An Integrated Approach for the Prediction of Fatigue Crack Paths in Face Gears for Aerospace Transmissions

Mauro Filippini[1], Marco Giglio, Carlo Gorla and Francesco Rosa

[1] Politecnico di Milano, Dipartimento di Meccanica, Via La Masa 34, I-20158 Milano, Italy, Email: mauro.filippini@polimi.it

ABSTRACT. *In the present paper, an integrated approach for the analysis of fatigue crack propagation paths of cracks originating from tooth root in face gears for aerospace transmissions is presented. A modelling tool has been developed, in order to automatically generate FEM meshes suitable for the calculation of the stress field at the tooth root, once geometrical characteristics of gears are given. 3D planar cracks at the tooth root have been modelled and the stress intensity factors have been derived by means of FEM calculations. Three different types of gear blank design have been analysed: in all cases, the predicted crack paths are directed towards the removal of a single tooth while the rim does not seem to be affected by crack propagation.*

INTRODUCTION

Durability and structural integrity assessment of highly loaded components is becoming more and more important in gear technology. In fact, design of gears may benefit from tools, such as damage tolerance analysis, already employed in the aerospace industry for the design of structural components, in order to ensure that catastrophic failures are avoided, even if undesirable damaging events, such as nucleation of cracks at the root of teeth might occur. In many applications, especially in the aerospace field, the failure of a single tooth by fatigue crack propagation may be considered as a minor failure event with respect to the loss of the complete gear. In fact, if a single tooth fails by fatigue, the power transmission is not necessarily interrupted and an emergency landing might be still possible, while a rim or web failure generally determines a catastrophic event. At the same time, this kind of analyses may be of great help for the lightweight design of gears, because the gear blank may be designed by imposing that no crack propagation through the rim and web should occur. In this way, reduction of weight of the gears may be accomplished by balancing both stiffness and strength needs.

Within the frame of the European Community funded BRITE-EURAM programme titled “The development of face gears for use in aerospace transmission” (FACET), lasting from February 1998 until March 2002, a comparative analysis of different types of face gear designs has been carried out [1]. Three different types of gear design have been analysed, each corresponding to a different geometrical configuration designed by the industrial partners of the programme. In addition, a laboratory test gear design has been also considered. The aim of part of the work performed within the FACET programme has been to ascertain that, for the face gears designed for FACET tests, web and rim breakage are not likely to occur and that bending fatigue failure is therefore limited to teeth breakage.

MODELLING TOOLS FOR CRACK PATHS ANALYSIS

Bending Fatigue Failure of Gears

Bending fatigue failure of gears generally takes places with the propagation of a crack, that could be either nucleated or already existing, under the action of repeated loads. The crack propagates up to the breakage, which occurs when the maximum stress intensity factor of the growing crack reaches the critical value for the given material. The consequences of the failure can be quite different according to the breakage type, that is to say according to the crack propagation path, which can be limited to the tooth root only or, on the contrary, could interest the gear blank, determining a catastrophic event, caused by the interruption of the power transmission [2]. On the basis of such considerations, a gear blank design that avoids the occurrence of rim and web failure is necessary.

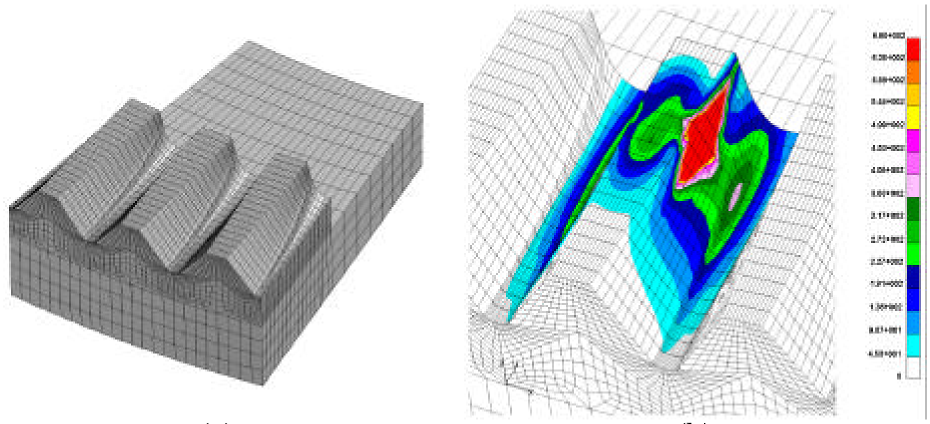
The geometry of a growing crack is generally complicated and its front shape is influenced by the boundary conditions imposed by the surrounding geometry. Even though a semi–elliptical crack front shape may be more appropriate for 3D crack propagation analyses, a planar semicircular crack front has been adopted throughout. Of course, such assumption represents a severe hypothesis on the crack geometry but it may be considered adequate for studying the paths of cracks in the middle part of the tooth. Furthermore, for case hardened gears, the material discontinuity between the hardened skin and the supporting softer material is known to affect crack growth. This effect has been neglected in the present study: the tooth material has been considered as continuous and homogeneous.

