

Evaluation of the Crack Growth for an Ellipsoidal Surface Crack Subjected to Fretting Loading

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ABSTRACT. *A curved surface crack was produced during a fretting contact experiment in titanium. Constant normal contact and material bulk loads were combined with a cyclic tangential load. The overall crack shape became a part of an ellipsoid. It was assumed that the fatigue crack growth followed the direction that resulted in pure mode I. Thus, the loading on the curved ellipsoidal crack remained constant as the crack shape was transformed into that of an elliptical edge crack. A parametric crack growth description procedure for the elliptical edge crack was used to model the crack growth. The growth equations were obtained in terms of the change of ellipse's semi axes. Finally, the number of load cycles from an initial small edge crack to the final ellipsoidal crack was determined. A crack path prediction based on the largest normal stress range is included.*

INTRODUCTION

With the term fretting is meant the deteriorating process that is observed at the contact region between mechanical components where a normal load is combined with a small relative tangential displacement. A typical example where fretting is an important design issue is the contacts between blades and rotor discs in the compressor stages of gas turbines. The small relative tangential displacement, referred to as slip, may originate from cyclic small-amplitude variations in the mechanical loading. Slip is observed in a part of the overall contact, typically at the contact boundary. The rest of the contact is characterized by a no-relative displacement, referred to as stick. Hills and Nowell [1] give an extensive description of the fretting phenomena.

The aim of the current work was to predict crack propagation life of an ellipsoidal crack that was produced during fretting experiments. The growing crack was subjected to a complex multiaxial and non-proportional stress state that had its origin in the fretting slip phenomenon. Crack growth was then simulated numerically using a few parameter description procedure based on linear elastic fracture mechanics. This led to the problem of how to analyse a fairly complicated crack growth process in three dimensions.

BACKGROUND

In order to reproduce fretting fatigue in well-defined laboratory conditions, a new experiment was designed. A schematic representation of the experiment principles is presented in Fig. 1a. The specimen had a rectangular cross section with dimensions 15 x 7.5 mm and was loaded with a constant bulk load F . A spherical indenter with radius $R = 400$ mm was pressed against the flat surface of the specimen with a constant normal force $P = 5.44$ kN. The resulting mean contact pressure of 200 MPa is representative for blade-disc applications in gas turbine compressors. According to the Hertz theory, the contact radius a was 3 mm. To reproduce slip at the contact, an alternating cyclic tangential load Q , with peak value Q_{\max} chosen such that no global sliding occurred, was then applied to the indenter. A series of experiments were conducted using different bulk and peak tangential loads. The case with $F = 85$ kN and $Q_{\max} = 2.5$ kN, which corresponds to a bulk stress of 750 MPa and to a stick zone $c/a = 0.76$ (Fig. 1b), is here presented. More details about the experimental set-up can be found in Alfredsson and Cadario [5].

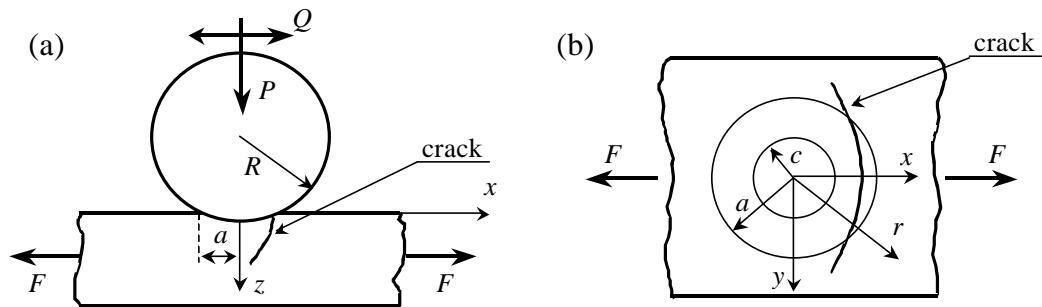


Figure 1. Schematics of the fretting experiment including fretting cracks. a) Lateral view. b) Top view.

Crack Detection

Cracks were detected during fretting tests by strain gauges and acoustic emission measurements. Two strain gauges were positioned outside the contact boundaries in proximity to the expected crack initiation sites. The presence of cracks was captured by the strain gauges through variations in the surface strains due to crack shielding effects.

Acoustic emissions measurements detected transient waves generated during crack propagation due to rapid release of the elastic energy stored in the material. The waves propagated through the material and were transformed into electrical signals by a piezoelectric transducer. An event was counted when the electrical signal exceeded a fixed threshold value. The number of events (total emission count) was then monitored as function of the number of cycles. More details about the acoustic emission technique are found in Miller and McIntire [6].

