Fatigue Crack Propagation and Path Assessment in Industrial Structures

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ABSTRACT*. The present work is part of a development of a complete and fast method, which will allow to determine the crack path in structures submitted to fatigue. Firstly a complete numerical method allows us to calculate the Stress Intensity Factor (SIF), the branching angles, enabling to determine the crack path and the crack growth curves. Secondly, we are developing a simplified approach which will assess, on the one hand the crack path, using a maximum principal stress criterion, and on the other hand, the crack growth, based on the Line Spring method. This approach will determine the crack propagation for part-through crack and is extended to full crack propagation in shell structure. This simplified approach is then used in a global scheme with a multi-initiation criterion based on a multiaxial fatigue assessment by local approach. It's to be noted that the adopted methodology doesn't need any remeshing or numerous iterative FEM calculations in order to obtain the SIF.*

INTRODUCTION

Fatigue crack growth behavior in industrial structures is an important parameter for fatigue life assessment. The crack bifurcation criterion needs a lot of finite element calculations which are time consuming for industrial purpose.

In the present paper, we suggest a few methods to assess the crack propagation under mixed mode fatigue loading in bidimensionnal medium.
The following approaches have been studied :

- A step-by-step complete remeshing method and the determination of the crack growth path by elastic method under monotonic loading. It must be noted that this kind of approach cannot be used in industrial applications because of numerous iterative FEM calculations.
- A very simplified determination of the crack path thanks to the maximum of principal stresses. This is often used in industrial purpose.
- A numerical analyses method, which allows calculating the stress intensity factors and the branching angles based on the use of the Line Spring Method. In this case, the crack path has to be known. The previous criterion has been used

to determine it. However, by calculating K_{II} along the crack path, it is possible to check the chosen path.

Further, a large set of fatigue and static experimental results in mixed mode will be carried out on steel and aluminium alloys. Using these results, we will obtain the crack path and the crack growth curves, which will be compared with the simplified method we present.

STEP BY STEP REMESHING

Plenty of fatigue crack growth studies are concentrated on the pure mode-I loading condition in elastic material where different methods and criteria have been proposed since 1960s.

It is known that when the plastic zone near the crack tip is none negligible and while the crack grows gradually, the dimension of the plastic zone increases. The mechanical characteristics of the plastic zone change also as the number of cycles increases. In this paper, a numerical method which not considering the presence of the plastic zone near the crack tip is used. Our recent developments allow us to neglect the presence of this plastic zone if the loading is less than a third of the material Yield Stress. This doesn't represent a limitation, as far as we are concerned, because we work under endurance fatigue loading.

In order to determine the crack growth path under mixed mode loading, one can use different criteria to calculate the bifurcation angle. For example, the maximum circumferential stress $\sigma_{\theta\theta max}$ criterion (Erdogan and Sih [1]), the maximum energy release rate criterion (Palasniswamy and Knauss [2]), the stationary strain energy density criterion (Sih [3]), the J_{II}=0 (Pawliska et al. [4]) and K_{II}=0 (Cotterell and Rice [5]) criteria (J_{II} is the value of the J-Integral corresponding to pure mode II and K_{II} is the value of the stress intensity factor corresponding to pure mode II), the crack tip opening displacement (or angle) criterion (Sutton et al. [6]), and so on.

In the case of a crack in elastic material, the $\sigma_{\theta\theta max}$ criterion is more often used. According to this criterion, the crack propagates always in the direction of the maximum circumferential stress. Consider the equation of the circumferential stress $\sigma_{\theta\theta}$ as follow:

$$\sigma_{\theta\theta} = \frac{1}{4\sqrt{2\pi r}}\left[K_I(\cos\frac{\theta}{2}+\cos\frac{3\theta}{2}) - 3K_{II}(\sin\frac{\theta}{2}+\sin\frac{3\theta}{2})\right] \tag{1}$$

where r and θ are the polar coordinates from the crack tip.

When $\sigma_{\theta\theta}$ is maxium ($\frac{\partial\sigma_{\theta\theta}}{\partial\theta}=0$), one can obtain:

$$K_I \sin\theta_0 + K_{II}(3\cos\theta_0 - 1) = 0 \tag{2}$$

The bifurcation angle θ_0 can be determined after calculating the values of the stress intensity factors K_I and K_{II} :

$$tg\left(\frac{\theta_0}{2}\right) = \frac{1}{4}\left(\frac{K_I}{K_{II}}\right) \pm \frac{1}{4}\sqrt{\left(\frac{K_I}{K_{II}}\right)^2+8} \qquad (3)$$

In order to determine the crack propagation in mixed mode linear elasticity, we have carried out numerous iterative FEM calculations. A complete remeshing of the structure has been realized at each crack propagation increment of 0.5 mm.

