Crack Growth in Bimaterial Joints with Crack Perpendicular to Interface

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ASTRACT. *This paper regards the determination of the SIF in plate composed of two materials with crack starting from free edge and growing in direction perpendicular to the interface. The RGB photoelastic method was used for the following capabilities: full field investigation, rapidity, appreciable precision. The result shows that the influence of the interface is limited within a little area around interface then a greater part of crack path is good fitted by the homogeneous Irwin-Westergard formulation. The general form of the relation between SIF and a-dimensional crack length presents a maximum value of* K_I *for* a/h_1 *dependent to the geometric* h_1/h_2 *ratio. The next goal is to find an analytical relation that describe correctly the stress field at crack tip in a zone near the interface.*

INTRODUCTION

In the present work, the crack growth through bimaterial joints is investigated under uniform displacement (Fig. 1) and for different geometric conditions in order to determine the influence of the interface discontinuity upon the stress field at the crack tip. Several numerical works are reported in the literature [3], while only few experimental works have been found [7], [9]. After a preliminary FEM analysis, an experimental stress analysis approach has been adopted in this work. In fact, the correct choice of parameters to build the FEM model is of difficult estimation for some mechanical cases, while an experimental approach would not be affected by these problems. The influence of the interface over the stress field at the crack tip is investigated.

White light automatic photoelasticity has been chosen allowing rapid, real time analyses without requiring special skills. The comparison of experimental and numerical results allow to improve modelling the problem.

The photoelastic fringes have been analysed by Newton-Rapson algorithm and the overdeterministic Sanford-Dally algorithm, which allow sufficient rapidity and precision in analyses with great number of points.

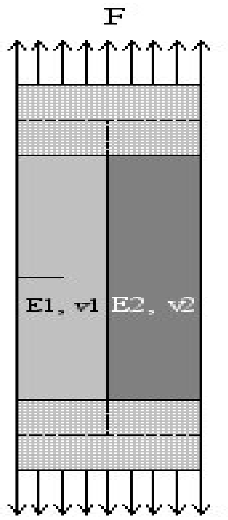


Figure 1. Uniform displacement load condition.

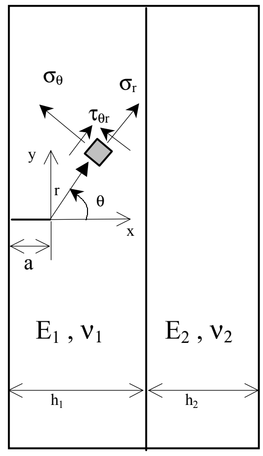


Figure 2. Component of stress in polar coordinate.

FORMULATION OF THE PROBLEM

As already done in several papers, the general problem of investigation of the stress field in dissimilar media can be approached by eigenfunction [1] to determine the singularity character of the extensional stress near crack tip.

Starting from a short crack length departing from a free edge, it has been simulated a crack growth by artificially cutting the material with a very thin blade, whose stress singularities have been analysed in all steps and finally stress intensity factor was evaluated. In this work it has been determined also the crack length at which the stress field is too different from that of homogeneous semi-infinite plates, represented by Irwin equations [2]:

$$
\begin{aligned}
\sigma_x &= \frac{1}{\sqrt{2\pi \cdot r}} \cdot \left[K_I \cdot \cos\frac{\vartheta}{2} \cdot \left(1 - sen\frac{\vartheta}{2} \cdot sen\frac{3\vartheta}{2}\right) - K_{II} \cdot sen\frac{\vartheta}{2} \cdot \left(2 + \cos\frac{\vartheta}{2} \cdot \cos\frac{3\vartheta}{2}\right)\right] - \sigma_0 \\
\sigma_y &= \frac{1}{\sqrt{2\pi \cdot r}} \cdot \left[K_I \cdot \cos\frac{\vartheta}{2} \left(1 + sen\frac{\vartheta}{2} \cdot sen\frac{3\vartheta}{2}\right) + K_{II} \cdot sen\frac{\vartheta}{2} \cdot \cos\frac{\vartheta}{2} \cdot \cos\frac{3\vartheta}{2}\right] \\
\tau_{xy} &= \frac{1}{\sqrt{2\pi \cdot r}} \cdot \left[K_I \cdot sen\frac{\vartheta}{2} \cdot \cos\frac{\vartheta}{2} \cdot \cos\frac{3\vartheta}{2} + K_{II} \cdot \cos\frac{\vartheta}{2} \left(1 - sen\frac{\vartheta}{2} \cdot sen\frac{3\vartheta}{2}\right)\right]
\end{aligned} \qquad (1)
$$

