Modeling of Fatigue Crack Growth in Concrete

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ABSTRACT. *The fracture behaviour of concrete subjected to mode I crack opening under low-cycle loading was investigated. Two widely accepted non-linear methods were used: the Cohesive Crack Model to analyse the evolution of the process zone and the Continuous Function Model (CFM) to analyse the local hysteresis loop under unloading-reloading. In the original formulation, the CFM assumes that the* tension-softening *law is independent of the number of cycles and that no damage occurs during the so called* inner loops. *The above mentioned hypotheses entail an unrealistically high endurance limit. A more realistic behaviour of the numerical model is obtained by rescaling the tension-softening law with the number of cycles. The numerical results obtained by varying the size ratio and keeping all other geometrical and mechanical dimensionless parameters constant show that the endurance limit is an increasing function of size.*

INTRODUCTION

The performance of concrete structures under cyclic loading is fundamentally affected by the behaviour of the material after cracking. It is well known that concrete presents a diffused damage zone within which micro-cracking increases and stresses decrease as the overall deformation increases. This results in the softening of the material in the so called *fracture process zone* (FPZ), whose size can be compared with a characteristic dimension of the structure. This dimension is not constant and may vary during the evolutionary process. In this context, a numerical method has to be used together with the *cohesive* or *fictitious* crack model as shown by Hillerborg [1].

The interaction between strain-softening and fatigue behaviour is analysed by modeling the hysteresis loop under unloading-reloading conditions.

DESCRIPTION OF THE MICROMECHANICAL MODEL FOR THE PROCESS ZONE

In each point of the fictitious process zone a micromechanical approach to tension softening is used according to a strategy proposed in [2, 3]. Tension softening behaviour appears when the damage in the material has localized along possible fracture planes. This behaviour has been successfully modelled using two- and three-dimensional micromechanical models.

All models provide a relationship between the residual tensile stress carrying capacity and crack opening displacement (COD) as a function of known concrete microstructural parameters (included in factor β), e.g., aggregate volume fraction V_f, Young's modulus E_c, ultimate tensile strength f_t and fracture toughness of the homogenized material K_{Ic}^{hom} (see Fig. 1, left). According to these models, the function is assumed to be:

$$\frac{w}{w_c} = \frac{(K_{Ic}^{hom})^2}{E_c(1-V_f)f_t} \frac{f_t}{\sigma} \left[1 - \left(\frac{\sigma}{f_t}\right)^3\right] = \beta \frac{f_t}{\sigma} \left[1 - \left(\frac{\sigma}{f_t}\right)^3\right]. \tag{1}$$

Figure 1 (left) shows the unloading and reloading loop, according to the so-called *Countinuous Function Model* presented by Hordijk [4]. The unloading and reloading loops are magnified in Fig. 1 (right).

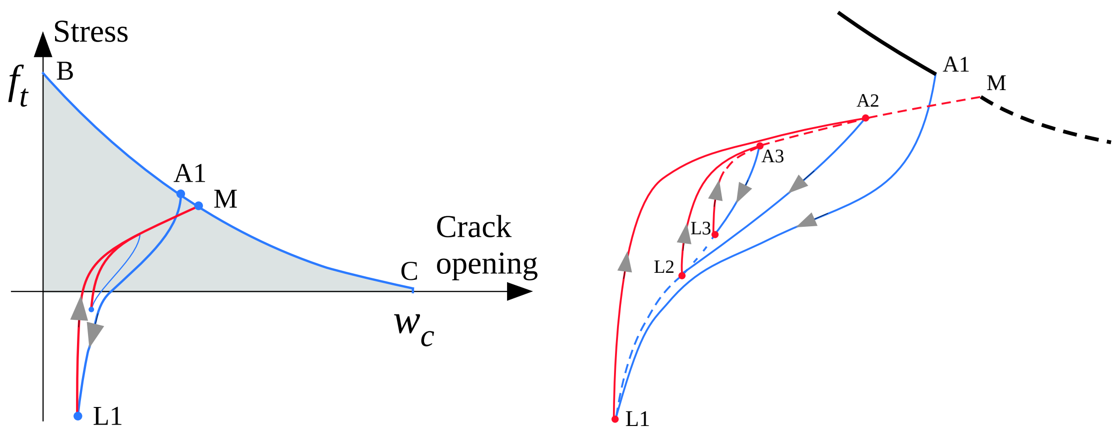

Figure 1. Cohesive stress-COD law (left) and hysteretic unloading and reloading loop according to the Countinuous Function Model (right).