Stress Field

According to the Hertz theory, a sphere pressed onto a half plane with a normal force P causes an elliptical pressure distribution $p(r)$ in the contact region, see Fig. 2a. When a cyclic tangential load Q is applied, slip is expected to occur at the edge of the contact. Slip covers an annular region with outer radius a and inner radius c' . The inner slip radius c' varies continuously as function of Q from $c' = a$ to $c' = c$ when Q varies from Q_{\max} to $Q_{\min} = -Q_{\max}$ and back, see Mindlin [3] for details. The surface shear stress distribution $q(r)$ in the contact region at a general instant during the load history is presented in Fig. 2a. It can be shown that the same contact shear stress distribution is obtained by superposition of the shear stresses due to three sliding spheres of same radius and contacting the plane in concentric regions with radius a , c' and c respectively. Hamilton [2] derived explicit equations for the stresses beneath a sliding sphere. Thus, the stress solution for the fretting configuration shown in Fig. 1b can be obtained by superposition of the solution for three sliding spheres.

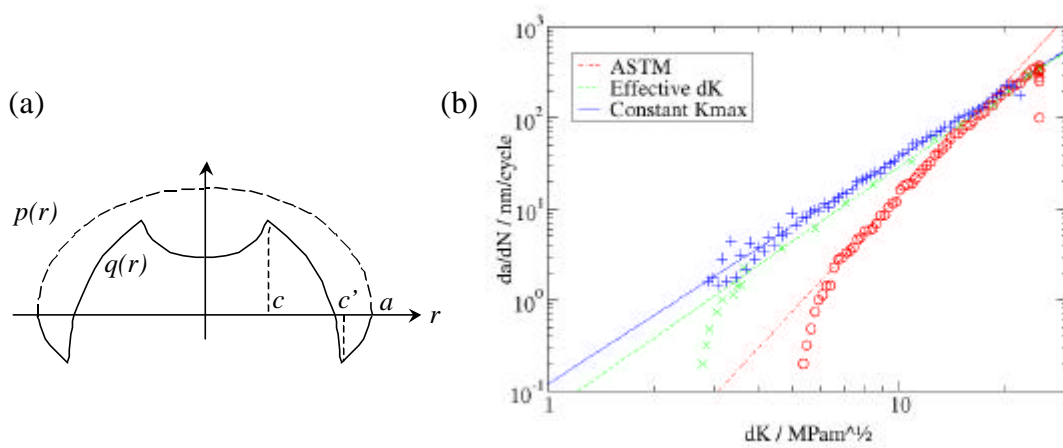


Figure 2. a) Normal and tangential stress distributions at the contact.
b) Experimental crack growth law curves.

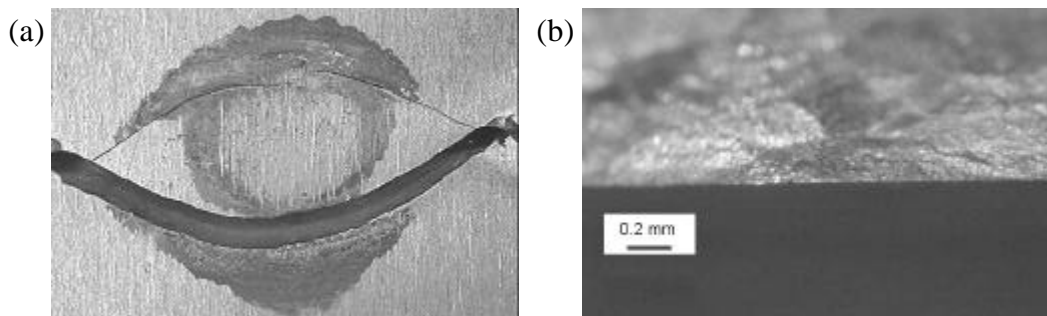


Figure 3. Experimental crack. a) Top view. b) Crack surface at initiation.

Materials

Two $\alpha+\beta$ titanium alloys were used in the experiments. The indenter was manufactured from Ti-6Al-4V and the specimen from Ti-17 which corresponds to a blade-disc contact.