and the dependence of this critical length on the geometry aspect ratio a/h1 (Fig. 2) is also investigated. To do this we have considered three different configurations of bimaterial joint with three different h1/h2 value: 1/3, 1, 3. For all geometry, when a/h1 =1 the stress field can be expressed according with K.Y. Lin e J. W. Mar [3]. The use of a complex variable approach (Muskehelishvili [5]) has demonstrated that the stress field at the crack tip can be expressed by:

$$\sigma_x + \sigma_y = 4\,\mathrm{Re}\{\phi^{|}(z)\} \quad \sigma_y - \sigma_x + 2i\tau_{xy} = 2\cdot\{\bar{z}\phi^{||}(z) + \psi^{|}(z)\} \tag{2}$$
$$u + iv = \frac{1}{2\mu}\{\eta\phi(z) + z\overline{\phi^{|}(z)} - \overline{\psi(z)}\}$$

then generalised stress and displacement can be written as:

$$\phi_1(z) = a_1 z^{\lambda},\ \eta_1(z) = c_1\lambda z^{\lambda-1},\ W_1(z) = c_1 z^{\lambda} \tag{3}$$
$$\phi_2(z) = a_2 z^{\lambda},\ \eta_2(z) = c_2\lambda z^{\lambda-1},\ W_2(z) = c_2 z^{\lambda}$$

where a_1, a_2, c_1, c_2, are complex coefficients that can be evaluated through the satisfaction of boundary conditions. λ represents a minimum value solution of the following eigenfunction:

$$\lambda^2\left(-4\alpha^2 + 4\alpha\beta\right) + 2\alpha^2 - 2\alpha\beta + 2\alpha - \beta - 1 + \left(-2\alpha^2 + 2\alpha\beta - 2\alpha + 2\beta\right)\cos\lambda\pi = 0 \tag{4}$$

Finally, the Cartesian stress can be expressed by:

$$\sigma_x = \sum_n Re\{\lambda_n r^{\lambda_n - 1} b_n [(2f_R - g_R)\cos(\lambda_n - 1)\vartheta - (2f_I - g_I)sen(\lambda_n - 1)\vartheta +$$
$$-(\lambda_n - 1)(f_R\cos(\lambda_n - 3)\vartheta - f_I\, sen(\lambda_n - 3)\vartheta)]\}$$

$$\sigma_y = \sum_n Re\{\lambda_n r^{\lambda_n - 1} b_n [(2f_R + g_R)\cos(\lambda_n - 1)\vartheta - (2f_I + g_I)sen(\lambda_n - 1)\vartheta +$$
$$+(\lambda_n - 1)(f_R\cos(\lambda_n - 3)\vartheta - f_I\, sen(\lambda_n - 3)\vartheta)]\} \tag{5}$$

$$\tau_{xy} = \sum_n Re\{\lambda_n r^{\lambda_n - 1} b_n [g_R\, sen(\lambda_n - 1)\vartheta + g_I\cos(\lambda_n - 1)\vartheta +$$
$$+(\lambda_n - 1)(f_R\, \mathrm{sen}(\lambda_n - 3)\vartheta + f_I\cos(\lambda_n - 3)\vartheta)]\}$$

Until now there was found no formulation of the stress field for the case of a crack lying entirely in one material, therefore it is important to investigate this case to understand the critical condition of a bimaterial joint.

