crack loaded by a Herztian contact. An iterative procedure was applied for studying the conditions of partial crack closure. The efficiency of the method allowed for a parametric study to be performed including: the position of the load, the direction of the movement, the friction between the crack edges, the inclination of the crack and the ratio between the crack length and the extension of the contact. A basis for the evaluation of the fatigue crack growth has been established. The proposed method can be easily adapted for studying more complex situations including the presence of residual stresses and friction or elasto-hydrodinamic conditions on the contact. A model including the effect of trapped fluid is under study.

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