FINITE ELEMENT ANALYSIS

In this work, the continuum surrounding the process zone is taken to be *linear elastic*. All non-linear phenomena are assumed to occur in the *process zone*. When the fictitious crack tip advances by a pre-determined length, each point located along the crack trajectory is split into two points. The virtual mechanical entity, acting on these two points only, is called *cohesive element*: the local behaviour of such an element follows the rules mentioned in the previous section. Each cohesive element interacts with the others only through the undamaged continuum, external to the process zone.

According to the finite element method, by taking the unknowns to be the n nodal displacement increments, $\Delta\boldsymbol{u}$, and assuming that compatibility and equilibrium conditions are satisfied at all points in the solid, we get the following system of n equations with $n+1$ unknowns ($\Delta\boldsymbol{u}$, $\Delta\lambda$):

$$(\boldsymbol{K}_T + \boldsymbol{C}_T)\,\Delta\boldsymbol{u} = \Delta\lambda\,\boldsymbol{P}, \tag{2}$$

where:

- $\boldsymbol{K}_T$: positive definite tangential stiffness matrix, containing contributions from linear elastic (undamaged) elements and possible contributions from cohesive elements having (σ, w) below the curve of Fig. 1;
- $\boldsymbol{C}_T$: negative definite tangential stiffness matrix, containing contributions from cohesive elements with (σ, w) on the curve of Fig. 1;
- $\boldsymbol{P}$: external load vector;
- $\Delta\lambda$: load multiplier increment. During the numerical analysis the stresses follow a piece-wise linear path. To obtain a good approximation of the non linear curves shown in Fig. 1, $\Delta\lambda$ increments have to be small enough.

During the loading phase the stress paths of the cohesive elements are forced to stay on the curve $B-A1$ of Fig. 1 (left), whereas during the cyclic loading phase they are forced to stay on the curves shown in Fig. 1 (right). The stress path $A1-L1-A2$ is called external loop, while the path $A3-L3$ internal loop [4].

Fatigue rupture is reached when the smallest eigenvalue of the tangential stiffness matrix becomes negative: this condition means that the external load cannot reach the upper value P_{upper} any longer.

NUMERICAL RESULTS

The loading procedure analysed is based on two phases. In the first, the external load grows from zero to the fatigue upper level (P_{upper}), a fraction of the peak load (P_{peak}). In the second, a cyclic loading condition is applied, from P_{upper} to P_{lower} and vice versa. In the case of three point bending test, the global response in the nondimensional load-CMOD plane, is shown in Fig. 2.

As the fictitious crack grows, the undamaged ligament reduces and structural compliance increases. The previously described fatigue rupture condition is achieved approximately when the global load path reaches the post-peak branch of the static curve. The results shown in Fig. 2 are obtained for the dimensionless parameters presented in Table 1 where L/H represents the span to depth ratio, a_0/H the notch to depth ratio, $\Delta H/H$ the mesh size ratio, ν Poisson's ratio, $l_{ch} = \frac{E\mathcal{G}_F}{\sigma_u^2}$ Hillerborg's characteristic length, $(H-a_0)/l_{ch}$ the ligament length to characteristic length ratio,

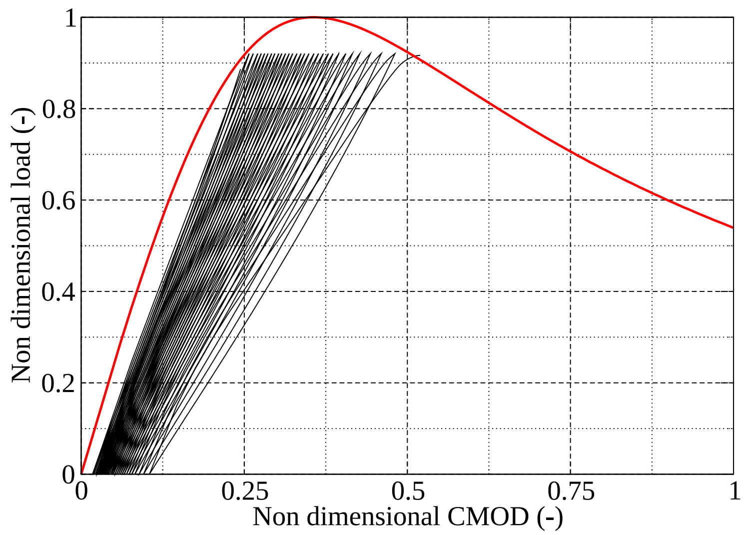

Figure 2. Load vs. CMOD for case $\frac{P_{upper}}{P_{peak}} = 0.92$.