The fatigue crack growth properties, presented in Fig. 2b, were determined according to a power law form with three alternative methods, see for instance Månsson *et al.* [4]. The first method followed ASTM E647. In the second method the crack growth rate was determined as a function of the effective ΔK_I that takes into consideration the crack closure effects. In the third method, $K_{I,max}$ was kept constant, while $K_{I,min}$ was stepwise increased, thus minimizing crack closure effects. The third method did not show any tendency for crack closure effects. It was thus considered the most appropriate for the current analysis, since the large bulk load kept the crack open constantly. Thus, the growth law

$$\frac{da}{dN} = C \Delta K_I^m \quad (1)$$

where $C = 0.12$ and $m = 2.49$ if crack growth rate is expressed in nm/cycle and K_I in $\text{MPa}\sqrt{\text{m}}$, was used in predictions.

In Alfredsson and Cadario [5] the coefficient of friction is determined for the current material combination. The coefficient of friction in the slip zone increases with the number of cycles until it reaches a steady state value after a few thousands cycles. The static coefficient of friction for the unfretted contact and the steady state value in the slip zone were measured to $\mu = 0.45$ and $\mu = 0.83$ respectively.

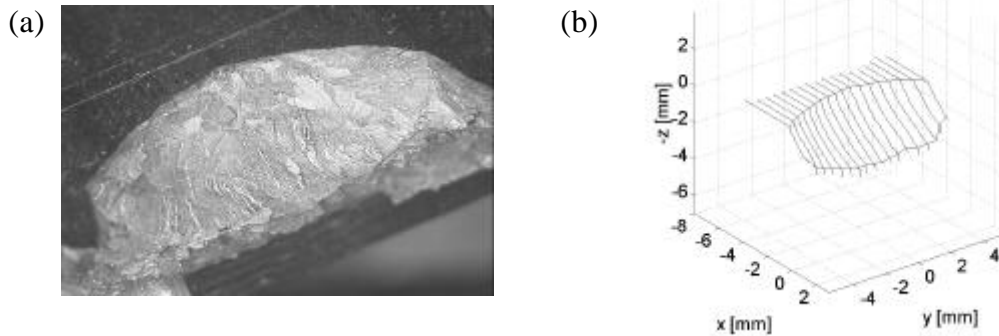


Figure 4. a) Experimental crack surface and b) measured crack profiles.

EXPERIMENTAL RESULTS

One or two symmetric cracks were observed in most of the fretting tests. Figure 3a shows the contact region including two fretting cracks. The central stick region with

unaffected surface structure is surrounded by an annularly shaped fretting scar. The initiation site was determined by investigating the crack surfaces through an optical microscope. An example of the crack initiation site is presented in Fig. 3b.

Figures 3a and 4a present a fretting crack obtained experimentally. The location for initiation was determined by optical microscope investigations of the crack surface. The cracks initiated invariably in the slip zone and in most cases at two locations at either side of the symmetry line (x -axis in Fig. 1b). In the top view the crack propagated towards the specimen edges and in the cross-section it propagated under the contact region, in both cases the propagation followed a curved line. The cracks arrested under the contact close to the plane $x = 0$.

A series of Talysurf profile measurements were performed in order to determine the geometry of the crack surface. The reconstruction of the crack visible in Fig. 4a is presented in Fig. 4b using the measured profiles. It was found that the crack shape could be well approximated by a part of ellipsoid. The coefficients of the ellipsoidal crack description were determined by least square fit to the measured profiles.

Figure 5 presents the experimental results from crack detection. Figure 5a shows the cyclic minimum value of the radial strain at $y = 0$ versus the number of cycles. Figure 5b shows the acoustic emission count versus the number of cycles.

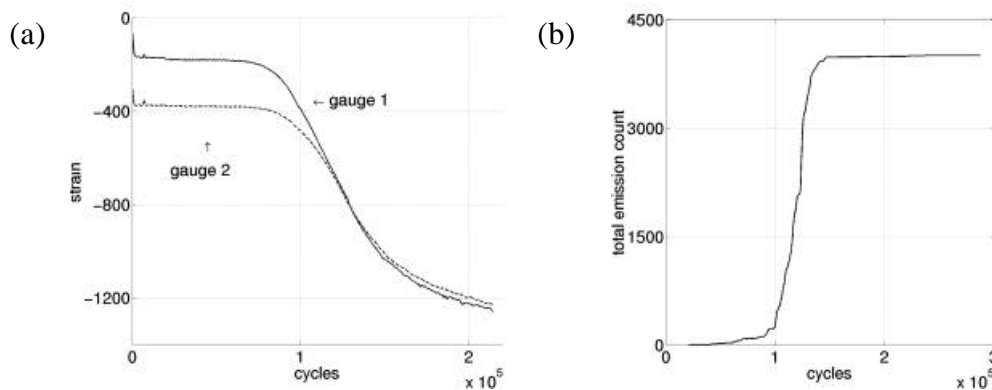


Figure 5. Crack detection measurements. a) Radial surface strain at $y = 0$ just outside the contact. b) Total acoustic emission count.

