Fatigue and brittle fracture propagation paths of planar tunnel-cracks with perturbed front under uniform remote tensile loadings

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ABSTRACT. *The aim of the paper is the determination of the propagation path of an in-plane perturbed tunnel-crack embedded in an infinite isotropic elastic body loaded in pure mode I through some uniform stress applied at infinity. The crack advance is supposed to be governed by the stress intensity factor, through Paris' law in fatigue and Irwin's criterion in brittle fracture. In practice, the advance is computed in both fatigue and brittle fracture by a Paris' type law, Irwin's criterion being regularized by a procedure analogous to the "viscoplastic regularization" in plasticity. The necessary determination of the stress intensity factor along the front for all the stages of propagation is achieved by successive iterations of Bueckner-Rice weight-function theory, that gives the variation of the stress intensity factor along the crack front arising from some small arbitrary coplanar perturbation of the front. It is closely linked to previous numerical works of Bower and Ortiz [1] and revisited by Lazarus [2] for closed crack fronts. It is adapted here to the tunnel-crack, that is to two crack fronts. In fatigue, two kinds of propagation paths can be distinguished depending on the width of the perturbation. If this width is less than a critical value, the perturbation vanishes, so that the front becomes rectilinear (stable case). Otherwise, the perturbation increases so that the front becomes more and more perturbed (unstable case). The numerical study allows us, however to study the non-linear effects due to the finite size of the perturbation. It is noticed that these effects enhance the instability and slacken the come-back to the rectilinear configuration in the stable case. In brittle fracture, it appears that the perturbation increases in width and then in amplitude; that is, it behaves in a kind of unstable manner whatever the initial perturbation.*
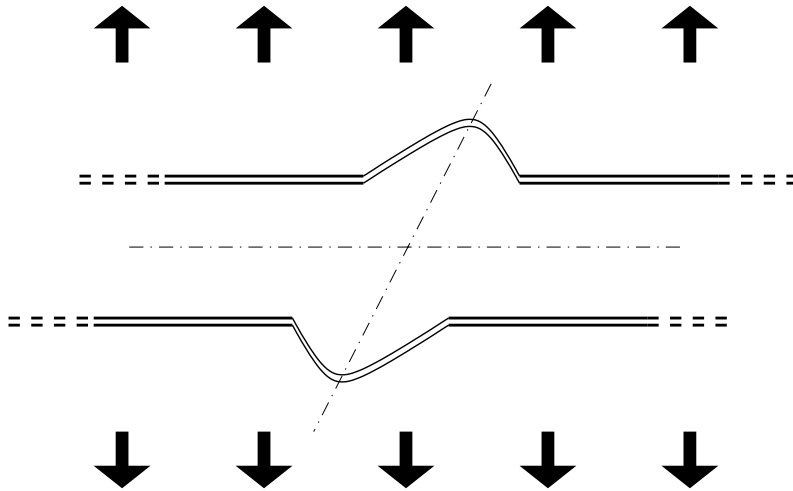
INTRODUCTION

Figure 1: Perturbed crack front

Let us consider a planar tunnel-crack with perturbed front, embedded in an infinite isotropic elastic body loaded in pure mode I through some uniform stress applied at infinity (see figure 1). The aim of this paper is to study the in-plane propagation path in both fatigue and brittle fracture, and in particular the stability of the rectilinear configuration of the crack front versus inplane perturbations. The crack advance is supposed to be governed by the stress intensity factor (SIF), through either Paris' law in fatigue, or Irwin's criterion in brittle fracture.

The first part briefly describes the numerical method developped : Paris' type law is written so as to deal with both fatigue and brittle fracture and the Bueckner-Rice weight function theory is used to compute the necessary determination of the SIF along the front at all stage of propagation. The main advantage of this iterative method (see Lazarus [2]) is that only one dimensional integrals along the crack front are involved so that only the one dimensional meshing of the crack is needed, instead of the 3D meshing of the whole body as in the FEM. At last, to illustrate the method, we present the results obtained for the stability problem of the straight configuration of the front, in which the non-linearity effects can be investigated through this numerical approach.