β the Huang-Li tension-softening constant and ε_u the ultimate tensile strain. The upper loading level is $\frac{P_{upper}}{P_{peak}} = 0.92$ and the lower loading level is $\frac{P_{lower}}{P_{peak}} = 0.0$.

Table 1. Geometrical and material parameters.

L/H (-)	a_0/H (-)	$\Delta H/H$ (-)	ν (-)	l_{ch} (-)	β (-)	ε_u (-)
8	1/3	1/160	0.1	0.71	0.055	$7.8\,10^{-5}$

Figures 3, 4, 5, 6 show the stress path in the (σ, w) plane related to a cohesive element near the notch, for four values of the upper loading level. For a high load level (Fig. 3) the number of inner loops is small. Every time a stress path achieves point M in Fig. 1 the damage grows. The sequence continues until collapse occurs. The same comment can be made with reference to Fig. 4. It is worthwhile noting that the number of inner loops, for each external loop, grows when the load level decreases. In the case of Fig. 5, a condition is achieved in which no cohesive element achieves point M in Fig. 1. According to the original Continuous Function Model, no damage evolution occurs and therefore an infinite loop condition occurs. From a theoretical point of view, it means that no cyclic crack growth occurs and therefore the structure can sustain an infinite number of cycles. This physical condition is called *endurance limit*. The comments on Fig. 4 and Fig. 5 show that the endurance limit range is between 0.81 and 0.85.

Since the experimental results of Slowik et al. [5], obtained with a frequency of 3Hz (similar to the frequenzy induced by an earthquake), show an endurance limit range from 0.52 and 0.67, it is possible to conclude that the original formulation of the CFM predicts an unrealistically high endurance limit. A more realistic behaviour of the numerical model is obtained by rescaling the tension-softening law with the number of cycles.

Since the numerical code is able to follow the mechanical quantities up to collapse, the infinite loop condition is overcome by reducing the Huang-Li constant in order not to stop the evolution of damage. From a physical point of view it means that the time scale is changed. For load level 0.81 collapse is achieved for β=0.00539, 2/1000 less than the time independent value assumed (0.0055). For load level 0.76 collapse is achieved for β=0.00528, 4/1000 less than the time independent value assumed (0.0055).

The above mentioned analyses were repeated for $(H - a_0)/l_{ch}$=0.355 (half the previous size). The time independent endurance limite range remains unchanged (between 0.81 and 0.85) but the β reduction, strictly necessary to obtain the collapse, reduces (1/1000 for load level 0.81 and 3/1000 for load level 0.76). It is therefore possible to conclude that the endurance limit is an increasing function of size.

CONCLUSIONS

- The *Continuous Function Model*, developed in the context of the Multi-Layer Beam Model to describe the cyclic behaviour of damaged concrete in the Fracture Process Zone, is useful also in the more general context of the Cohesive Crack Model.

- From a theoretical point of view, if the upper fatigue load level is smaller than the so called *endurance limit* no cyclic crack growth occurs and therefore the structure can sustain an infinite number of cycles.

- In the original formulation, the CFM assumes that the *tension-softening* law is independent of the number of cycles and that no damage occurs during the so called *inner loops*. The above mentioned hypotheses cause an unrealistically high endurance limit.

- A more realistic behaviour of the numerical model is obtained by rescaling the tension-softening law with the number of cycles.

- The numerical results obtained varying the size ratio and keeping all other geometrical and mechanical dimensionless parameters constant show that the endurance limit is an increasing function of size.

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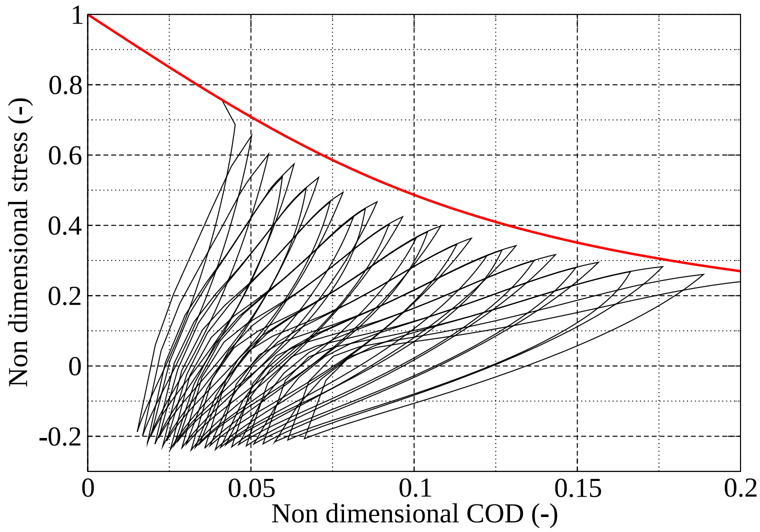

Figure 3. Stress paths for case $\frac{P_{upper}}{P_{peak}} = 0.92$.