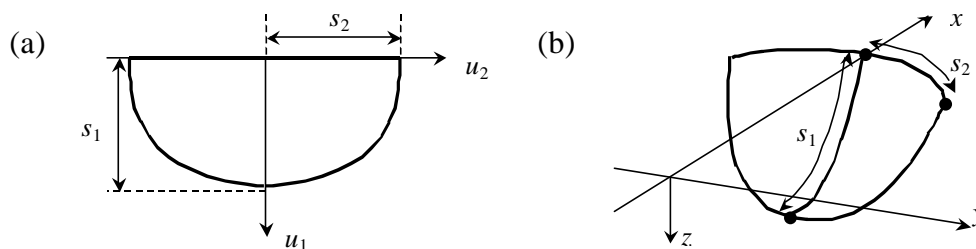


Figure 6. a) Equivalent elliptic crack. b) Ellipsoidal crack.

ANALYSIS

The fatigue growth of the ellipsoidal crack subjected to the fretting loading was simulated numerically. It was assumed that the crack propagated following the direction of pure mode I. Thus, the growth of the ellipsoidal crack was considered analogous to the growth of an equivalent plane edge crack loaded with the normal component of the stresses acting on the real crack surface. The propagation of the equivalent plane crack was simulated numerically by a parametrical crack growth procedure developed by Nilsson [7]. The approximation was made that the geometry of the equivalent crack was reasonably well described by a succession of elliptical configurations. Eq. 1 was used to determine the crack growth rate. The growth driving forces (*i.e.* K_I) were determined along the equivalent crack front using a K_I -database. Nilsson [8] compiles the K_I -database from a large number of FEM computations for plane elliptical edge cracks. By least squares approximation of the crack advance rate, the growth equations could be expressed in terms of the ellipse axes. The crack growth estimations were then transferred back to the three-dimensional ellipsoidal crack.

An initial crack was required as a starting point for the simulations. Its size was determined by inspection of the crack surface. It was possible to distinguish a different surface microstructure in a small region corresponding to the initiation location and therefore determine the crack size at which stage II-mode I crack growth presumably began, see Fig. 3b. The initial crack dimensions were set to 0.25 mm depth and 1.1 mm width.

The crack growth simulations followed seven steps:

1. the analytical form for the ellipsoidal crack was determined and used as description of the experimental crack;
2. an initial crack, shaped as a part of the ellipsoidal crack, was placed on the x -axis at the position where the experimental crack met the contact surface;
3. the stress component normal to the current ellipsoidal crack surface were computed for the load cycle;
4. the equivalent plane crack, with semi axes s_1 and s_2 equal to the crack lengths in cross-sectional and top views, was considered, see Fig. 6;
5. DK_I was computed along the equivalent crack front using the stresses from step 3;
6. the crack growth was determined in terms of s_1 and s_2 using a fixed time step and the ellipsoidal crack was updated;
7. steps from 3 to 6 were repeated until one of the crack dimensions s_1 or s_2 reached the values corresponding to the real crack at the end of the fretting test.

The number of cycles for crack propagation was predicted to $N = 120\ 000$ cycles. The computed crack shape is compared to the measured one in Fig. 7.

DISCUSSION

The results from crack detection were only used qualitatively in the sense that no correlations were done between experimental data and crack size or crack growth rate.

So far, the aim has been to detect the presence of the crack and its growth. From the strain gauges measurements in Fig. 5a it is clearly noticeable that the presence of the crack influenced the strain field by crack shielding. In particular, the larger the crack the larger its influence. The total emission count in Fig. 5b showed a significant increase of event rate during a portion of the experiment, which could be related to crack growth. In general, the measurements indicated the presence of a crack at approximately 50 000 fretting cycles and a propagation life of approximately 100 000-150 000 cycles. Thus, the numerically predicted crack growth life is comparable to the experimentally observed one. The numerical estimate may be regarded as conservative, since crack closure-free growth properties were used in the simulations. The numerical results were sensitive to the initial crack size although the differences were rather small. If an initial crack depth of 0.05 mm is used, then the estimated growth life increased by less than 10%. An explanation for this is presented in Fig. 8a. The crack growth rate is almost constant throughout the simulation and since a change in initial length is very small compared to the total crack length, it should not influence the total growth life significantly.