NUMERICAL PROCEDURE

Let us study the propagation path of a planar crack with arbitrary front $\mathcal{F}$ subjected to uniform remote loading σ_∞ (see figure 2). For numerical purpose, the propagation path is described by very closed to each other steps. $\delta a(s; \tau)$ denotes the advance between steps

τ and $\tau + 1$ and $K(s; \tau)$ the SIF at step τ, both at point s of the front.

Next section shows how the advance $\delta a(s; \tau)$ is obtained in both fatigue and brittle fracture by a Paris' type law, involving the SIF. Second section shows how to update this SIF using Bueckner-Rice weight function theory.
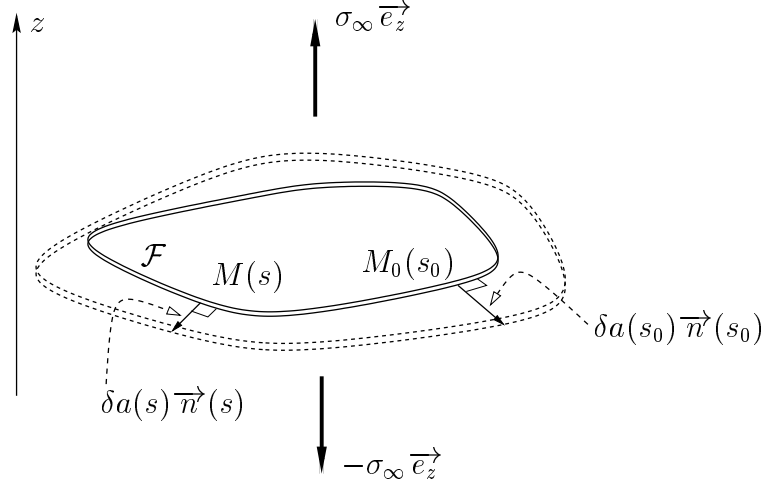

Figure 2: Small (magnified on the figure for the sake of visibility) perturbation of the crack front $\mathcal{F}$.

Propagation Laws

In *fatigue*, propagation can be described by the Paris' law :

$$\frac{\partial a}{\partial \tau} = C(\Delta K)^n$$

where $\partial a / \partial \tau$ denotes the rate (τ could be interpreted as "kinematical time") of crack advance at any position on the crack front, ΔK the amplitude of the cyclic mode I SIF at that point, and C and n positive material constants.

This could be written :

$$\delta a(s; \tau) = \delta a_{max}(\tau) \left(\frac{K(s; \tau)}{\|K\|_\infty (\tau)} \right)^n \tag{1}$$

where $\|K\|_\infty (\tau) = \sup_{s \in \mathcal{F}} K(s; \tau)$, $\delta a_{max}(\tau)$ the maximum distance between step τ and $\tau + 1$.

In *brittle fracture*, Irwin's criterion reads :

$$\begin{cases} K < K_c & \Rightarrow \quad \text{no propagation } (\delta a \equiv 0) \\ K = K_c & \Rightarrow \quad \text{possible propagation } (\delta a \geq 0) \end{cases}$$

The K_c factor is known as tenacity of the material.

To avoid such a sequential and irregular criterion, one could adapt the Paris' law, letting the exponent go to infinity :

$$\delta a(s, \tau) = \delta a_{max}(\tau) \left(\frac{K(s; \tau)}{\|K\|_{\infty}(\tau)} \right)^n , \ n \to +\infty \tag{2}$$

So that $\delta a(s; \tau)$ tends to 0 with n if $K(s; \tau)$ is smaller than a value linked to K_c (here $\|K\|_{\infty}(\tau)$) or could be non zero if $K(s; \tau)$ is equal to this value.

Bueckner-Rice Weight Function Theory
Let us suppose now that $K(s; \tau)$ is known at step τ. The advance is then given by (1) or (2). Rice[3] has shown that the SIF at step $\tau + 1$ changes by the amount $\delta K(s_0)$ given, to the first order in the perturbation, by the formula :

$$\delta K(s_0) = \frac{1}{2\pi} PV \int_{\mathcal{F}} \frac{W(s, s_0)}{D^2(s, s_0)} K(s) \big[\delta a(s) - \delta a_*(s)\big] \, ds \tag{3}$$

where D denotes the Cartesian distance, W a function of two points s_0 and s (which also depends upon the entire geometry of the body and the crack) linked to the weight function of the crack.