Three bending point and four bending point tests determined by Tohgo K. [7] have been used for our numerical simulations (see Fig. 1). Different loads are applied in order to simulate various mixed mode cases, from pure opening mode (Beam A) to pure shear mode (Beam E). Characteristics and loads of bending tests are given in Fig. 1. Results are given in Table 1 and Figs 2 and 3.

Two angles are determined (see Fig. 4):

- $\alpha 1$: initial direction of the crack
- $\alpha 2$: global direction of the crack

It is to be noted that during 8 mm out of 16 mm of crack growth, the direction of the crack doesn't vary.

COARSE ANGLE DETERMINATION

The following method aims to define a very simple criterion to assess the global path of a crack. It is based on the determination of the maximum principal stresses considering that a crack will always tend to grow in mode I. This approach is commonly used in industrial structures but no criterion has ever been defined.

So we have performed FEM calculations on a very less fine meshing of the beams. Following previous observation using step-by-step remeshing, the direction of the crack doesn't really vary for an 8 mm growth in the beams. The length of the elements we have chosen is about a quarter of the crack length equal to 2 mm. It allows us to have enough elements to assess the crack direction.

The Finite Elements used are linear elastic. The maximum principal stress values determining the crack path correspond to values at the Gauss integration points.

Different values have been determined:

- α_r : angle of the maximum principal stress given by the element at the right of the crack tip.
- α_l : angle of the maximum principal stress given by the element at the left of the crack tip.
- global assessment of the angle by taking into account more elements and assuming a weight which depends on the distance between elements and the crack tip (see Fig. 5).

Table 2 shows the obtained results. It appears that the angle given by the first element at the right of the crack tip gives a good approximation of the crack direction given by the step-by-step method. Furthermore the angle given by a more global assessment is more precise. But there is a limitation, depending on the value of K_I/K_{II}. Indeed when K_{II} is more important that K_I, it's no longer possible to take into account the presence of a large shear component.

A possible industrial approach can be developed in order to determine the crack path in a structure:

- Firstly, we determine the position where the crack can initiate by a multi-axial fatigue damage assessment based on local approach.
- Secondly, we introduce a crack in the structure without remeshing: just split the nodes of the crack.
- Thirdly, we perform a FEM calculation

Two cases can be expected:

- If the angle (α) is greater than 45°, we can assess the crack path thanks to this very coarse approach.
- If the angle (α) is about 45 degrees or less, one notes that K_{II} is greater than K_I. In this case, as the crack will grow in I mode, one deduces that the chosen path is not the correct one. So another crack can be introduced in the model and then calculations are performed again until α>45°.

LINE SPRING METHOD APPROACH

The Line Spring model was introduced in 1972 by Rice and Levy [8,9] in order to estimate the stress intensity factors due to tension and bending in large plates containing part-through surface cracks. The line-spring model is based on the fact that there exists a relationship, at each point along the cut, between local displacements and the loading : the compliance coefficients. Then, from the point of view of the plate, the surface crack is modeled as a through-crack with a continuous distribution of generalized springs connected across the crack : the line springs. It must be noted that compliance coefficients are determined thanks to Tada coefficients [10].

The program used to assess the SIF and the crack propagation is named SISIF, developed by Bureau Veritas and Elf Aquitaine [11]. SISIF aims to evaluate fatigue life in welded structures. The direction of the crack has to be determined a priori. This problem is resolved in considering the maximum principal stress criterion. Using Sisif, one can verify by calculating K_I and K_{II} stress intensity factors that the crack path is the real one i.e. K_{II} tends to zero.

CONCLUSIONS

The present work aims to develop an industrial approach to assess the fatigue life and the crack growth in welded structures.

Using maximum circumferential stress criterion, one notes that the crack path is determined during the first increments of the crack and it's almost corresponding to numerical results obtained by the coarse method. Then this method can assess the crack path when the shear component of the loading is not very important. The chosen path can then be used to compute the crack growth by a fast method based on the Line Spring Approach. This method allows to evaluate the K_{II} along the crack, and verify that it is negligible compared to K_I.

Further developments will deal experimental test investigation in order to validate the chosen method and then to calculate the fatigue life in industrial structures in relation with the crack growth path.

AKNOWLEDGMENTS

The present work is part of a global study performed among a research partnership between GIAT-Industries, CETIM, Bureau Veritas and LERMES.

We especially thank all the other members of our cooperation, which are not mentioned in this article: Michelle Serror, Guy Parmentier and Stéphanie Maherault from Bureau Veritas, Isabel Huther from CETIM and Xiao-Bing Zhang from LERMES.