EXPERIMENTAL SETUP

Aluminium 6061 T6 as high Young modulus material and PSM-1 polycarbonate as other material have been chosen to manufacture test specimens. These materials have been chosen because their joint presents the same Dundurs bimaterial constants of that in classical epoxy/glass-fiber composite materials and because they can be joined with epoxy Araldide® D glue that has the same elastic characteristic of PSM-1, thus permitting to neglect the presence of a third material. An appropriate grip-head was realised in test specimens to avoid non homogeneous load distribution. Material properties are reported in Table 1.

Table 1. Elastic properties

Material	Young module [Mpa]	Poisson ratio
6061-T6	64000	0.33
PSM-1	3220	0.35

Tests have been performed by using an tensile test machine together with a classical white light circular polariscophe. Photoelastical images have been recorded with a Nikon D1 digital camera which was fully driven by a computer.

PHOTOELASTIC METHOD

The fotoelastic fringes where then analysed with the *frange 3.0* software, which was developed by the authors [10], [11], providing a rapid, simple, full photoelastic analysis. This software, which implements a Sanford and Dally least square method with the overdeterministic Newton-Rapson approach, permits to evaluate stress intensity factor value(SIF). This method is based on the assumption to express the stress function with a Cartesian formulation:

$$\sigma_x (K_I, K_{II} , \sigma_o, r , \theta) \quad \sigma_y (K_I, K_{II}, \sigma_o, r , \theta) \quad \tau_{xy} (K_I, K_{II}, \sigma_o , r,\theta) \tag{6}$$

we can write the maximum in plain shear stress as:

$$(\sigma_y - \sigma_x)^2 + (2\tau_{xy})^2 = (2\tau_{max})^2 \tag{7}$$

the fundamental photoelastic relation is:

$$2 * \tau_{\max} = \frac{N * f}{t} \tag{8}$$

Then it can be written:

$$f_j (K_I , K_{II} , \sigma_o) = (\sigma_y - \sigma_x)^2 + (2\tau_{xy})^2 - \left(\frac{Nf}{t} \right)^2 = 0 \tag{9}$$

if we give an estimation of K_1, K_2, σ_o we are sure to obtain $f_j (K_1 , K_2 , \sigma_o) \neq 0$ but we can evaluate the correction with expansion of this relation to a Taylor series:

$$(f_j)_{i+1} = (f_j)_i + \left(\frac{\delta f}{\delta K_I} \right)_i * \Delta K_I + \left(\frac{\delta f}{\delta K_{II}} \right)_i * \Delta K_{II} + \left(\frac{\delta f}{\delta \sigma_0} \right)_i * \Delta \sigma_0 \tag{10}$$

Now we must have $\left(f_j\right)_{i+1} = 0$ and then we rewrite the previous equation in a matrix compact form for n points:

$$|f| = \|A\| * |\Delta K| \tag{11}$$

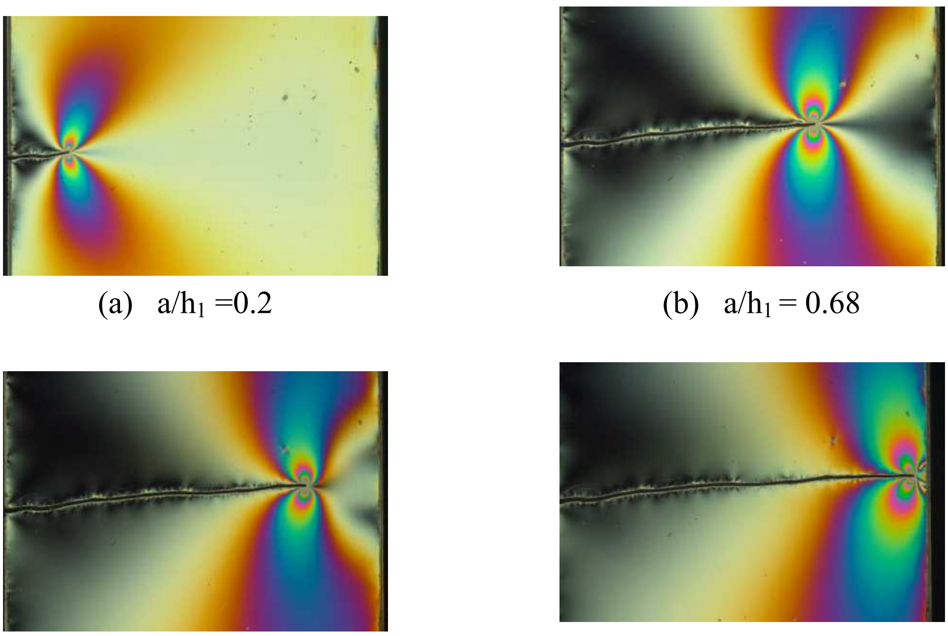
if n > 3 then ||A|| is not square and then we can solve the equation with the least square method:

$$|d| = \|c\| * |\Delta K| \tag{12}$$

Equation (a) gives the value of correction by means of which the values of K_I, K_I, σ_o can be evaluated. After some iterations this method allows to evaluate a correct value of the SIF.

EXPERIMENTAL RESULTS

To obtain the SIF we have chosen experimental sample points lying along a line with 0.3< r < 3 mm and θ = ±45°. In fig. 3: (a), (b), (c), (d) there are reported the white light isochromatic fringes for the h_1/h_2=3 test specimen, loaded at σ_{Nom} = 0.17 MPa.



(a) a/h_1 =0.2

(b) a/h_1 = 0.68

(c) a/h_1=0.8

(d) a/h_1 = 0.98

Figure 3. Isocromatic fringes pattern for h_1/h_2 =3 case.

In this test crack has grown along 14 loading steps, obtaining K_I, K_{II}, σ_o for each step using a Irwing-Westergard formulation Eq. (1) which is plotted in Fig.4. In this graph we can note that the highest value of K_I is around 0.7 a/h_1. At $a/h_1 \approx 0.8$ the values of K_{II} and σ_o start to grow and an analysis upon the singular exponent $(\lambda_n\text{-}1) \approx 0.6$ (Eq. 4) shows that the influence of the interface is relevant, see Fig. 5, therefore the simple formulation of the homogeneous case (Eq. 1) is no longer sufficient. In Fig 5 it is reported the relation between the singular exponent $(\lambda_n\text{-}1)$ and the dimensionless crack length we can note a rapid increment of the $(\lambda_n\text{-}1)$ at $a/h_1 = 0.7$. Over this value, the singular exponent $(\lambda_n\text{-}1)$ decrease to a value of 0.328, which represents the theoretical

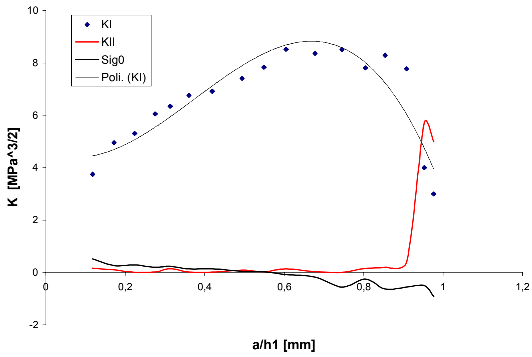


Figure 4. Relation between the SIF and the a-dimensional crack length $h_1/h_2 = 3$.

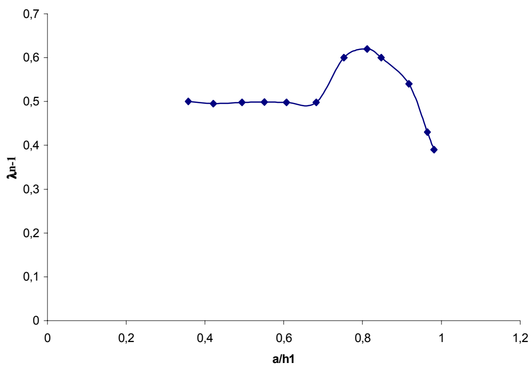


Figure 5. Relation between the λ and the a-dimensional crack length $h_1/h_2 = 3$.