A similar formula for the amount $\delta W(s_1; s_2)$ can be stated :

$$\delta W(s_1, s_2) = \frac{D^2(s_1, s_2)}{2\pi} PV \int_{\mathcal{F}} \frac{W(s, s_1) W(s, s_2)}{D^2(s, s_1) D^2(s, s_2)} \big[\delta a(s) - \delta a_{**}(s)\big] \, ds \tag{4}$$

Formulae (3) and (4) are legitimate for special normal advances $\delta a_*(s)$ and $\delta a_{**}(s)$ that preserve the shape of the front and such that $\delta a_*(s_0) = \delta a(s_0)$, $\delta a_{**}(s_1) = \delta a(s_1)$ and $\delta a_{**}(s_2) = \delta a(s_2)$ so as to ensure the existence of the integrals as Cauchy principal value (PV). Here, these advances are built from a combination of translations, rotation and scaling (see [1] for example).

Formulae (1) or (2), (3) and (4) lead now to an iterative scheme that can be used to deal with many kinds of propagation problems : the following part study one of these.

APPLICATION TO THE PERTURBED TUNNEL-CRACK

Leblond [4] has shown that a sinusoidal slightly perturbed tunnel-crack front tends to recover the straight configuration if the wavelength is smaller than a critical value λ_c and tends to develop if it is bigger than λ_c.

The idea here is to extand these results to propagation path using the procedure depicted above. For numerical reasons, it was difficult to study sinusoidal perturbations. Instead
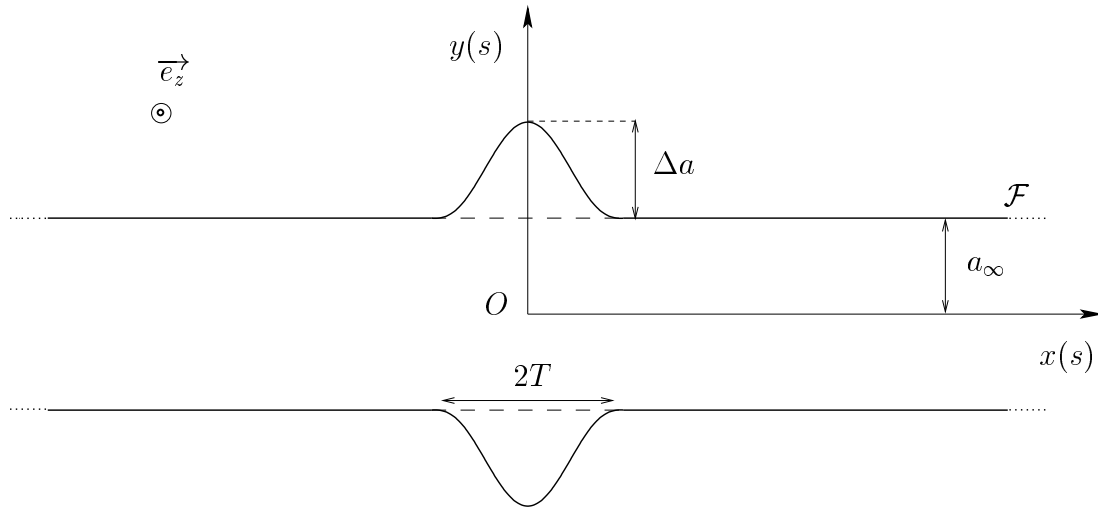
Figure 3: Perturbed tunnel-crack.

the propagation of perturbations depicted in figure 3 has been studied in both fatigue and brittle fracture for several values of Δa and T.

Adaptation Of The Method To Infinite Crack Fronts

The crack fronts are truncated: only the perturbation and a piece of straight front are meshed. Integrals along the meshed part of the front are evaluted by classical numerical linear interpolation (see Lazarus [2]). Integrals along the not meshed part are almost unchanged by the perturbation provided that the meshed straight part is sufficiently large toward perturbation size. Hence these integrals are evaluated by comparison with the known values of these integrals in the lack of perturbation.

Fatigue Propagation

Two kinds of propagation paths can be distinguished, depending on the initial width of the perturbation. For the narrow one, the perturbation decays in time, so that the front tends to get back, during its propagation, to the initial straight configuration (the "*Stable case*" figure 4), whereas for the large ones, an increase of the perturbation can be observed (the "*Unstable case*" figure 5). This obviously agrees with results of Leblond [4] described before.