Face Gears Design

Within the FACET programme, four different gear designs have been studied. The Agusta gear design is a on centre face gear with spur pinion with non-orthogonal shaft angle, with a medium gear ratio and medium power configuration. Eurocopter (France) gear is a on centre face gear with helical pinion, and it has been chosen for comparison with spiral bevel gear existing configurations. Westland Helicopters Ltd. gear design is a on centre face gear, 90 degrees shaft angle, suitable for high speed and high power configuration. Finally, ZFF (ZF Friedrichshafen) has designed a on centre face gear with spur pinion, suitable for laboratory testing for deriving basic design data on face gears. All the designed gears have been produced and tested in test rigs. Due to confidentiality agreement with the programme partners, further details about geometrical characteristics and arrangement of gears cannot be given here.

Gear Teeth FEM Modelling

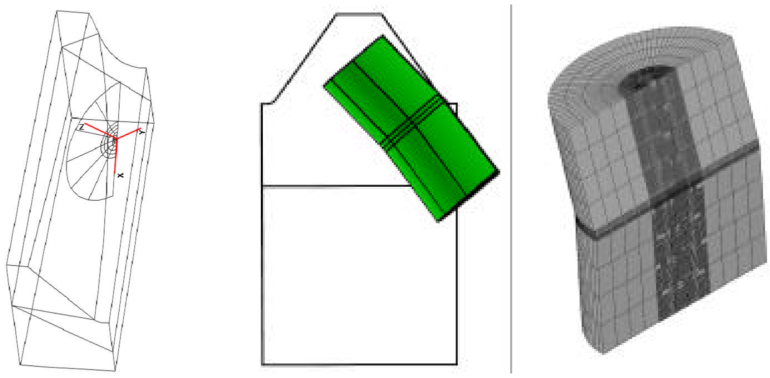
In order to speed up the analysis and design process, an integrated modelling tool has been developed for generating gear teeth FEM models. From the face gear geometrical data (i.e. characteristics of shaper, face gear, pinion, assembly configuration), tooth surfaces (tooth flanks, fillets) are automatically generated and a 'meshable' geometrical model for MSC–Patran is built by means of a Matlab code. Then, the FEM mesh of a convenient portion of the face gear is generated, boundary conditions are imposed and stress field of the (uncracked) gear is calculated (Fig. 1). Due to the need of capturing the (theoretical) stress field singularity at the crack tip, a very fine mesh in the zone surrounding the crack is required. Therefore, the mesh definition would require long time and also the solver runs would be time consuming. Besides, it would be necessary to repeat the procedure for each growth step. For these reasons, the crack feature has been inserted in the model by employing a convenient sub–modelling technique.



(a) (b)

Figure 1. FEM mesh (a) and von Mises stress field (b) of the Agusta face gear (uncracked "global" model).

When the dimensions of the crack are small compared to the face gear tooth, stress and displacement field are influenced by its presence only in a small zone surrounding the crack. Under this hypothesis, the stress and field displacement on a boundary enclosing the crack can be calculated for the "uncracked" global model. In a successive step, by means of the sub-model defined by the aforementioned boundary, the stress state given by the presence of the crack can be derived by imposing the displacement field calculated with the global model as boundary condition of the sub–model. The crack must be small enough so that the general stress and displacement field can be considered as independent from its presence, and the sub-model extension must be large enough so that the conditions on its boundary can be considered not influenced by the crack. These considerations limit the maximum dimension of the crack that can be studied with such a technique. Figure 2 shows the position and orientation of the initial crack: the former is chosen according to the position of the most stressed point at the tooth root and the latter is coincident with the maximum principal stress direction at such point. The sub-model is generated accordingly.



(a) (b) (c)

Figure 2. Position of the initial crack (a) and of the sub-model (b,c) for crack path evaluation; only the sub-model needs to be (re)generated at each growth step.

CRACK PATH EVALUATION

Crack Parameters Evaluation

Several studies have been conducted at the NASA Glenn Research Center, Cleveland, Ohio for assessing the occurrence of detrimental crack paths in gears. In that case, a bundle of fracture mechanics software developed by the Cornell Fracture Group has been used for the evaluation of crack path both in spur [3,4] and spiral bevel gears [5]. The BEM codes were developed to predict 3D crack trajectory by explicitly modelling cracks as geometry features [6]. In the work presented in [3–5], the SIFs are calculated at discrete points along the crack front using the displacement correlation method.