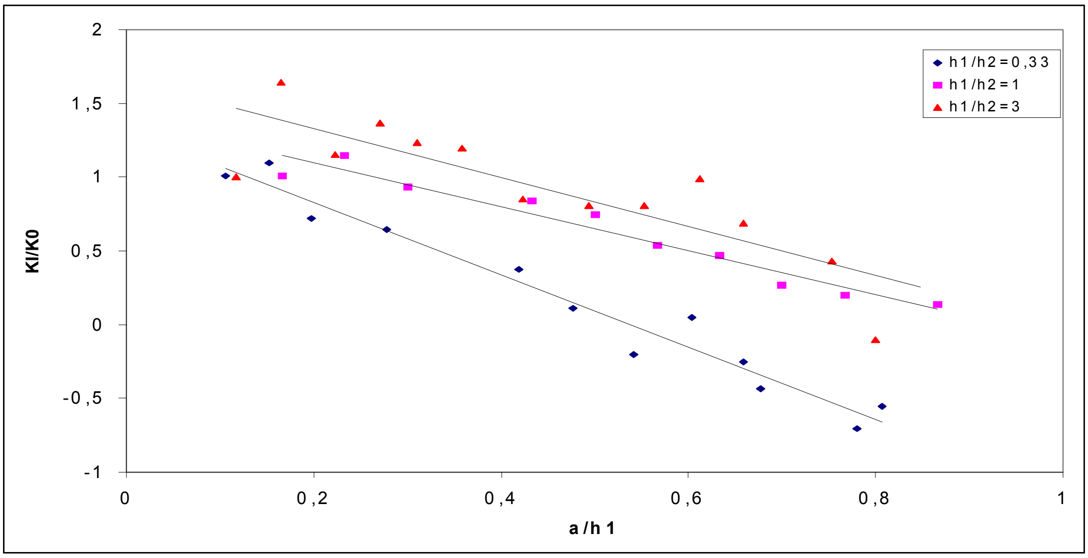


Figure 6. Relation between K_I/K_o and dimensionless crack length.

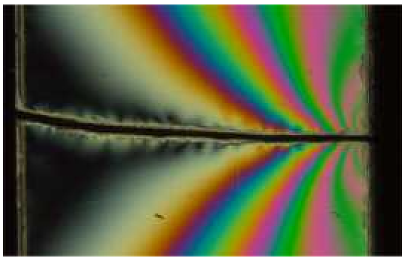


Figure 7. $a/h_1=1$ $h_1/h_2=3$.

value for this bimaterial specimen ($a/h_1=1$). The photoelastic fringe pattern for this last case is presented in Fig 7.

The relation between K_I/K_o are plotted in Fig 6 and the dimensionless crack length a/h_1 for all tree test specimens used, we can note that the general type of the relation is well fitted by a linear relation. In this figure we can note that the effect of the interface is to decrease K_I value in comparison to homogeneous case (in this case K_I/K_o increases his value with a/h_1) and that this effect is greater for the case where $h_1/h_2 = 0.33$.

CONCLUSIONS

In the present paper, the influence of bimaterial interface in the stress field near crack tip has been investigated. A defect growth along direction orthogonal to the interface has been considered, which grows until crack tip has reached the interface. Experimental photoelastic tests have been performed to study the behaviour of crack growth.

Results from tensile tests have shown that when whole h_1 thickness is cracked the following propagation step occurs along the direction parallel to the interface. The load value at which the above described propagation occurs, increases when the h_1/h_2 ratio will decrease.