Non-linear effects due to the finite size Δa of the perturbation are of two natures :

- a geometrical one : for a same perturbation width T, the bigger the amplitude Δa is, the more the top of the perturbation is shielded, that is the less the SIF is amplified. This suggests that a big perturbation shall advanced lower in comparison with the little one.

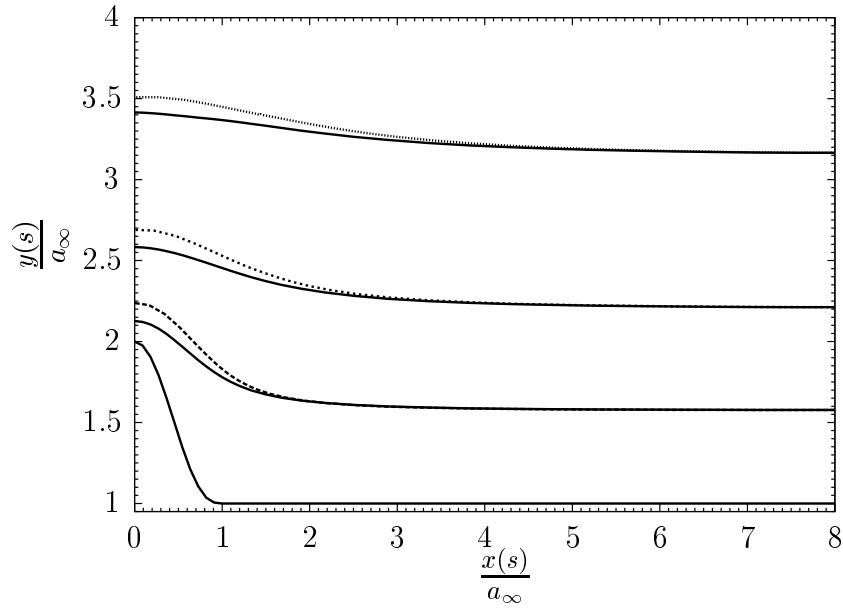- linked to the advance law : Paris' law is convex versus the SIF hence roughly

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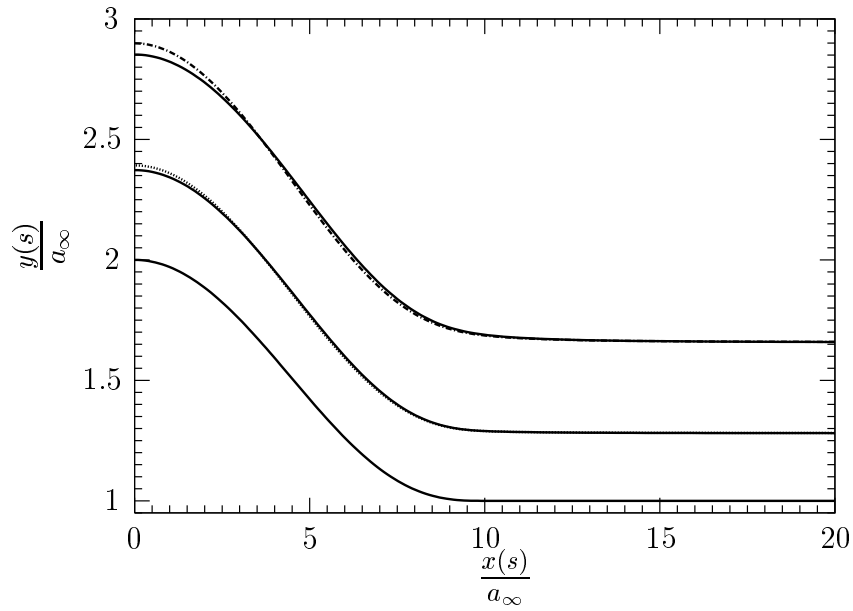Figure 4: Successive positions of the crack front in the stable case ($T/a_\infty = 1$)

Figure 5: Successive positions of the crack front in the unstable case ($T/a_\infty = 10$)

speaking, its linearized form is lower than the non-linear one. This implies that the advance rate is slower in the linear case, that is for a small perturbation than in the non-linear one for a large perturbation.

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Hence the two mechanisms are antagonist so could it be of interest to compare their relative influence.