In the present paper, the stress intensity factor K_I of Mode I is assumed as the parameter governing crack growth direction. For a 3D crack, the stress intensity factor K and the *J*-integral values are a function of the position along the crack front: K_I is therefore evaluated from the *J*-integral values which have been calculated versus the position along the crack front. In the case of pure elastic material behaviour, the relationship between pure Mode I Stress Intensity Factor *K* and *J*–integral:

$$J = \frac{K_I^2}{E^*} \qquad (1)$$

may be adopted for the calculation of Mode I SIFs, once *J*-integral is evaluated by means of Finite Element calculations. In the present work, the *J*-integral has been calculated by means of ABAQUS™ FEM solver procedure, where a virtual crack extension is adopted. Thus, it is possible to derive SIFs values for different growth directions (Fig. 3).

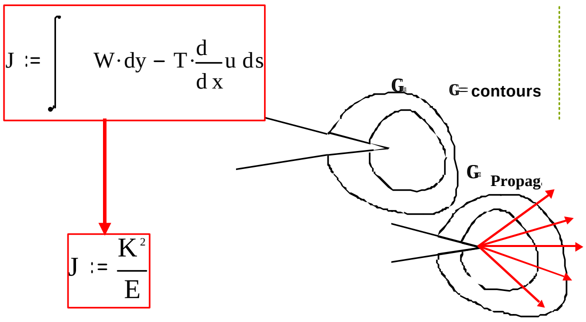


Figure 3. *J*-integral evaluation for crack path prediction.

It has been assumed that cracks may grow in the direction along which the Mode I SIF is maximum: the predicted directions of crack growth are different along the front, and this explains why the crack are not likely to remain in a plane. However, in this approach, where only plane cracks are considered, the crack is extended in the direction of the maximum *K* along the crack front.

The crack path is assessed in the following way: an sub-model of the initial crack with given radius (L_1 = 1.15 mm) is placed the position of the most stressed point at the tooth root and directed according to the maximum principal stress direction. A special routine permits to calculate the normal directions to the crack front and *J*-integral calculation in seven directions (virtual crack extension method), as shown in Figure 4. The direction with the highest value of K_I defines the propagation angle β. For a given value of L_2 (crack front propagation per step), the new value of the crack length L_3 and angle α are calculated. For the successive step, a new sub-model with radius L_3 and crack plane rotation of the angle α is generated.

After the determination of the growing angle, the crack front is extended in that direction of a discrete and constant step value; for the following step, a new larger semi-circular crack is considered, as shown in Fig. 4. It must be underlined that such procedure is not meant to determine the FCG rate. In Fig. 5 the calculated SIF for a given step for the Agusta face gear is plotted against position of nodes along crack front.

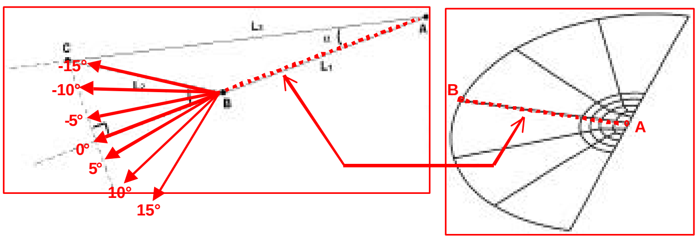
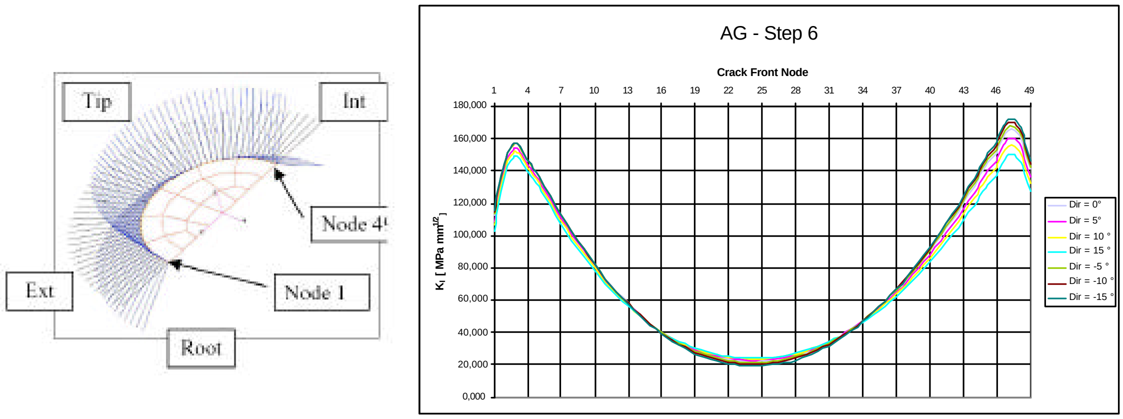


Figure 4. Procedure adopted in the crack path simulation.