In all photoelastic tests K_I values and for this reason crack growth velocity presents tree different phases: first, an increasing zone, then it stabilises and, at last, a decreasing zone; this last zone is smaller when the h_1/h_2 ratio will decrease. A decreasing SIF value has been observed when h_2 thickness increases. Results in term of fatigue life improvement are satisfactory.

For all width ratio of the two material joined, K_I values present a maximum value between $0.5 < a/h_1 < 0.7$, and up to an a-dimensional crack length of $a/h_1=0.8$, the singularity exponent is substantially the same of a homogeneous mono material sample (Eq. 1). Over this value the influence of the interface discontinuity became evident and it was not possible to use Irwin-Westergard formulas to calculate SIF. At the same time, until $a/h_1=1$, the K.Y. Lin e J. W. Mar [3] approach was not valid; in fact this approach gives $(\lambda_n\text{-}1) = 0.32$ and this value is evaluated only when material 1 is thoroughly cracked (Fig.7). In all tests we have found that the stress field have singular character up to a r < 3 mm.

Finite element method simulation of these tests have shown that stress field is singular up to r < 0.3 mm even if we have increased discretization up to $3.3{*}10^{-4}$ as $\frac{\text{lower length element}}{h_1}$. This result suggests that finite element method could not be the better one to analyse stress field with singular distribution.

The next goal will be to find an analytical relation which describes correctly the stress field at crack tip in a zone near the interface.

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Nomenclature

$z = x + iy = r^{i\vartheta}$ $\phi(z)$, $\psi(z)$ two complex function, $\phi^{|} = \frac{d\phi}{dz}$

μ = shear modulus, ν = poisson ratio

$\eta = 3 - 4\nu$ in plain strain $\eta = \frac{3-\nu}{1+\nu}$ in plain stress

$\alpha = \frac{\frac{\mu_1}{\mu_2} - 1}{1+\eta_1}$; $\beta = \frac{\mu_1}{\mu_2}\frac{1+\eta_2}{1+\eta_1}$ Dundurs bimaterial constant

Angular function in material 1:

$$f_R(\lambda_n) = \frac{\beta[1 + \alpha + (2\alpha\lambda_n - \alpha)\cos \lambda_n\pi]}{D(\lambda_n)} \quad f_I(\lambda_n) = \frac{\beta[(2\alpha\lambda_n - \alpha)sen\, \pi\lambda_n]}{D(\lambda_n)}$$

$$g_R(\lambda_n) = \frac{\beta\left[\lambda_n + \alpha\lambda_n + \left(2\alpha\lambda_n^2 + \alpha\lambda_n - \alpha\right)\cos \lambda_n\pi + (1+\alpha)\cos 2\lambda_n\pi\right]}{D(\lambda_n)}$$

$$g_I(\lambda_n) = \frac{\beta\left[\left(2\alpha\lambda_n^2 + \alpha\lambda_n - \alpha\right)sen\, \lambda_n\pi + (1+\alpha)sen\, 2\lambda_n\pi\right]}{D(\lambda_n)} \quad D(\lambda_n) = 1 + 2\alpha + 2\alpha^2 - 2\alpha(1+\alpha)\cos \lambda_n\pi - 4\alpha^2\lambda_n^2$$

Angular function in material 2:

$f_R = 1$, $f_I = g_I = 0$

$$g_R(\lambda_n) = \lambda_n - \cos \lambda_n\pi - \frac{\beta\left[\alpha + 2\lambda_n - \left(1 + 2\alpha - 4\alpha\lambda_n^2\right)\cos \lambda_n\pi + (1+\alpha)\cos 2\lambda_n\pi\right]}{D(\lambda_n)}$$

K_1, K_2 are generalised stress intensity factor

$K_o = \sigma\sqrt{\pi a}$

a= crack length; σ_o = remote tension; r,θ = polar co-ordinate of generic point

N = order of the isochromatic fringe; *f* = stress optical constant; t = thickness of the plate

$|d| = \|A\|^t * |f|$; $\|c\| = \|A\|^t * \|A\|$