In pratice, two specific situations are investigated : for a large amplitude $\Delta a/a_\infty = 1$ is taken, whereas for a small one $\Delta a/a_\infty = 10^{-2}$. Moreover, we chose the value 1 (*resp.* 10) of the T/a_∞ ratio to illustrate the stable (*resp.* unstable) case.

On figures 4 and 5, the front positions of the large (*resp.* small) perturbations are depicted in dashed (*resp.* full) lines. The amplitude of the small one is scaled by $1/10^{-2} = 100$ in order to make the two cases comparable. One can note that the large perturbation front is always above the tiny one. This means that the non-linear effects of the advance law are more important than the geometrical ones.
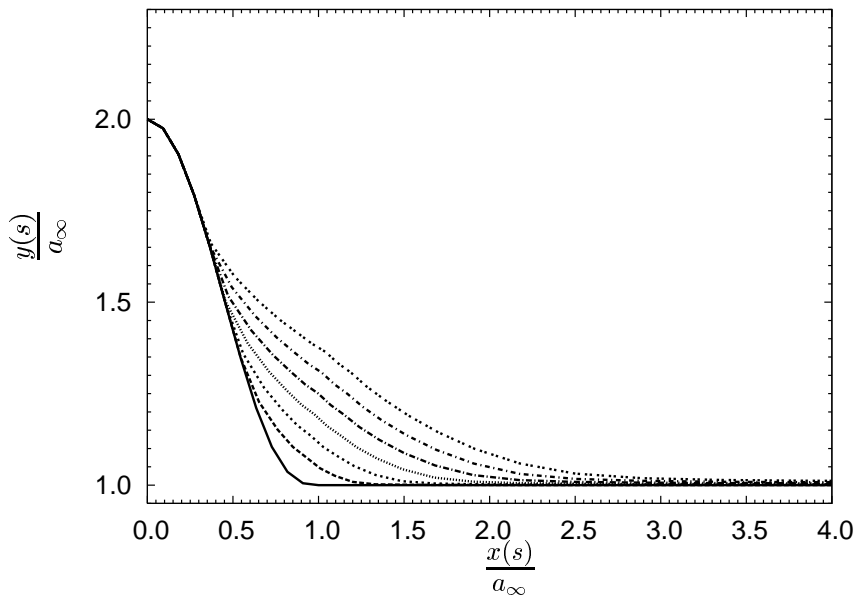
Brittle Fracture Propagation

Figure 6: "Brittle fracture" ($n = 30$)

Figure 6 shows the propagation path of a small width perturbation, the exponent in the Paris' law (1) being taken equal to 30. One can note that the SIF reaches a maximum between the top and the bottom of the perturbation so that the crack front tends to widespread. Unfortunately, due to numerical difficulties linked to the finite size of the meshing, one cannot go further in the calculations. It is probable that after a certain time, the SIF reaches its maximum at the top so that the unstable case is likely to appear.

Numerical simulations of large perturbations have been done, but rapidly the top of the perturbation becomes too sharp to allow any further calculations.

CONCLUSION

This numerical approach, besides the only requirement of 1D meshing of the crack front, gives us the opportunity to state a unique formulation describing both fatigue and brittle fracture propagation. Moreover, propagation upon large distances can be simulated. This allowed us to investigate some theoretical results about stability in the propagation paths of a tunnel-crack with perturbed front.

In fatigue, two domains can be distinguished depending on the perturbation width : for narrow ones, the perturbation vanishes during propagation (stable case); for large one, the perturbation increases (unstable case). It has also be shown that the non-linear effects increases the crack advance rate.

In brittle fracture, some further investigations are necessary to obtain satisfactory results.

REFERENCES

1. Bower, A. Ortiz, M. (1990) *Journal of The Mechanics and Physics of Solids* **38**(4), 443-480.

2. Lazarus, V. (2002) submitted to *International Journal of Fracture*.

3. Rice, J.R. (1989). In : *Fracture Mechanisms: Perspectives and Directions (twentieth symposium)*, pp.29-57, Wei, R.P., Gangloff, R.P. (Eds.), Philadelphia, USA.

4. Leblond, J.B. Mouchrif, S.E., Perrin, G. (1996) *European Journal of Solids and Structures* **18**(1), 135-145.