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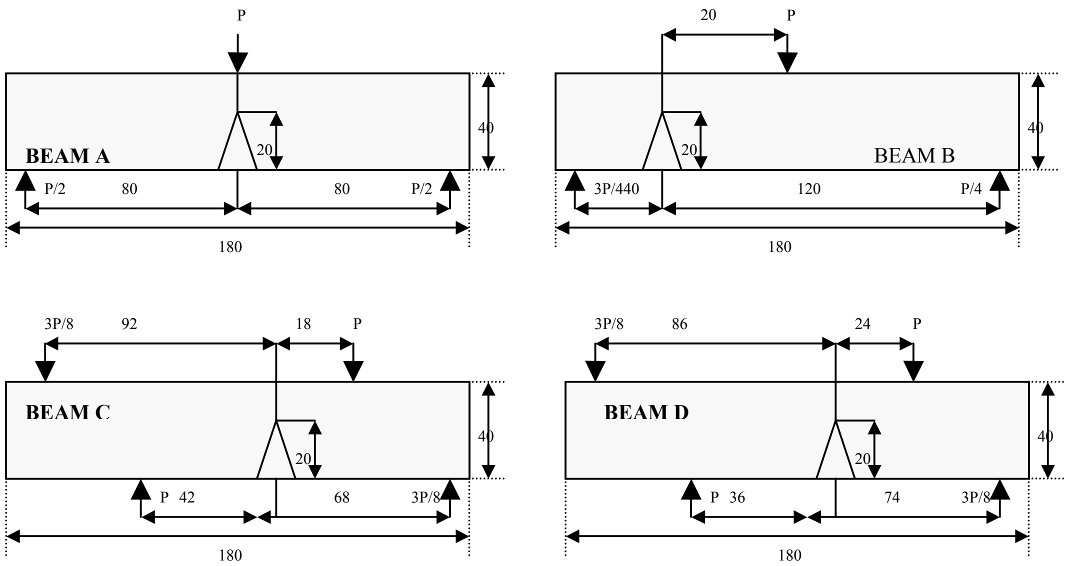

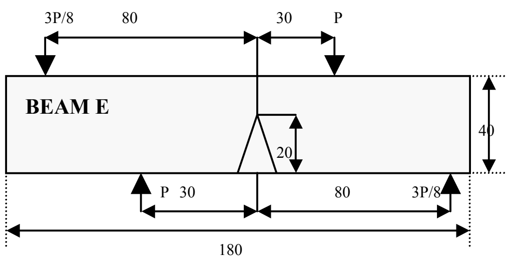

Beam number	KI/KII
A	Infinity (pure mode I)
B	3.3
C	2
D	1
E	0 (pure mode II)

Figure 1. Tests and loadings (dimensions in mm.).

Table 1. Numerical result for Beam C.

K is expressed in MPa√m.
θ is expressed in degrees.

Increment	1	2	3	4	5	6	7	8	9	10
K_I / K_{II}	2,00	108,36	-70,18	-907,99	-122,32	-162,57	-195,76	-119,81	-217,93	-144,33
θ	*-40,19*	*-1,057*	*1,632*	*0,126*	*0,936*	*0,705*	*0,586*	*0,956*	*0,526*	*0,794*
Increment	11	12	13	14	15	16	17	18	19	20
K_I / K_{II}	-177,95	-185,69	-164,56	-138,29	-168,96	-155,82	-164,6	-217,85	-134,69	-168,32
θ	*0,644*	*0,617*	*0,696*	*0,69*	*0,787*	*0,678*	*0,735*	*0,526*	*0,851*	*0,68*
Increment	21	22	23	24	25	26	27	28	29	30
K_I / K_{II}	-135,71	-182,06	-119,05	152,15	-143,86	-121,56	-123,65	-124,17	-145,56	-86,45
θ	*0,844*	*0,629*	*0,962*	*0,753*	*0,796*	*0,943*	*0,926*	*0,923*	*0,787*	*1,325*

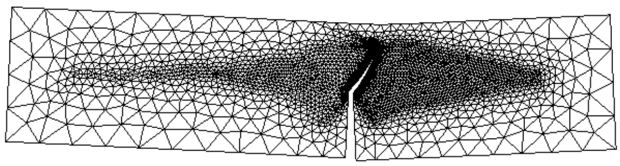

Figure 2. Crack path for Beam C.