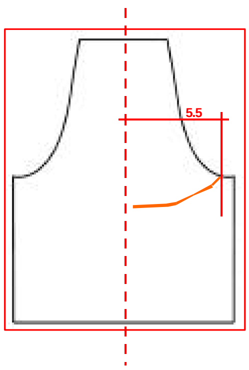


(a) (b)

Figure 5. Normal directions (a) along the crack front (only -15°, 0° and +15° normal directions are shown) and calculated Mode I SIF (b) (Agusta face gear).

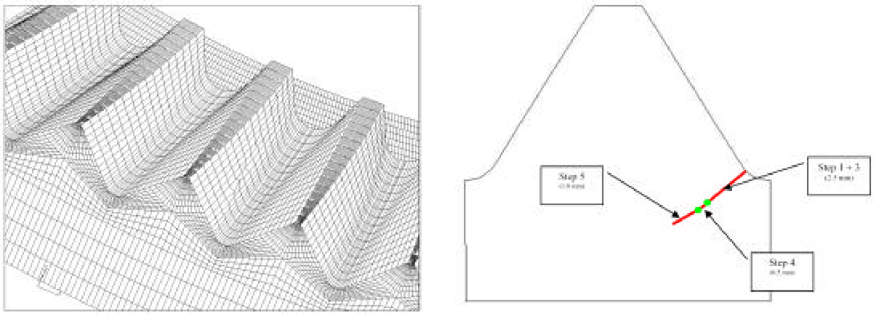
Crack Path Simulation Results

For all the face gears employed in the FACET programme the procedure described above has been applied. In all cases, from the preformed simulations, it may be deduced that the crack paths are directed upwards, involving a single tooth, while the gear blank is not affected by crack propagation (Figs 6 to 9). It may be observed that only for the geometrical configuration of the Agusta face gear it has been possible to evaluate the crack path up to the tooth axis, while in all other cases, the crack path simulation has been arrested after a few steps. The sub-modelling technique employed in this study, though useful from a design point of view, is not suitable for following the evolution of the crack path up to the final failure. Moreover, the procedure presented in this paper is not applicable to the case of corner cracks and the methodology developed in [6] or specialized LEFM FEM based software might be employed instead.



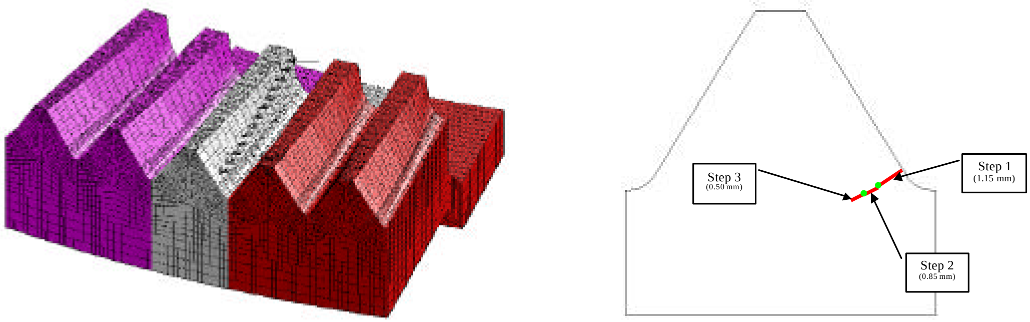
Step no.	Crack radius [mm]	Max K_I [MPa mm$^{1/2}$]	(Relative) Crack direction [deg]	Crack angle [deg]
1	1.15	204.71	-10	39
2	2.00	215.54	0	39
3	2.50	212.12	-10	29
4	3.00	207.42	0	29
5	4.00	191.86	-10	19
6	5.00	171.76	-5	14
7	6.00	159.10	-5	9

Figure 6. Schematic description of crack path in the Agusta face gear: the calculated SIF for each step (max values) and crack path directions are also given.



(a) (b)

Figure 7. FEM mesh of the ECF face gear (a) and schematic description of the crack path (b).



(a) (b)

Figure 8. FEM mesh of the Westland Helicopters Ltd. face gear (a) and schematic description of the crack path (b).

(a) (b)

Figure 9. FEM mesh of the ZFF face gear (a) and schematic description of the crack path (b).

CONCLUSIONS

A method to simulate the crack growth direction of tooth root cracks in face gears have been described and applied to the three face gears designed for FACET full-scale tests and for that designed for laboratory tests. In all cases, the predicted crack paths are directed towards the removal of a single tooth while the gear blank does not seem to be affected by crack propagation. Even if more work will be required for confirming the observed behaviour also in the case of crack propagation that might result from corner cracks nucleating in highly stressed regions at the base of teeth in some particular configurations, the result of the performed analyses validates the design of the face gears of the FACET programme and above all, demonstrates the validity of crack path analyses as an integrated and effective design tool.

ACKNOWLEDGEMENTS

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