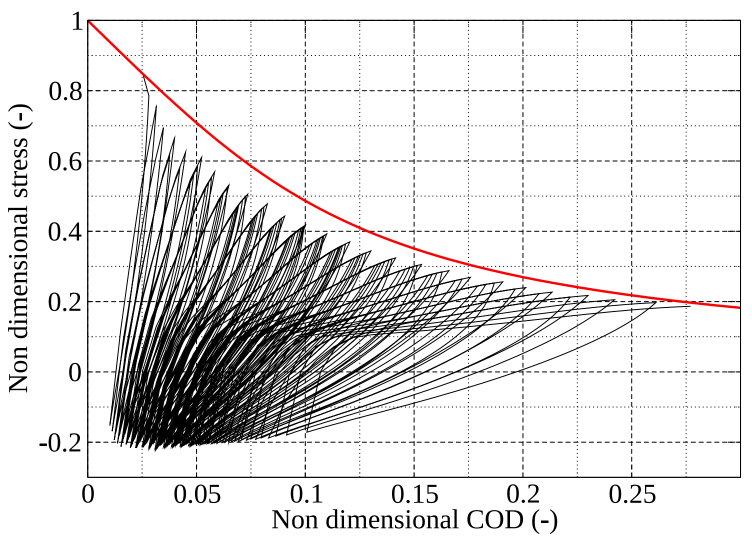

Figure 4. Stress paths for case $\frac{P_{upper}}{P_{peak}} = 0.85$.

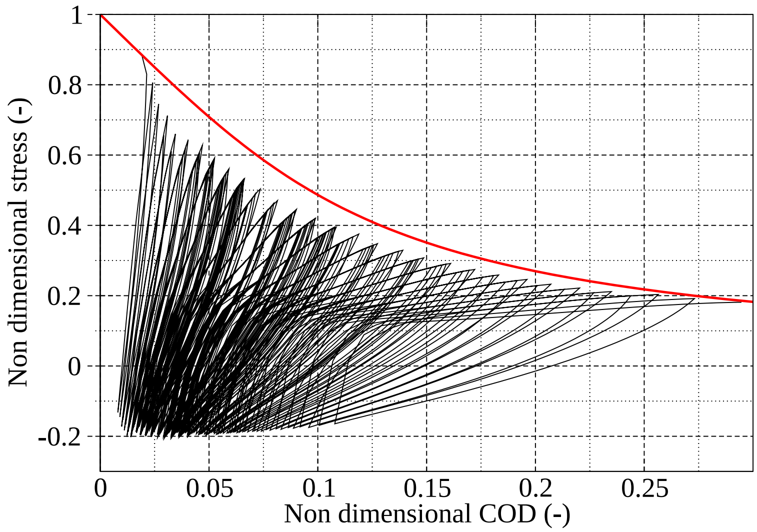

Figure 5. Stress paths for case $\frac{P_{upper}}{P_{peak}} = 0.81$.

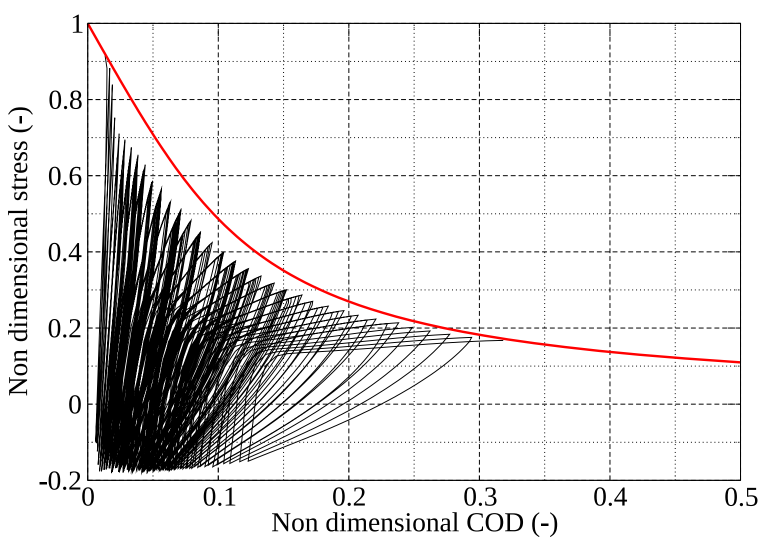

Figure 6. Stress paths for case $\frac{P_{upper}}{P_{peak}} = 0.76$.