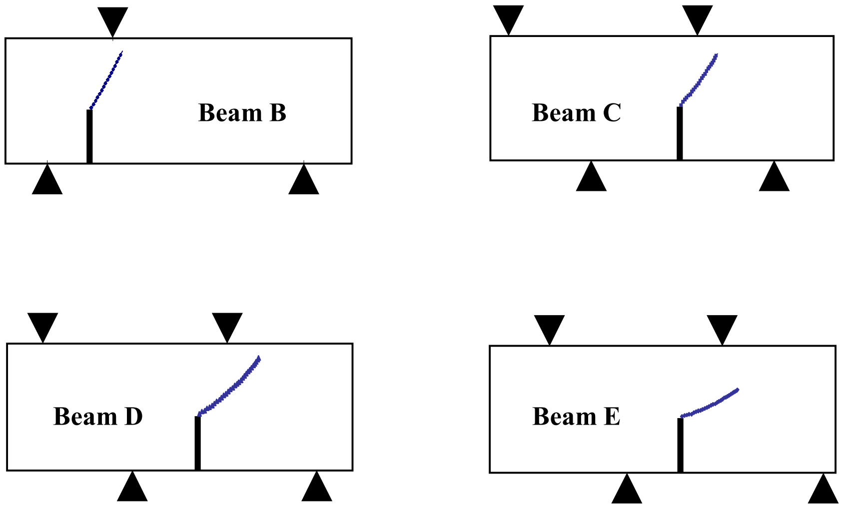

Figure 3. Crack path for Beam B to Beam E.

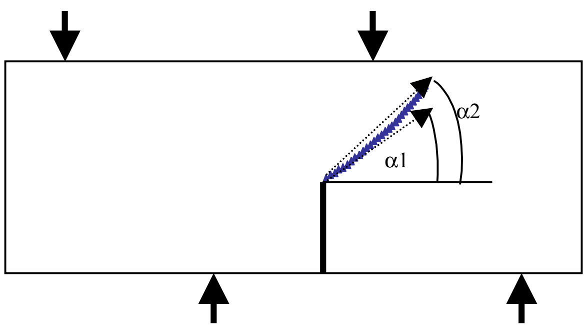

Figure 4. Angles determination α1 and α2.

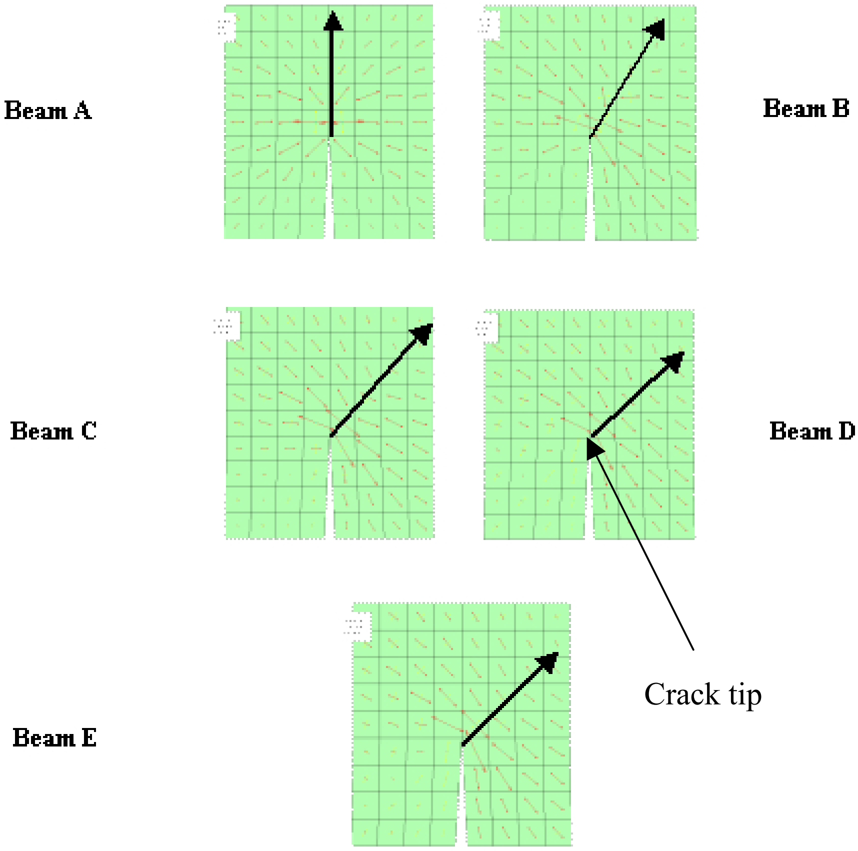

Figure 5. Estimation of the crack growth direction.

Table 2. Angle assessment (in degrees).

BEAMS	K_I/K_{II}	1.Step by step calculation		2.Maximum principal stress criterion		3.Maximum principal stress criterion Coarse Assessment
		$\alpha 1$	$\alpha 2$	α_r	α_l	
BEAM A	Infinity	90	90	90	90	90
BEAM B	3.2	60.88	65	65	71,6	60
BEAM C	2	49.81	57	51	69.45	50
BEAM D	1	37.88	43	46.5	63.44	45
BEAM E	0	19.20	20	43.5	64.11	45