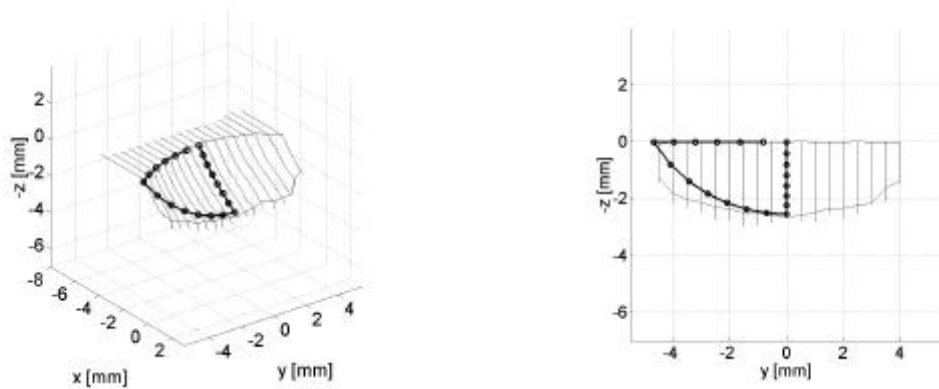


Figure 7. Comparison between the measured (thin line) and the predicted (thick line and circles) crack surface.

The computations presented above simulated the crack growth along the real crack path, *i.e.* the ellipsoidal surface. An approximate crack path prediction can be found by first assuming that the crack follows the direction of largest mode I range, given that $K_{I,max} > 0$. It is then straightforward to assume that crack path can be estimated by the trajectory perpendicular to the largest normal stress range, see Alfredsson and Olsson [9]. The crack path prediction in the surface is compared to some measured profiles in Fig. 8b.

CONCLUSIONS

The method for long crack growth estimations presented in this work provides a rapid estimate of the propagation of three-dimensional ellipsoidal cracks subjected to a

complex loading such as fretting. It can thus be considered as a useful engineering tool for an evaluation of the dangerousness of cracks in a fretted region.

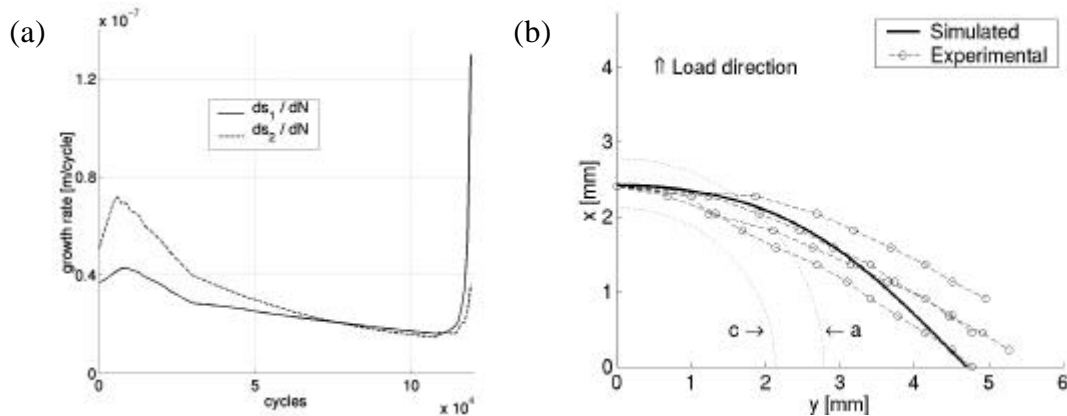


Figure 8. a) Predicted crack velocities. b) Path prediction.

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REFERENCES

1. Hills, D.A. and Nowell, D. (1994) *Mechanics of Fretting Fatigue*, Kluwer Academic Publishers, Dordrecht.
2. Hamilton, G.M. (1983), *Proc. Instn Mech. Engrs*, **197C**, 53–59.
3. Mindlin, R.D. (1949) *J. Appl. Mech.*, **16**, 259–268.
4. Månsson, T., Öberg, H. and Nilsson, F. (2002) Submitted for publication.
5. Alfredsson, B. and Cadario, A. (2003) Submitted for publication.
6. Miller, R.K. and McIntire, P. (1987) *Nondestructive testing handbook (2nd ed.) Vol. 5*, American Society for Nondestructive Testing.
7. Nilsson, F. (1992) *Int. J. Fract.*, **54**, 35–44.
8. Nilsson, L. (1998) SAQ/FoU-Report 98/10.
9. Alfredsson, B. and Olsson, M. (2003) To appear in *Fatigue Fract. Engng Mater. Struct.*